ON Semiconductor®

Small-Signal Modeling at Work with Power Converters

Christophe Basso

IEEE Senior Member

Course Agenda

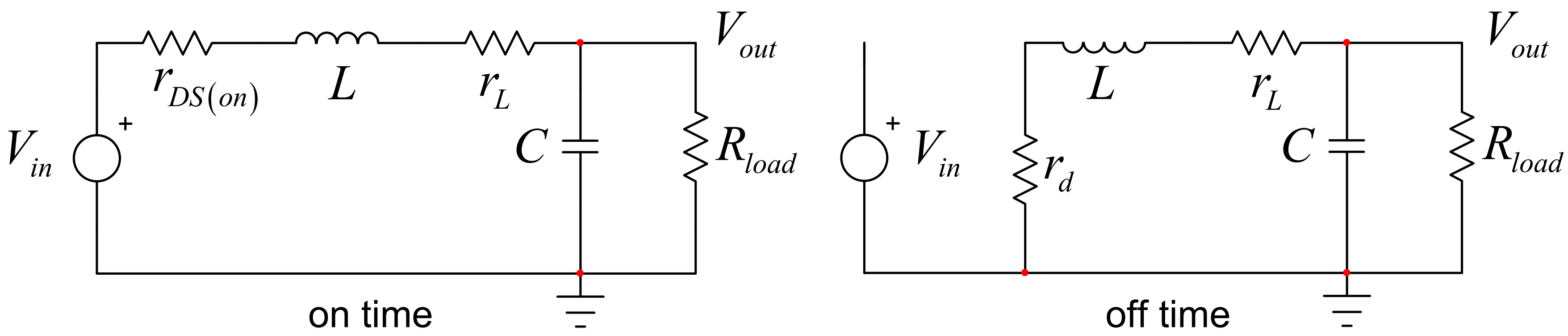
- ❑ Introducing the PWM Switch Model
- ❑ CCM, DCM and BCM in Voltage Mode
- ❑ Pulse Width Modulator Gain
- ❑ The PWM Switch Model in Current Mode
- ❑ PWM Switch at Work in a Buck Converter
- ❑ A Simplified Approach to Modeling a DCM Boost
- ❑ Transfer Function of a BCM Boost in Current Mode
- ❑ Small-Signal Model of The Active Clamp Forward

Course Agenda

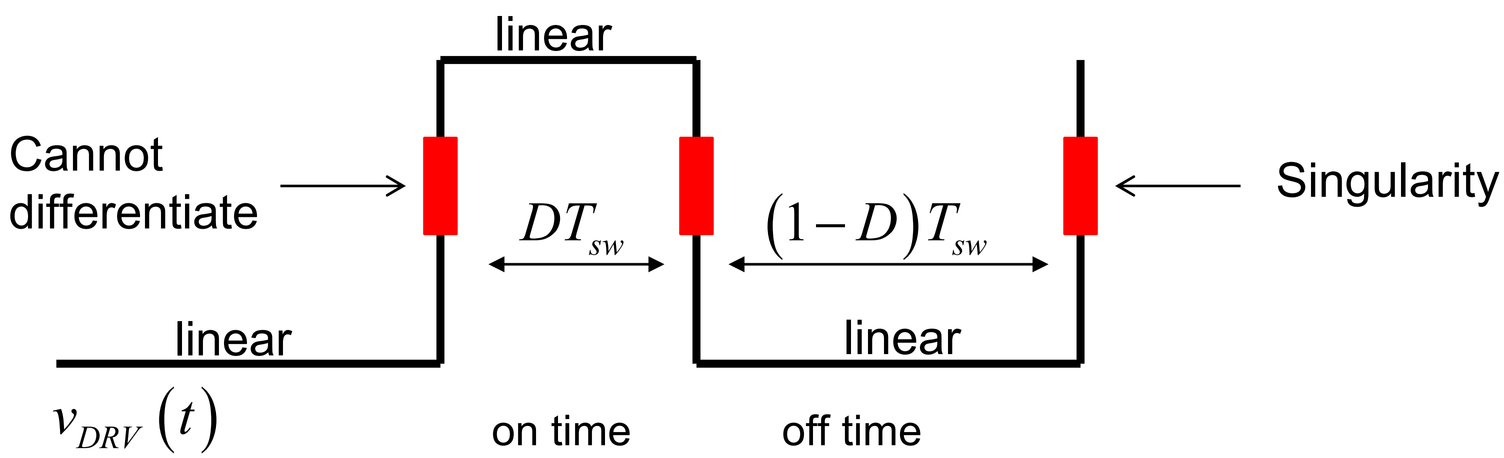
- ❑ **Introducing the PWM Switch Model**
- ❑ CCM, DCM and BCM in Voltage Mode
- ❑ Pulse Width Modulator Gain
- ❑ The PWM Switch Model in Current Mode
- ❑ PWM Switch at Work in a Buck Converter
- ❑ A Simplified Approach to Modeling a DCM Boost
- ❑ Transfer Function of a BCM Boost in Current Mode
- ❑ Small-Signal Model of The Active Clamp Forward

Manipulating Linear Networks

- A switching converter is made of linear elements!



- The non-linearity or discontinuity is coming from transitions



State Space Averaging (SSA)

- Despite linear networks, equation is discontinuous in time
- Introduced in 76, SSA weights on and off expressions

weighted during $(1\text{-}D)T_{sw}$

$$\dot{x} = \left[\mathbf{A}_1 D + \mathbf{A}_2\left(1 - D\right)\right]x\left(t\right) + \left[\mathbf{B}_1 D + \mathbf{B}_2\left(1 - D\right)\right]u\left(t\right)$$

weighted during DT_{sw}

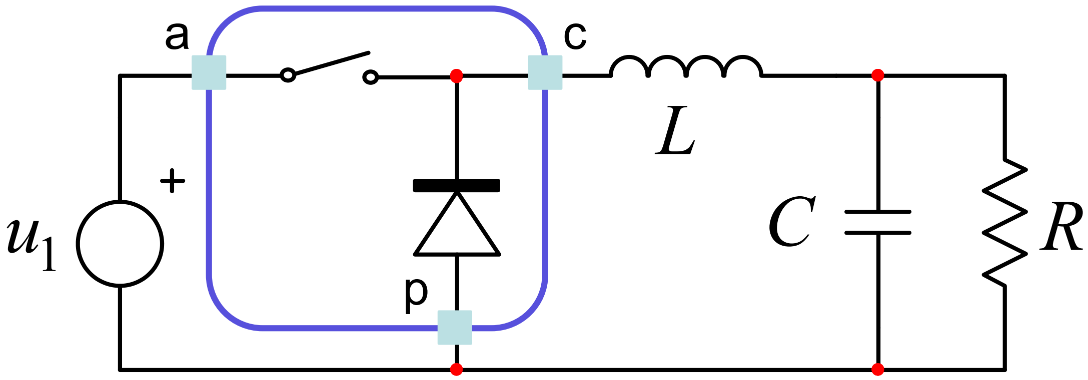
A is the state coefficient matrix
B is the source coefficient matrix

- This equation is now continuous in time: singularity is gone
- However, it became a non-linear equation
- ➢ You need to linearize it by perturbation (or differentiation)
- ➢ If you add a new element, you have to restart from scratch!

S.Ćuk, "Modeling, Analysis and Design of Switching Converters", Ph. D. Thesis, Caltech November 1976

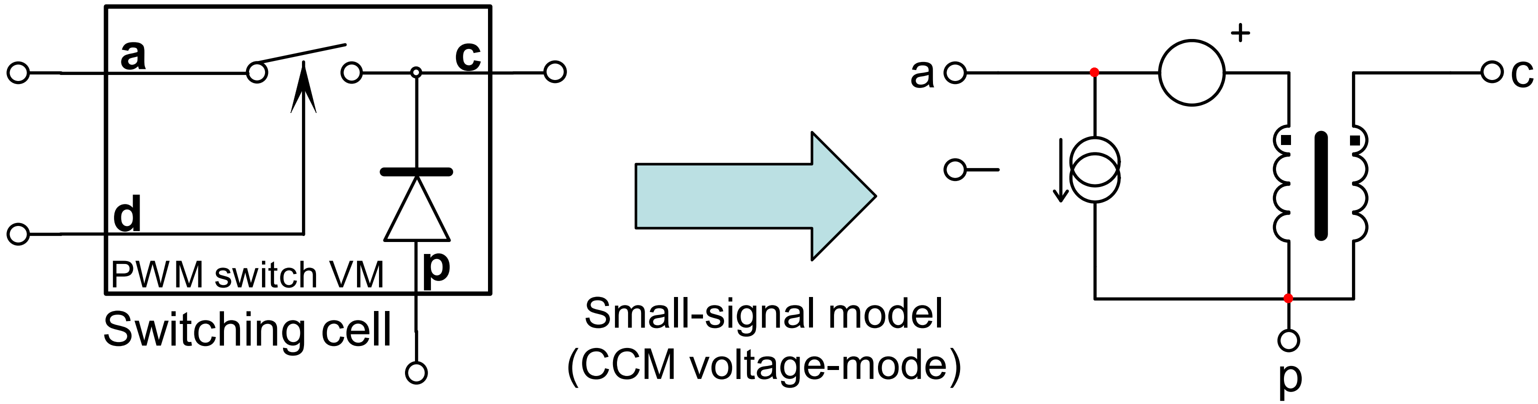
The PWM Switch Model in Voltage Mode

- We know that non-linearity is brought by the switching cell



a: active
c: common
p: passive

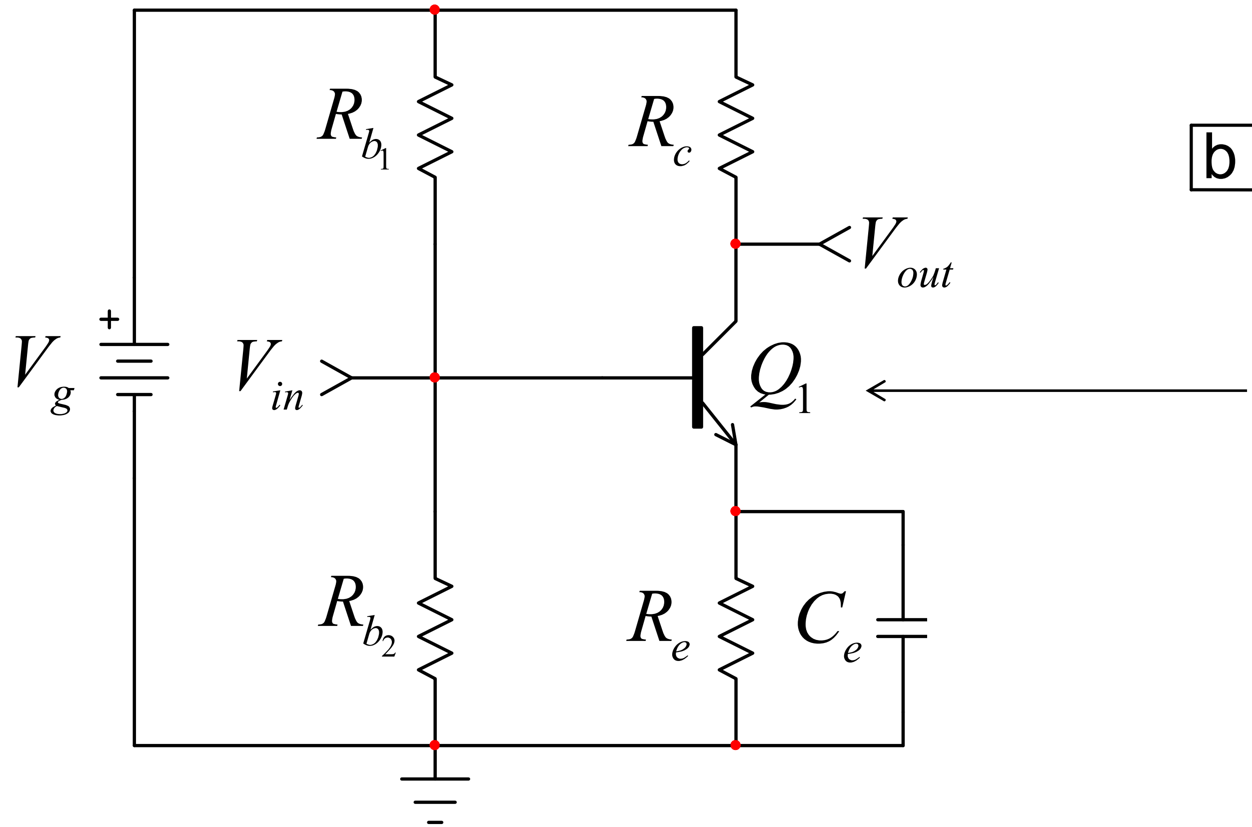
- Why don't we linearize the cell alone?

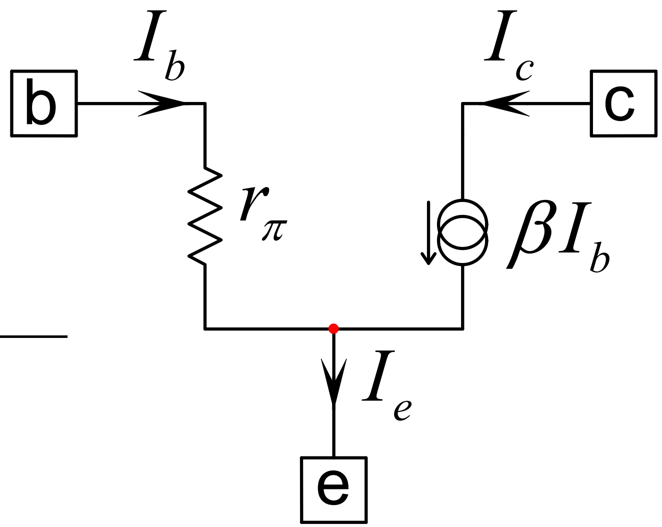


V. Vorpérian, "Simplified Analysis of PWM Converters using Model of PWM Switch, parts I and II" IEEE Transactions on Aerospace and Electronic Systems, Vol. 26, NO. 3, 1990

The Bipolar Small-Signal Model

- A bipolar transistor is a highly non-linear system
- Replace it by its small-signal model to get the response

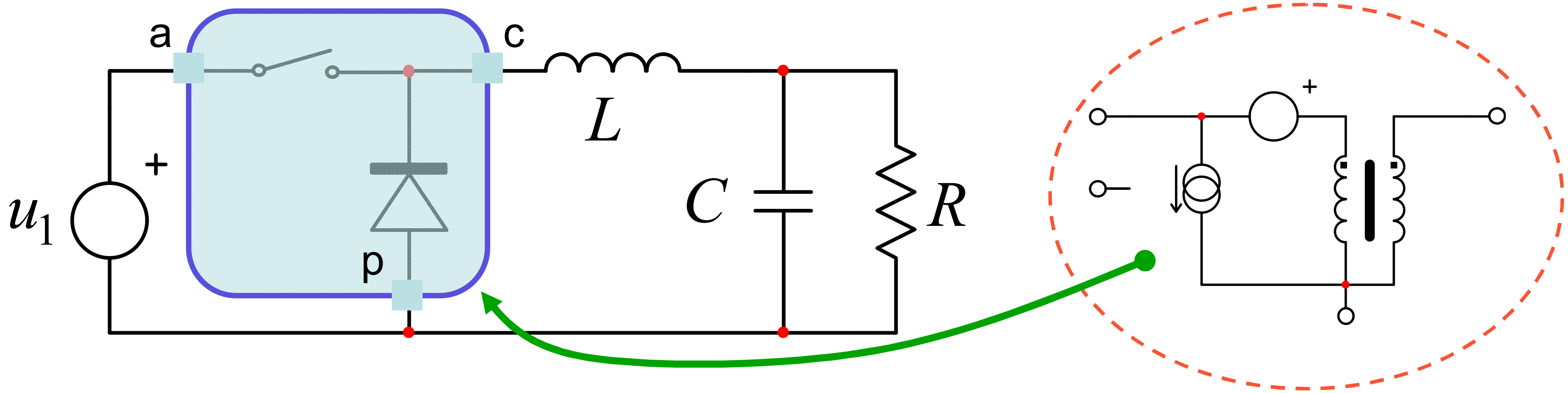




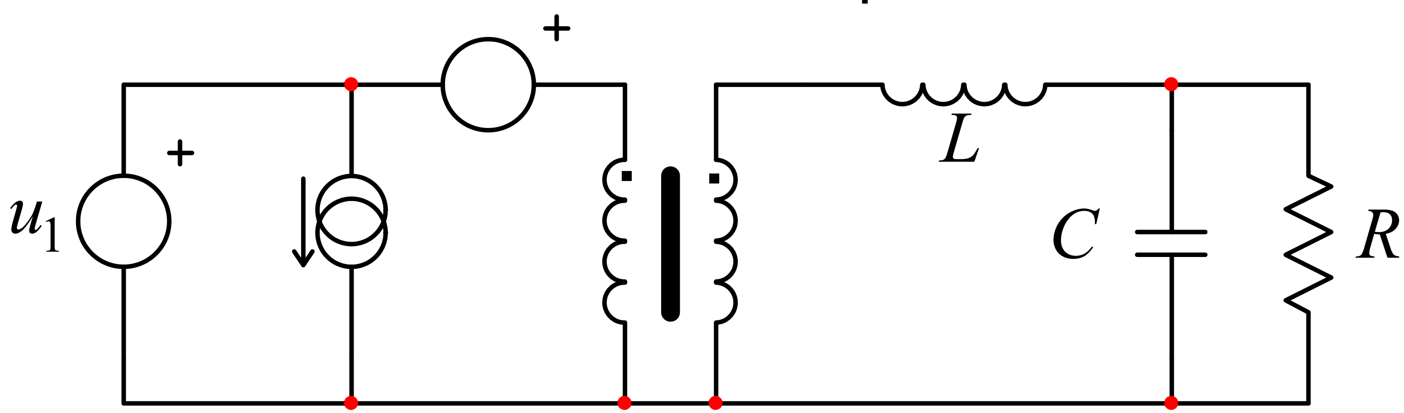
Ebers-Moll model

Replace the Switches by the Model

- ❑ Like in the bipolar circuit, replace the switching cell…

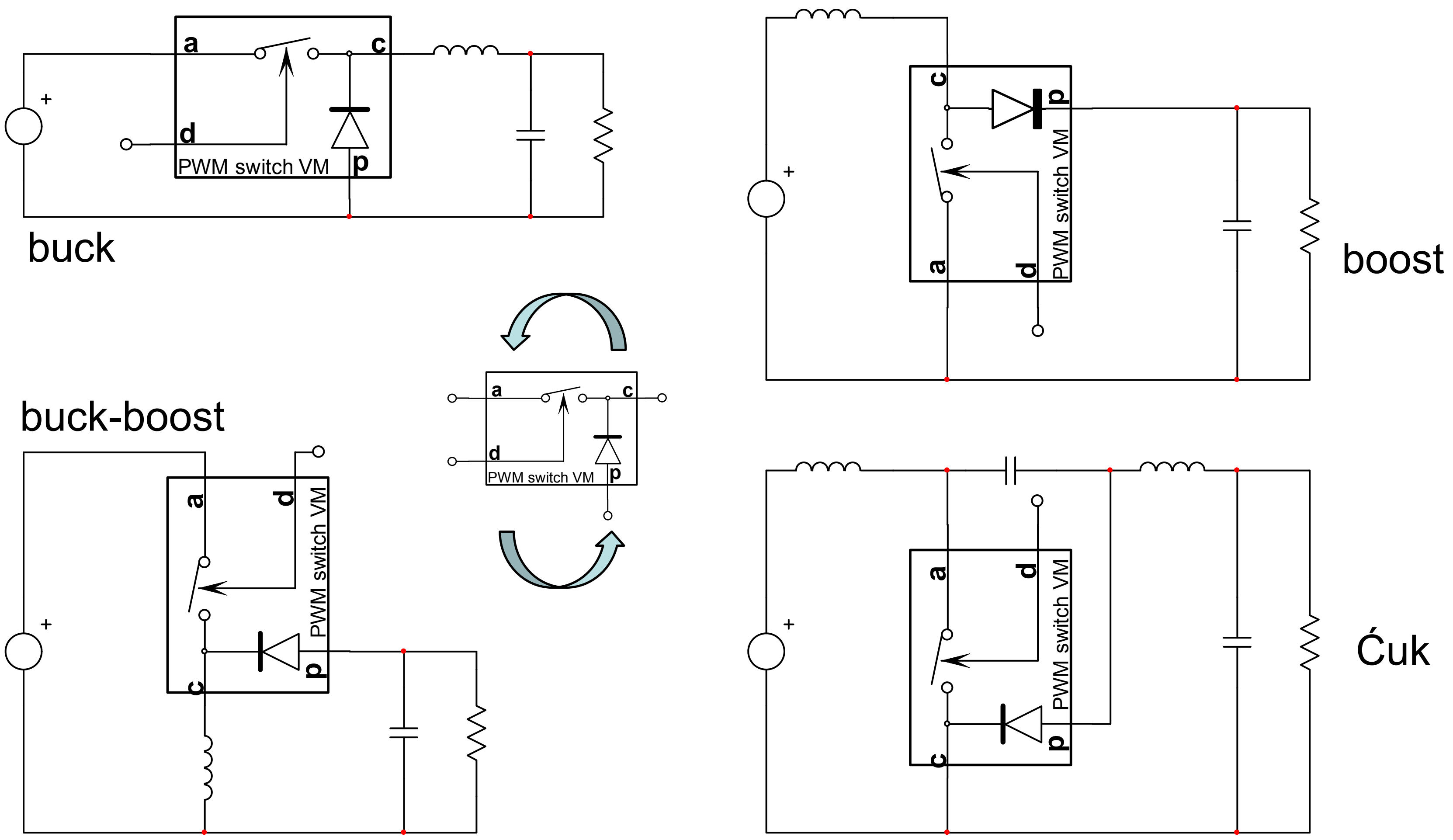


- ❑ …and solve a set of linear equations!



An Invariant Model

- The switching cell made of two switches is everywhere!

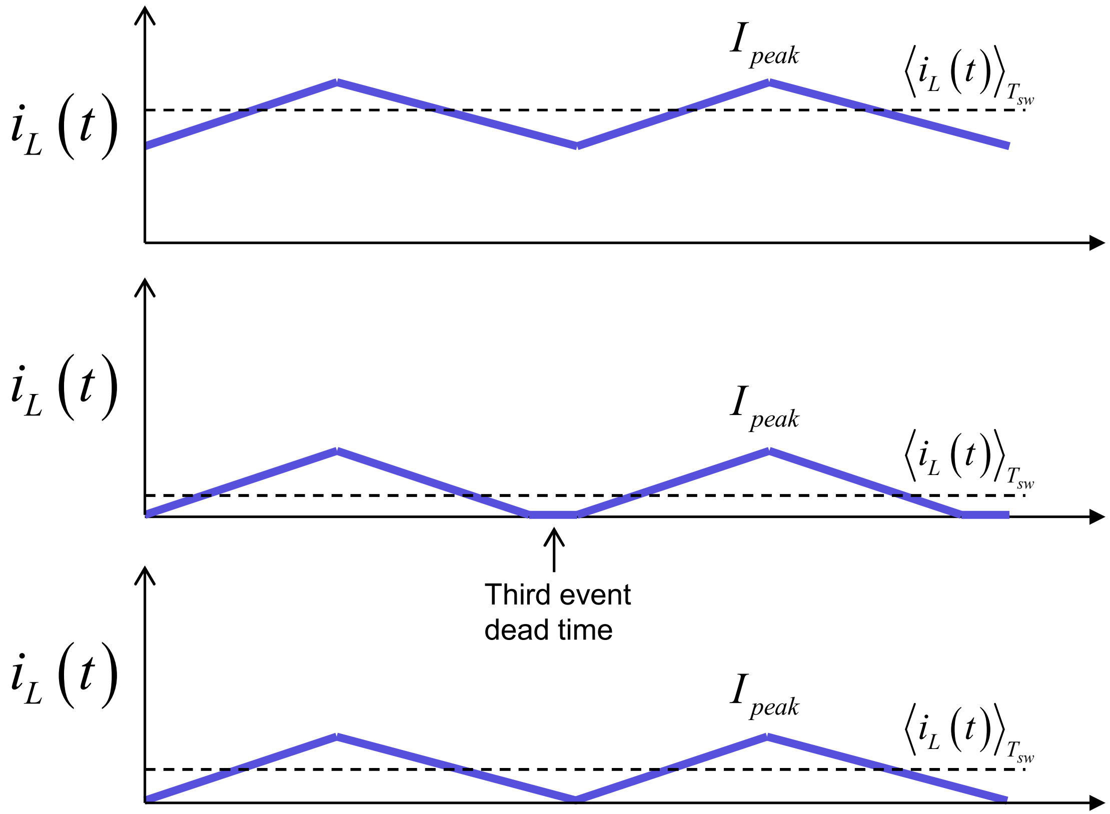


Course Agenda

- ❑ Introducing the PWM Switch Model
- ❑ **CCM, DCM and BCM in Voltage Mode**
- ❑ Pulse Width Modulator Gain
- ❑ The PWM Switch Model in Current Mode
- ❑ PWM Switch at Work in a Buck Converter
- ❑ A Simplified Approach to Modeling a DCM Boost
- ❑ Transfer Function of a BCM Boost in Current Mode
- ❑ Small-Signal Model of The Active Clamp Forward

CCM, DCM and BCM Operations

- Three types of conduction modes exist



Continuous **C**onduction **M**ode $\langle i_L(t)\rangle_{T_{sw}} > \frac{I_{peak}}{2}$

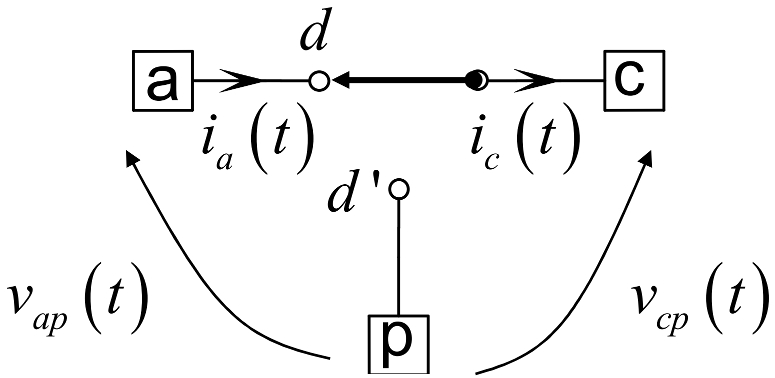
Discontinuous **C**onduction **M**ode $\langle i_L(t)\rangle_{T_{sw}} < \frac{I_{peak}}{2}$

Boundary **C**onduction **M**ode $\langle i_L(t)\rangle_{T_{sw}} = \frac{I_{peak}}{2}$

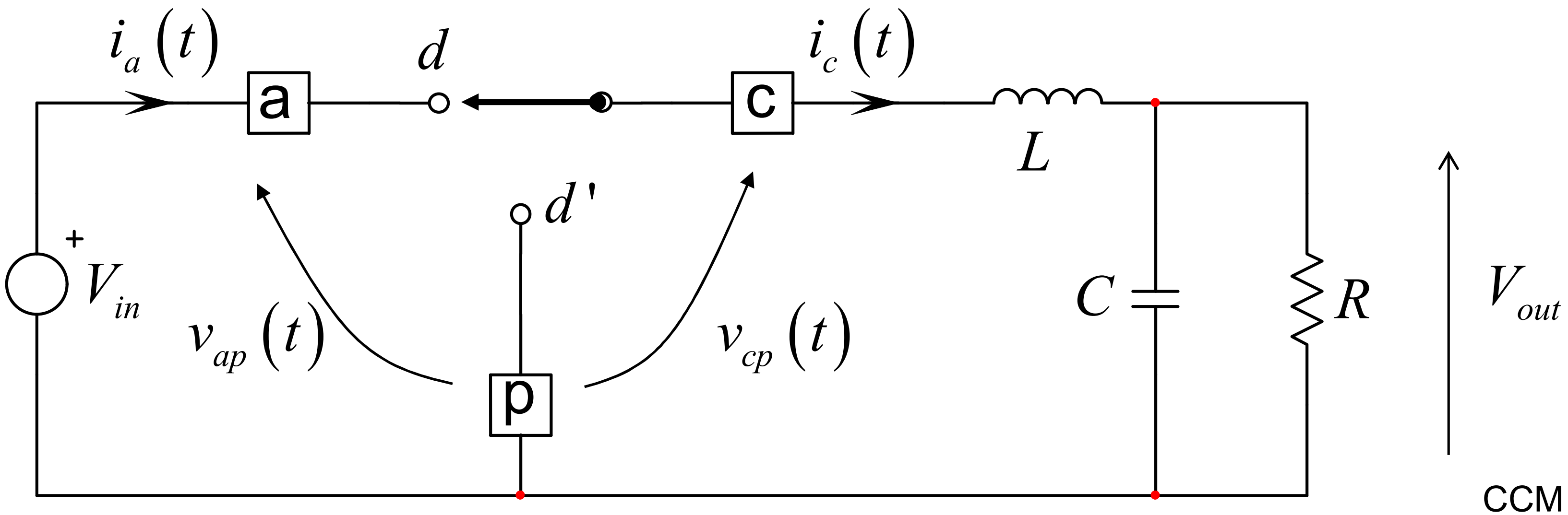
- Each mode has its own small-signal characteristics
  - A model is needed for these three modes!

CCM Common Passive Configuration

- The PWM switch is a single-pole double-throw model



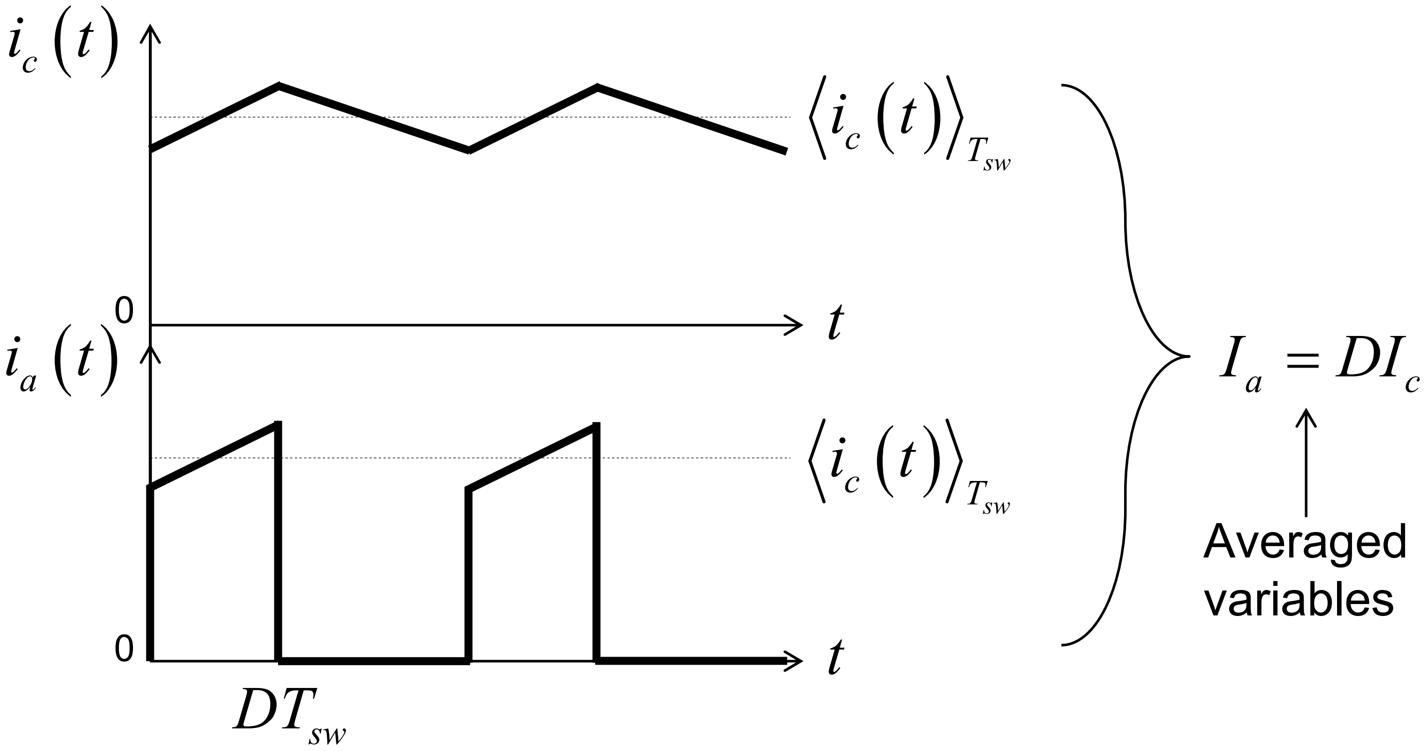
- Install it in a buck and draw its terminals waveforms



CCM VM

The Common Passive Configuration

- Average the current waveforms across the PWM switch

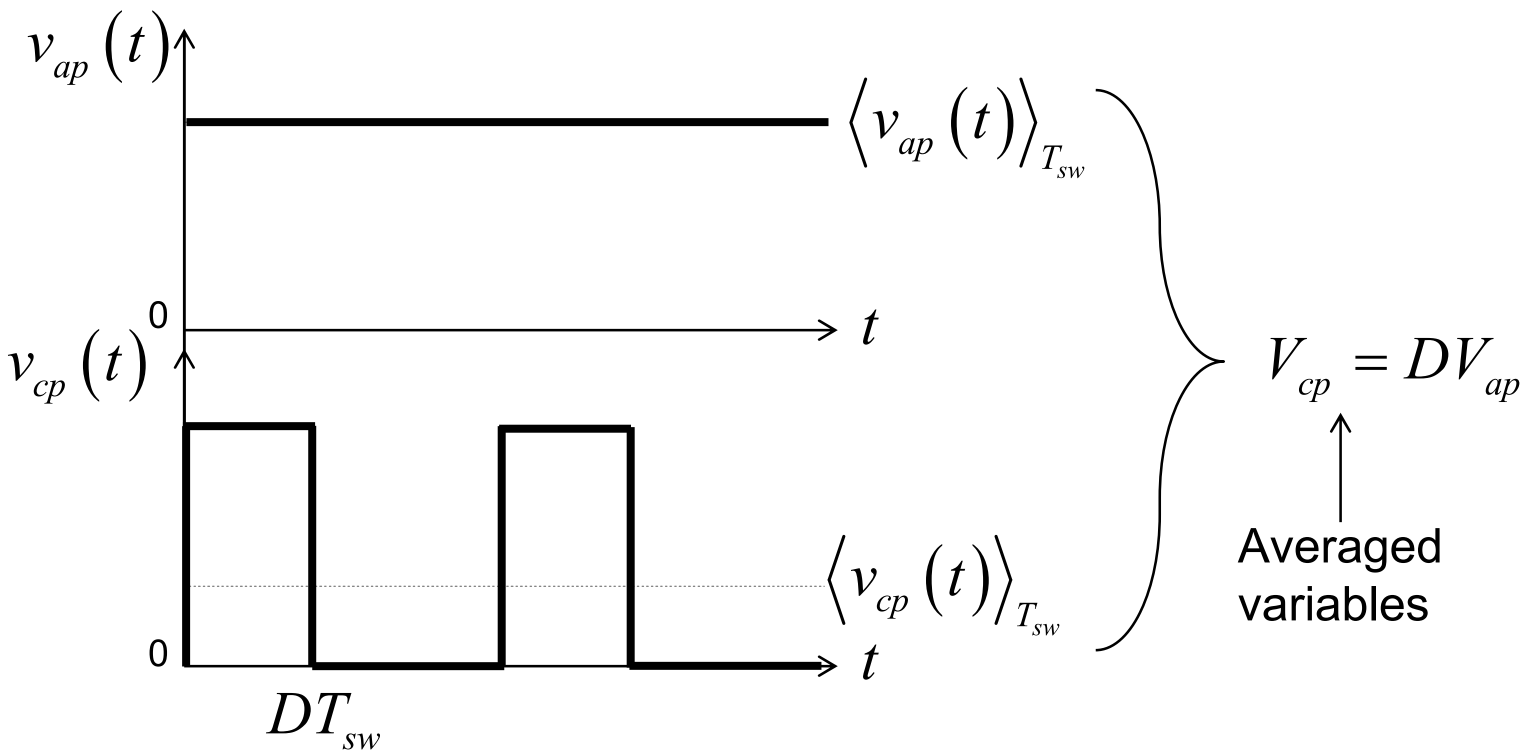


$$\langle i_a(t)\rangle_{T_{sw}} = I_a = \frac{1}{T_{sw}}\int_0^{DT_{sw}} i_a(t)\,dt = D\langle i_c(t)\rangle_{T_{sw}} = DI_c$$

CCM VM

The Common Passive Configuration

❑ Average the voltage waveforms across the PWM switch

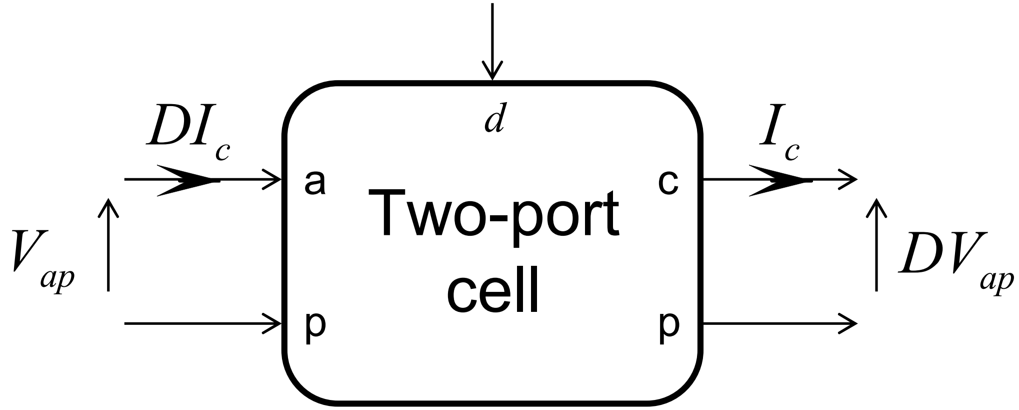


$$\langle v_{cp}(t) \rangle_{T_{sw}} = V_{cp} = \frac{1}{T_{sw}} \int_0^{DT_{sw}} v_{cp}(t)\,dt = D \langle v_{ap}(t) \rangle_{T_{sw}} = DV_{ap}$$

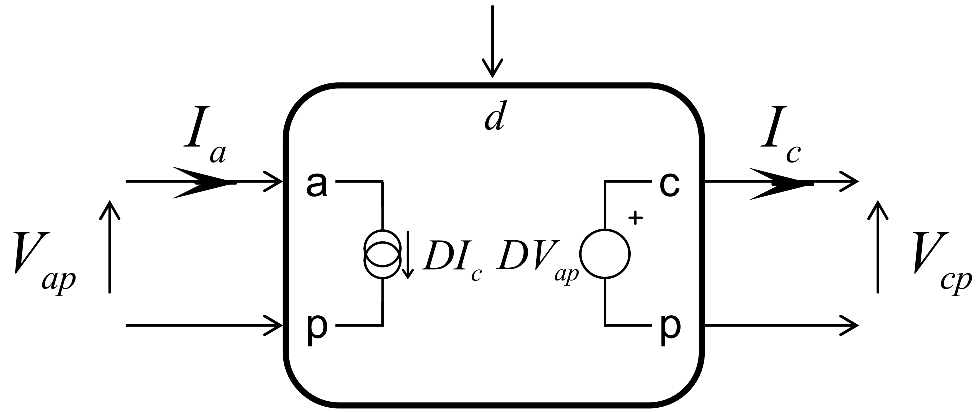
CCM VM

A Two-Port Representation

- We have a link between input and output variables



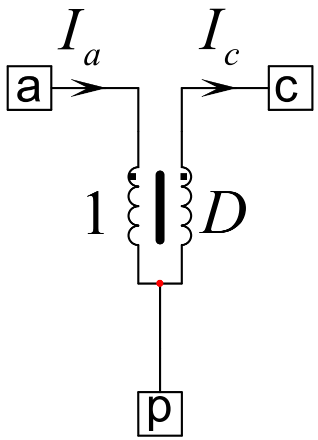
- It can further be illustrated with current and voltage sources



CCM VM

A Transformer Representation

- The PWM switch large-signal model is a dc "transformer"!

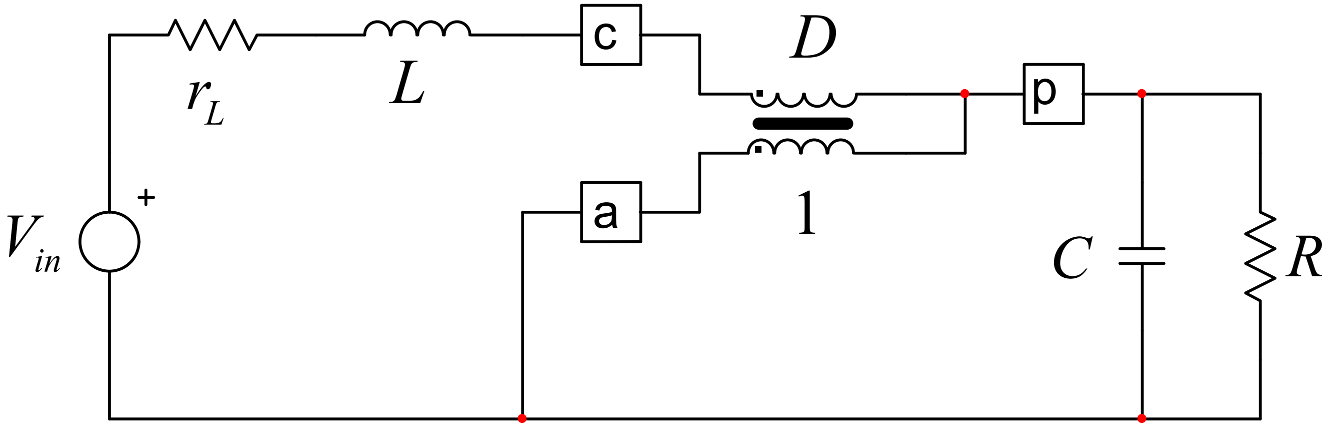


$$I_a = DI_c \qquad I_c = \frac{I_a}{D}$$

$$V_{ap} = \frac{V_{cp}}{D} \qquad V_{cp} = DV_{ap}$$

dc equations!

- It can be plugged into any 2-switch CCM converter

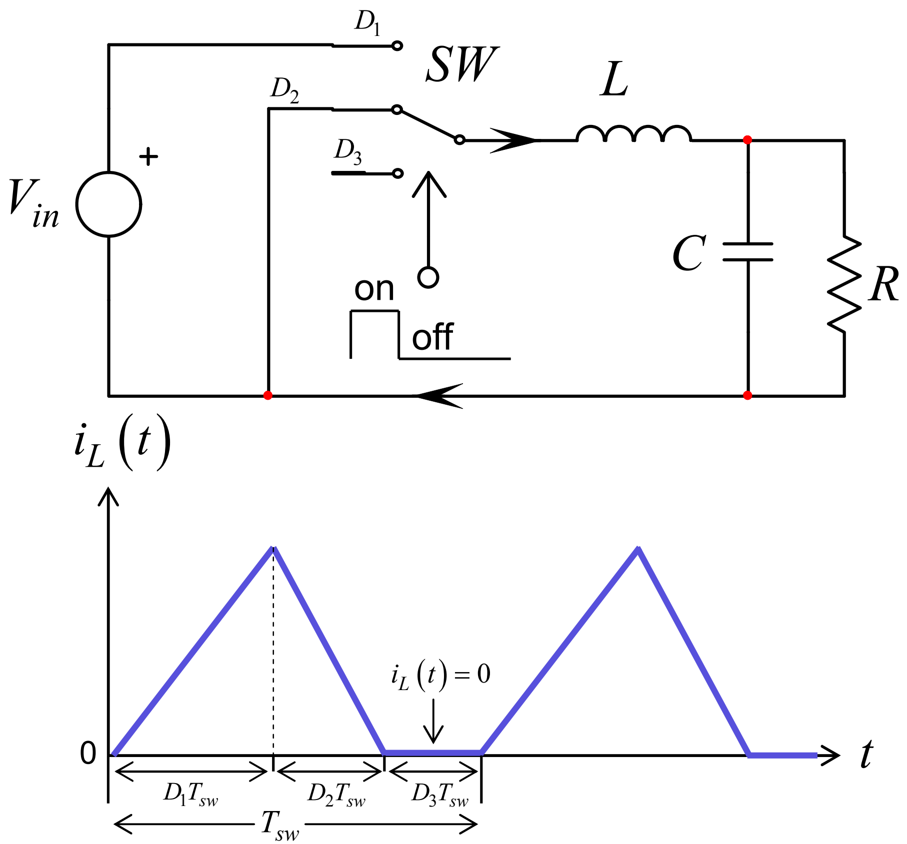


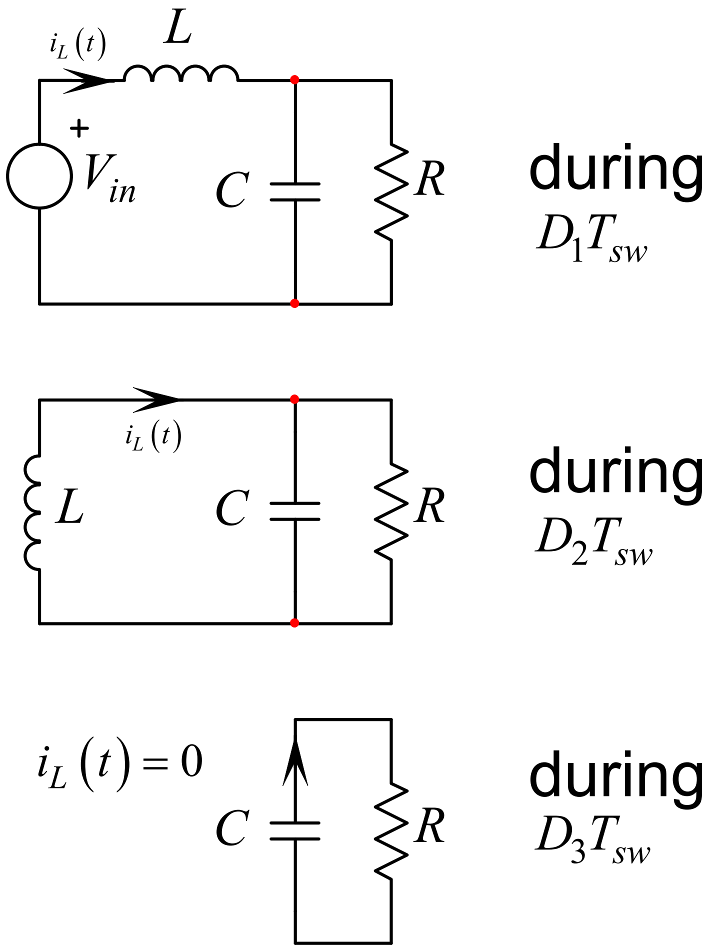
Dc bias point
Ac response

CCM VM

The Discontinuous Case

- In DCM, a third timing event exists when $i_L(t) = 0$

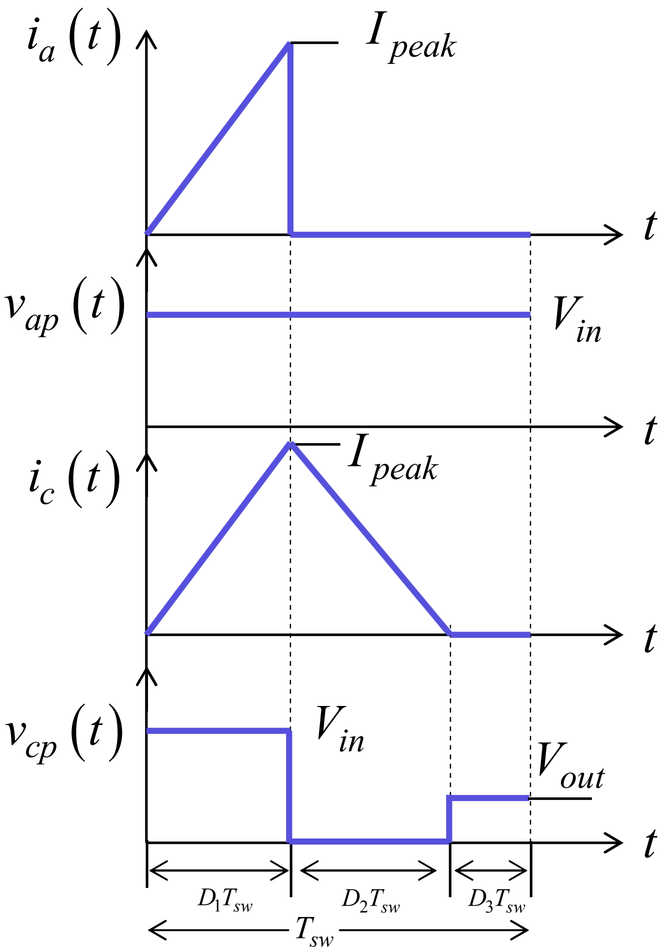


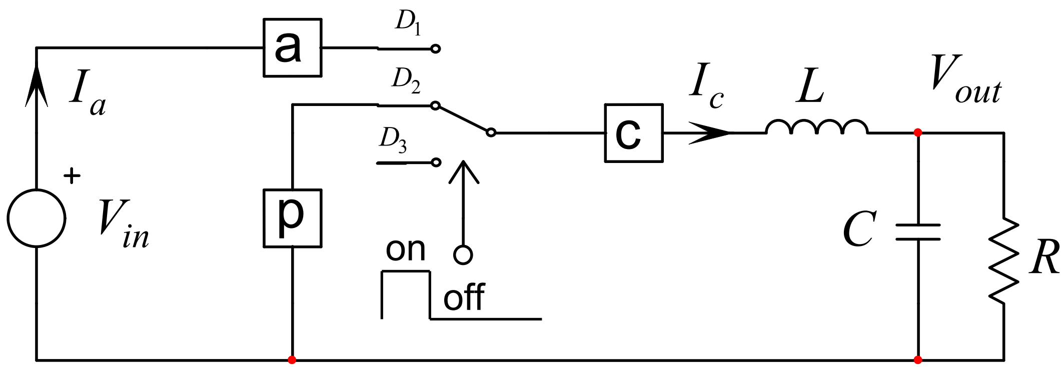


DCM VM

The Same Configuration as in CCM

- Draw the waveforms in the "common passive" configuration





- Average the waveforms:

$$I_a = \frac{I_{peak}}{2} D_1$$

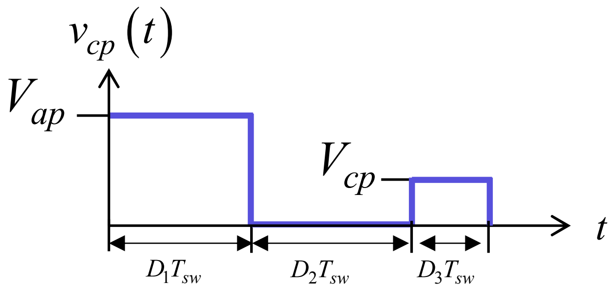
$$I_c = \frac{I_{peak}}{2} D_1 + \frac{I_{peak}}{2} D_2 = \frac{I_{peak}}{2} \left(D_1 + D_2 \right)$$

$$I_c = \frac{2 I_a}{D_1} \frac{D_1 + D_2}{2} = I_a \frac{D_1 + D_2}{D_1}$$

DCM VM

Derive V_{cp} to Unveil the New Model

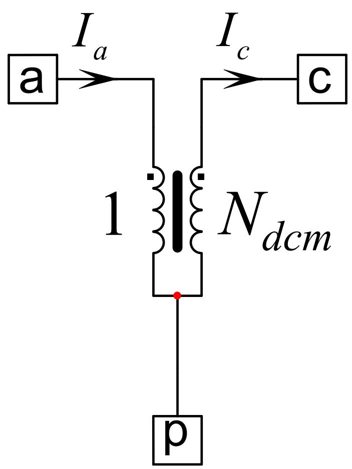
❑ The addition of the third event complicates the equations



$$V_{cp} = V_{ap}D_1 + V_{cp}D_3 \qquad D_1 + D_2 + D_3 = 1$$

$$V_{cp} = V_{ap}D_1 + V_{cp}\left(1 - D_1 - D_2\right)$$

$$V_{cp} = V_{ap}\frac{D_1}{D_1 + D_2}$$



$$I_a = N_{dcm}I_c \qquad I_c = \frac{I_a}{N_{dcm}}$$

$$V_{ap} = \frac{V_{cp}}{N_{dcm}} \qquad V_{cp} = N_{dcm}V_{ap}$$

Control input

$$N_{dcm} = \frac{D_1}{D_1 + D_2} \rightarrow f\left(D_1\right)$$

DCM VM

Finally, Get the D_2 Value

- In DCM the inductor average voltage per cycle is always 0

$$\longrightarrow \quad V_{cp} = V_{out}$$

- What is the averaged inductor peak current?

$$\left.\begin{aligned} I_{peak} &= \frac{\langle v_L(t)\rangle_{D_1T_{sw}}}{L} D_1 T_{sw} \\ \langle v_L(t)\rangle_{D_1T_{sw}} &= V_{ac} \end{aligned}\right\} \quad V_{ac} = L \frac{\boxed{I_{peak}}}{D_1 T_{sw}}$$

- The peak current uses a previous expression

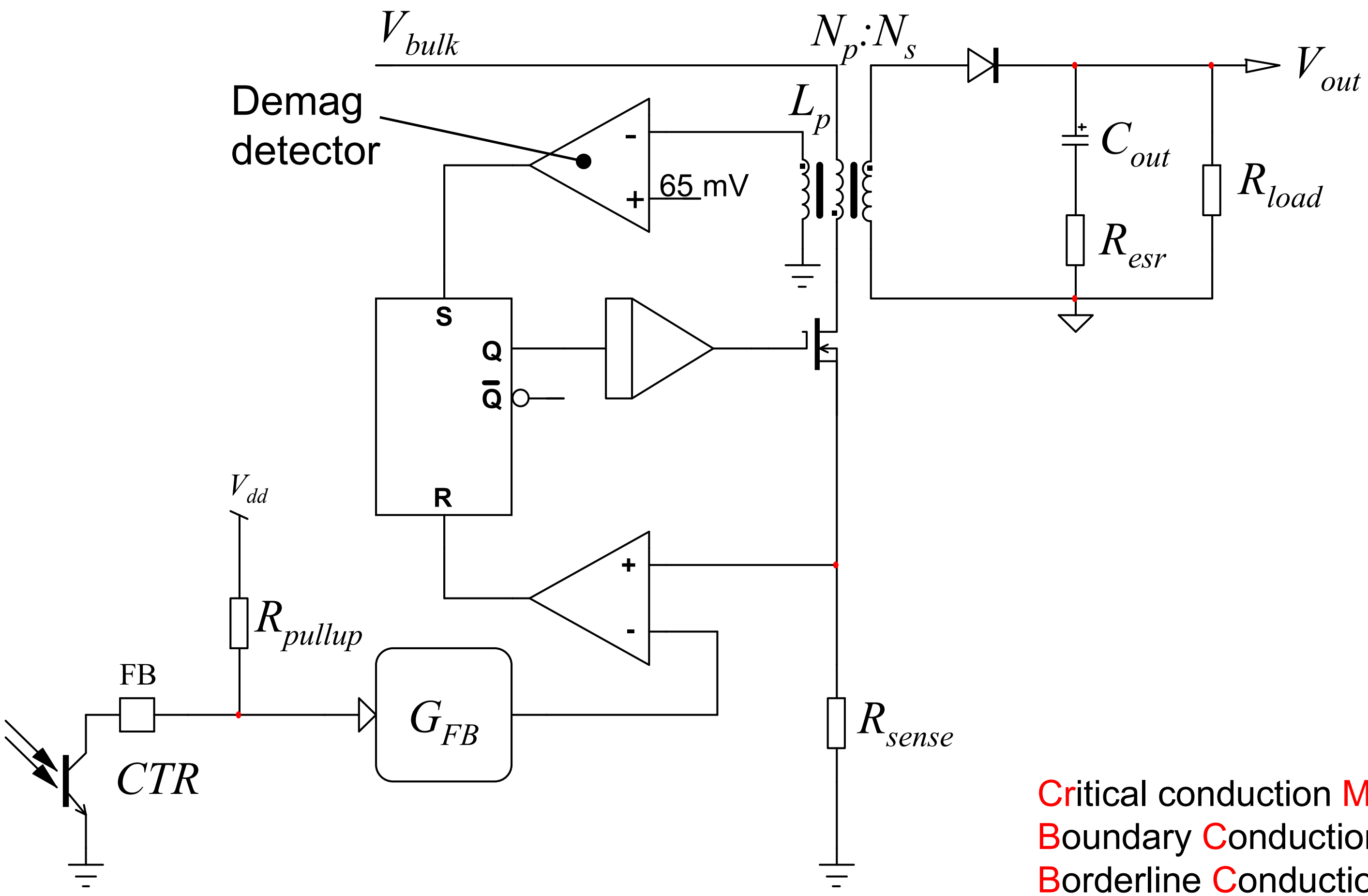
$$I_c = \frac{I_{peak}}{2}(D_1 + D_2) \longrightarrow \boxed{I_{peak}} = \frac{2I_c}{D_1 + D_2}$$

$$\Rightarrow \quad D_2 = \frac{2LF_{sw}}{D_1}\frac{I_c}{V_{ac}} - D_1$$

DCM VM

Variable Frequency Operation

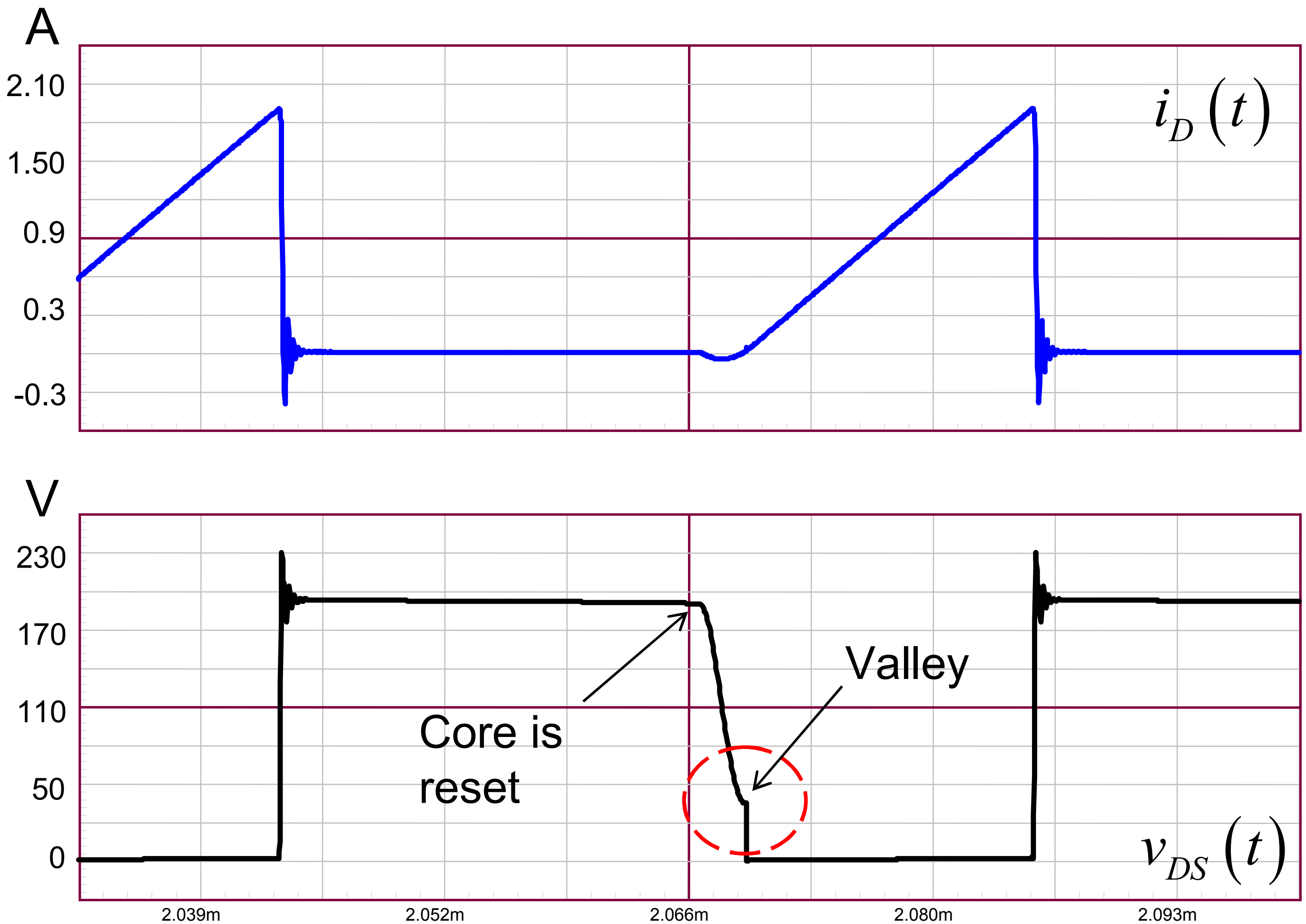
- The power supply operates in a self-oscillating mode



Critical conduction Mode (CRM)
Boundary Conduction Mode
Borderline Conduction Mode (BCM)

Benefits of Quasi-Resonance

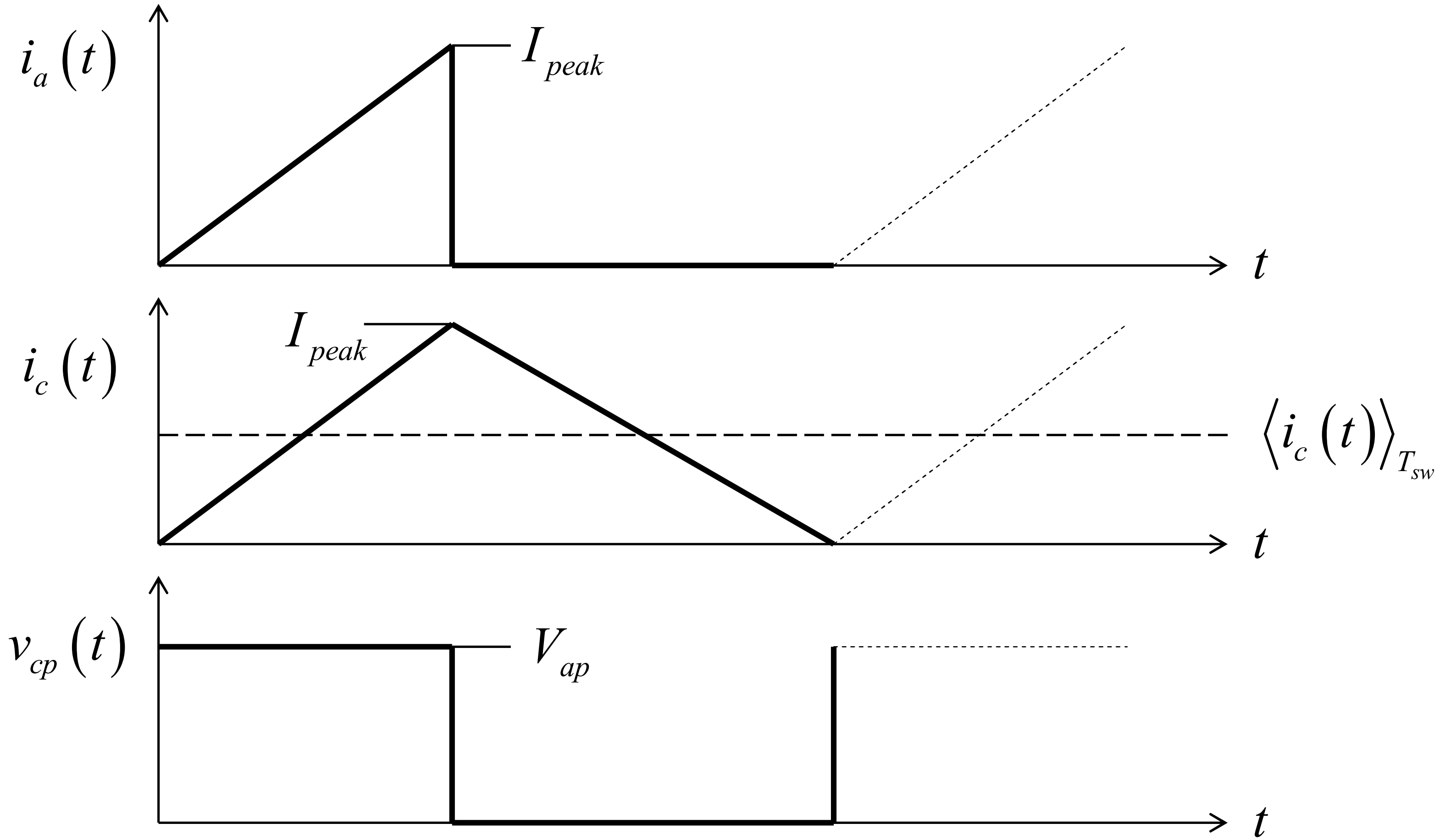
- ❑ Wait for core demagnetization and valley voltage



BCM VM

Idealized Waveforms

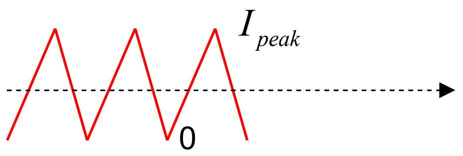
- Draw the PWM switch waveforms in a buck configuration



BCM VM

Derive Founding Equations

- Average current in terminal C is straightforward



$$\left\langle i_c(t)\right\rangle_{T_{sw}} = \frac{I_{peak}}{2}$$

- The off time depends on the voltage across the inductor

$$t_{off} = \frac{L}{V_{cp}} I_{peak} \qquad I_{peak} = 2I_c \qquad \longrightarrow \qquad t_{off} = \frac{2LI_c}{V_{cp}}$$

- Period and duty ratio come easily as t_{on} is imposed

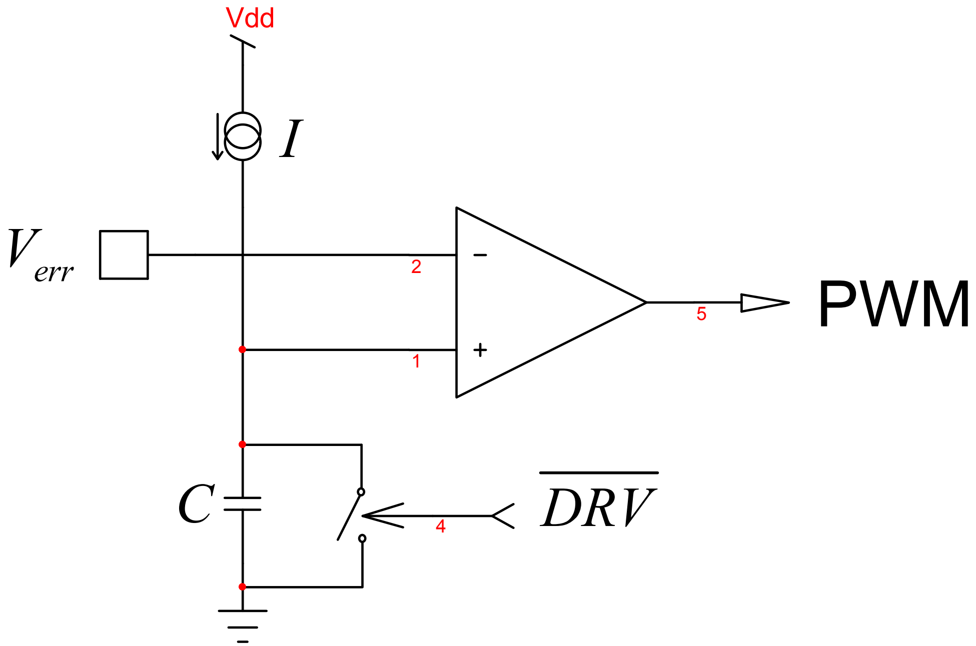
$$t_{on} + t_{off} = T_{sw} \qquad D = \frac{t_{on}}{T_{sw}}$$

➡ Need to transform the error voltage into time

BCM VM

Generating the On Time

- The on time modulator works as a PWM block



$$t_{on}\left(V_{err}\right) = \frac{V_{err} C}{I}$$

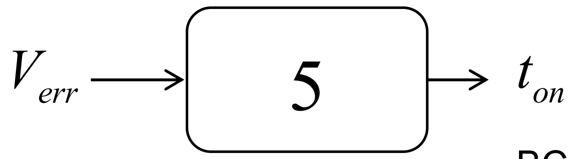
$$\downarrow$$

$$\frac{d}{dV_{err}} t_{on}\left(V_{err}\right) = \frac{C}{I}$$

- The modulator small-signal gain is a simple coefficient

$$\left.\begin{aligned} C &= 100\,\text{pF} \\ I &= 20\,\mu\text{A} \end{aligned}\right\} \frac{C}{I} = 5u$$

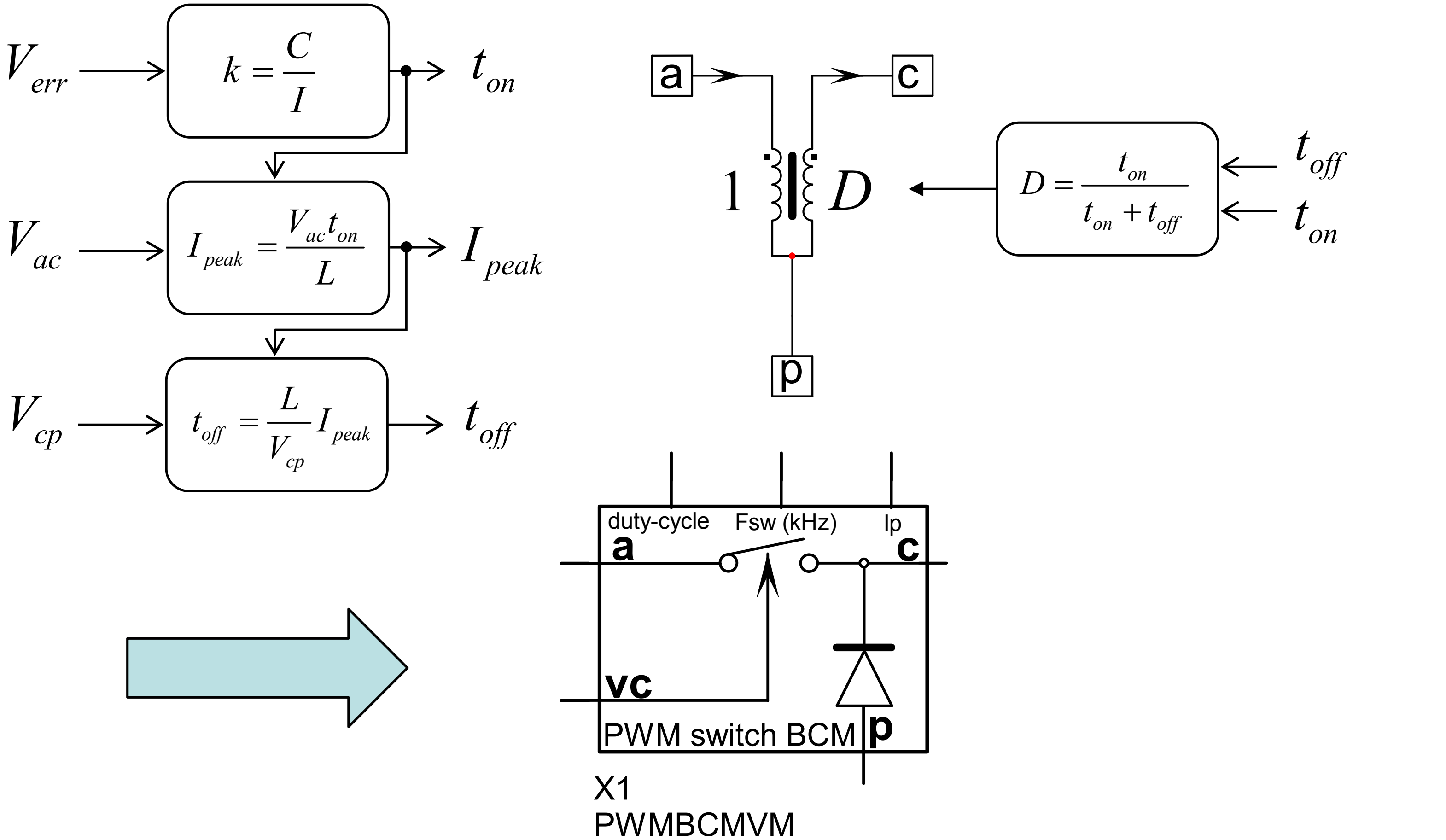
Assuming 1 V = 1 µs



BCM VM

Final PWM Switch Model in BCM

❑ Just add the on-time modulator to the VM PWM switch



C. Basso, "Switch Mode Power Supplies: SPICE Simulations and Practical Design", McGraw-Hill 2008

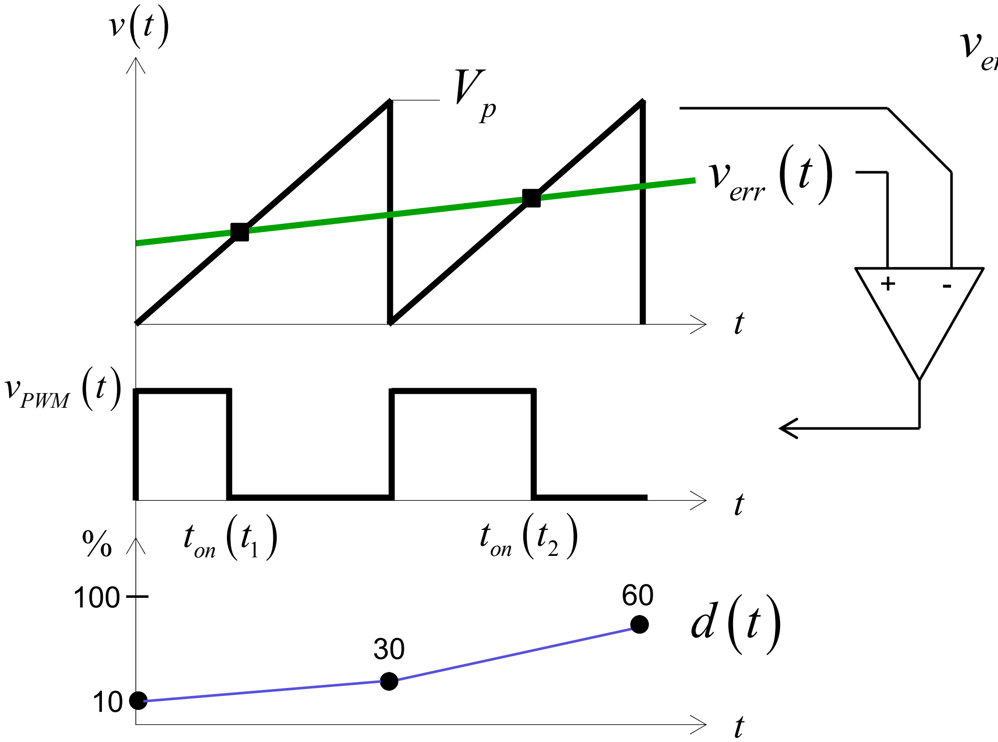
BCM VM

Course Agenda

- ❑ Introducing the PWM Switch Model
- ❑ CCM, DCM and BCM in Voltage Mode
- ❑ **Pulse Width Modulator Gain**
- ❑ The PWM Switch Model in Current Mode
- ❑ PWM Switch at Work in a Buck Converter
- ❑ A Simplified Approach to Modeling a DCM Boost
- ❑ Transfer Function of a BCM Boost in Current Mode
- ❑ Small-Signal Model of The Active Clamp Forward

What About the PWM Block?

- In voltage mode, the duty ratio depends on V_{err}



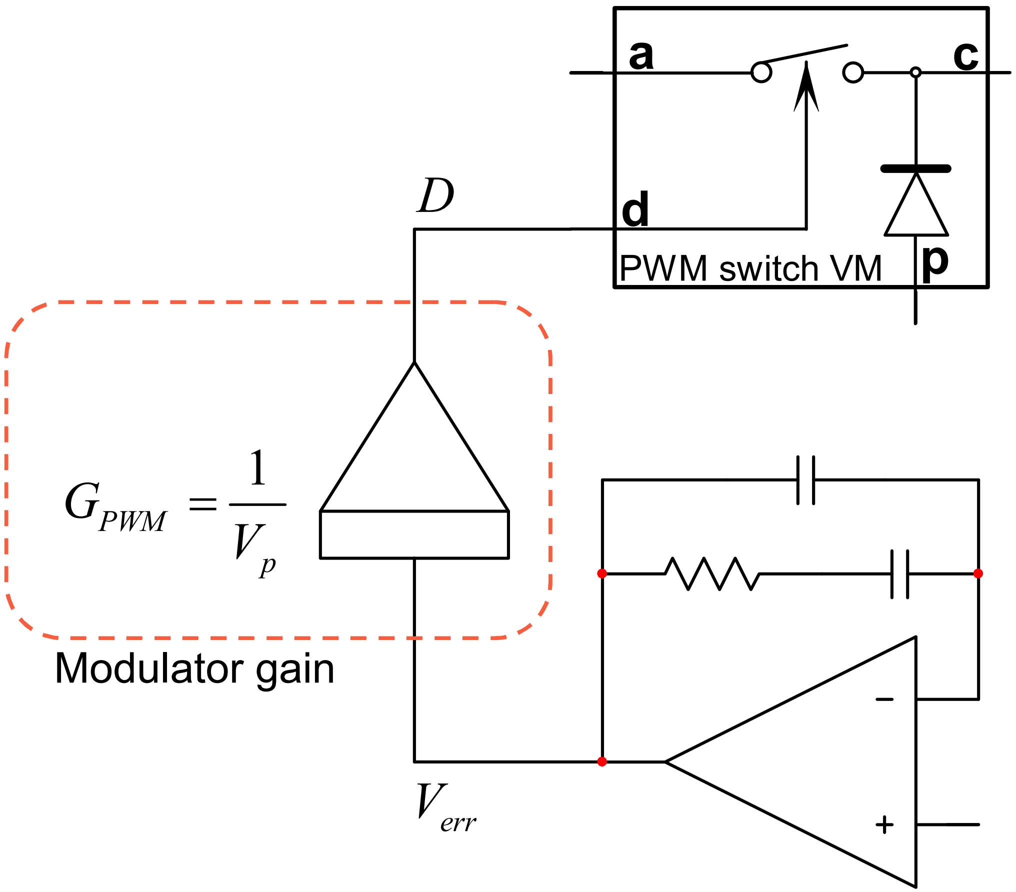
$$v_{err}(t) = V_p \frac{t_{on}(t)}{T_{sw}} \qquad D(t) = \frac{t_{on}(t)}{T_{sw}}$$

$$\frac{D(t)}{v_{err}(t)} = \frac{1}{V_p}$$

$$\frac{d}{dV_{err}} D(V_{err}) = G_{PWM} = \frac{1}{V_p}$$

Including the PWM Contribution

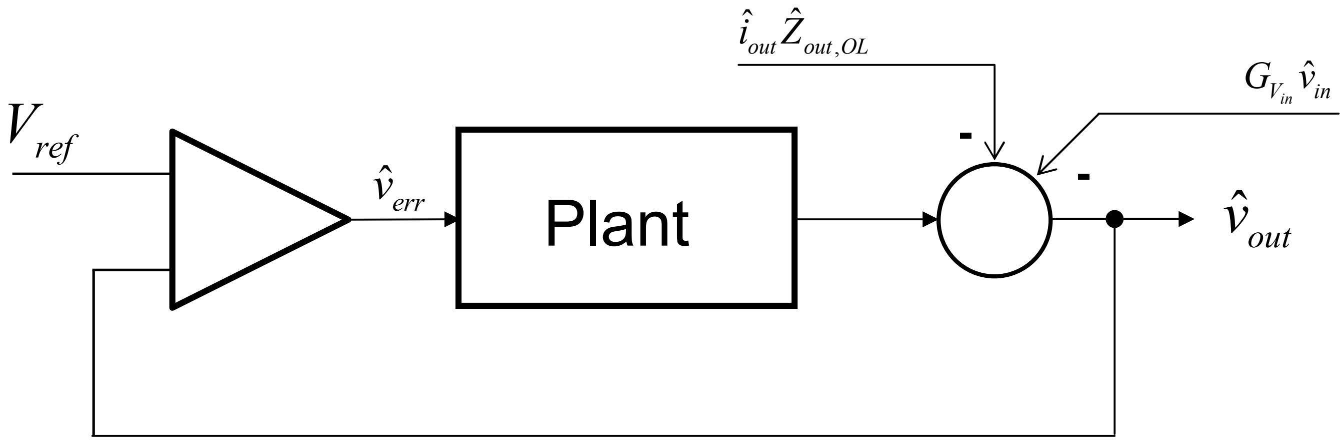
❑ In a simulation fixture, insert a gain block after V_{err}



PWM switch model in VM

Considering Feedforward

- A perturbation disturbs operations and must be fought
- In a power supply, the input voltage is a perturbation



- The perturbation must affect the output to trigger action

➢ Why not reacting before perturbation reaches the output?

✓ This is the principle of feedforward

Input Contribution in a Buck Converter

- The transfer function of a CCM buck converter includes V_{in}

$$H(s) = \frac{V_{in}}{V_p} \frac{1 + \frac{s}{s_{z_1}}}{1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_o}\right)^2}$$

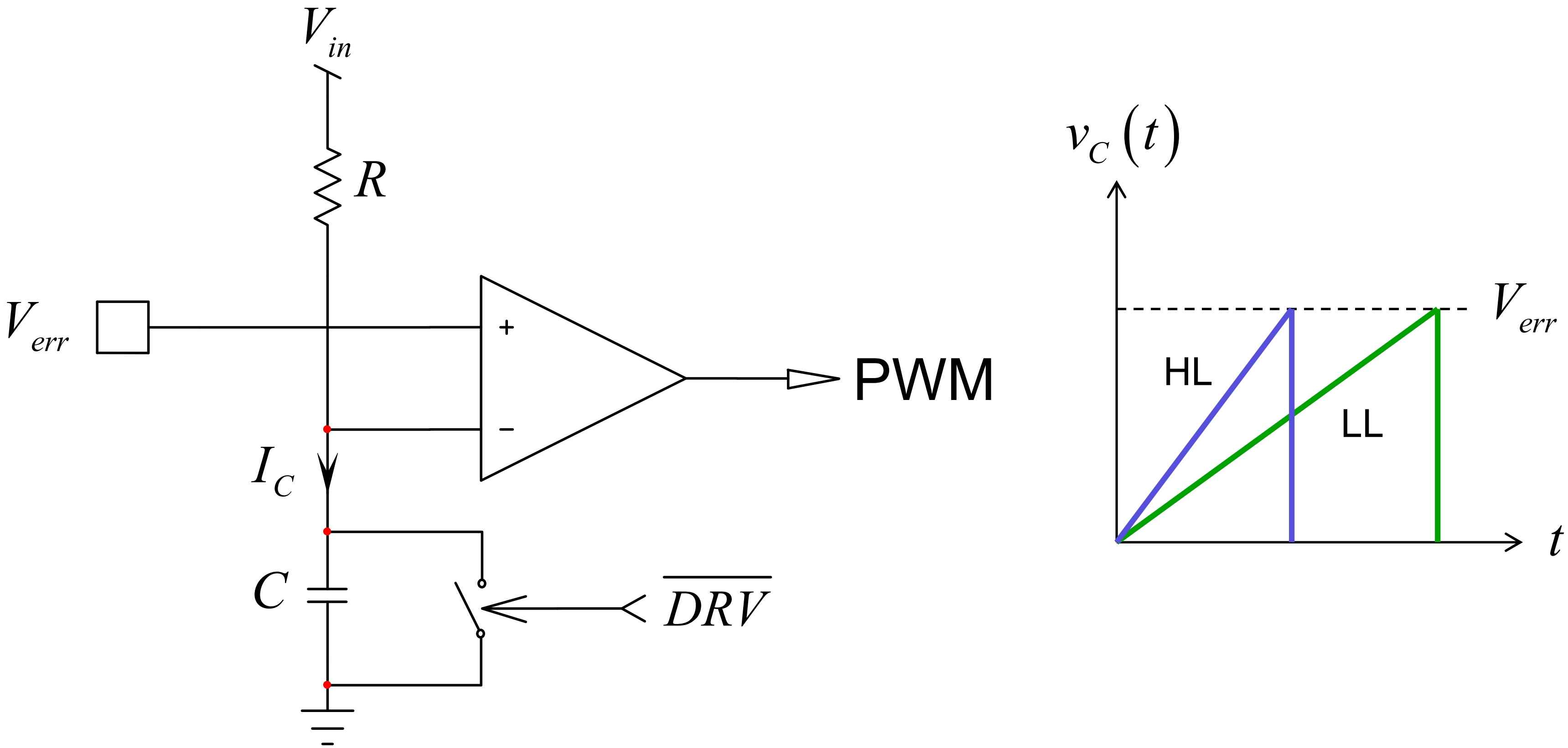
- Let's make V_p a function of V_{in}: $V_p(V_{in}) = k_{FF} V_{in}$

$$\Rightarrow \quad H(s) = \frac{V_{in}}{k_{FF} V_{in}} \frac{1 + \frac{s}{s_{z_1}}}{1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_o}\right)^2} = \frac{1}{k_{FF}} \frac{1 + \frac{s}{s_{z_1}}}{1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_o}\right)^2}$$

- The transfer function no longer depends on V_{in}

How to Make V_p a Function of V_{in}?

- Make the sawtooth capacitor current depend on V_{in}



HL = hi line
LL = lo line

Ramp Amplitude and Input Voltage

- We can neglect the sawtooth ramp amplitude

$$I_C \approx \frac{V_{in}}{R}$$

- The peak value, V_p, is linked to the time constant τ

$$V_p = \frac{I_C}{C_{ramp}} T_{sw} = \frac{V_{in}}{C_{ramp} R_{ramp}} T_{sw} = \frac{V_{in}}{\tau F_{sw}}$$

- In the time domain, the peak value will change

$$v_{ramp}(t) = V_p \frac{t}{T_{sw}} = \frac{V_{in}}{\tau F_{sw}} \frac{t}{T_{sw}}$$

- At $t = t_{on}$, the error voltage equals V_{ramp}

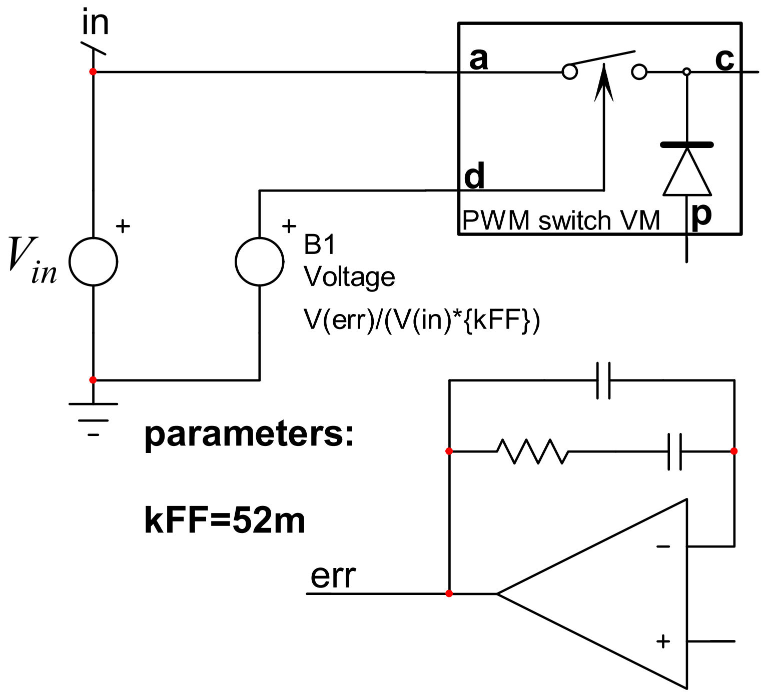
$$V_{err} = \frac{V_{in}}{\tau F_{sw}} \frac{t_{on}}{T_{sw}} = \frac{V_{in}}{\tau F_{sw}} D \quad \Rightarrow \quad D(V_{err}) = V_{err} \frac{\tau F_{sw}}{V_{in}}$$

An In-Line Equation to Include Feedforward

- Differentiate the expression to get the small-signal gain

$$G_{PWM} = \frac{\partial D(V_{err})}{\partial V_{err}} = \frac{\tau F_{sw}}{V_{in}} = \frac{1}{k_{FF} V_{in}} \quad \Rightarrow \quad k_{FF} = \frac{1}{F_{sw}\tau}$$

- The feedforward block requires an ABM source



$$R = 82\,\text{k}\Omega$$
$$C = 470\,\text{pF}$$
$$F_{sw} = 500\,\text{kHz}$$

$$k_{FF} = \frac{1}{500k \times 470p \times 82k} = 52m$$

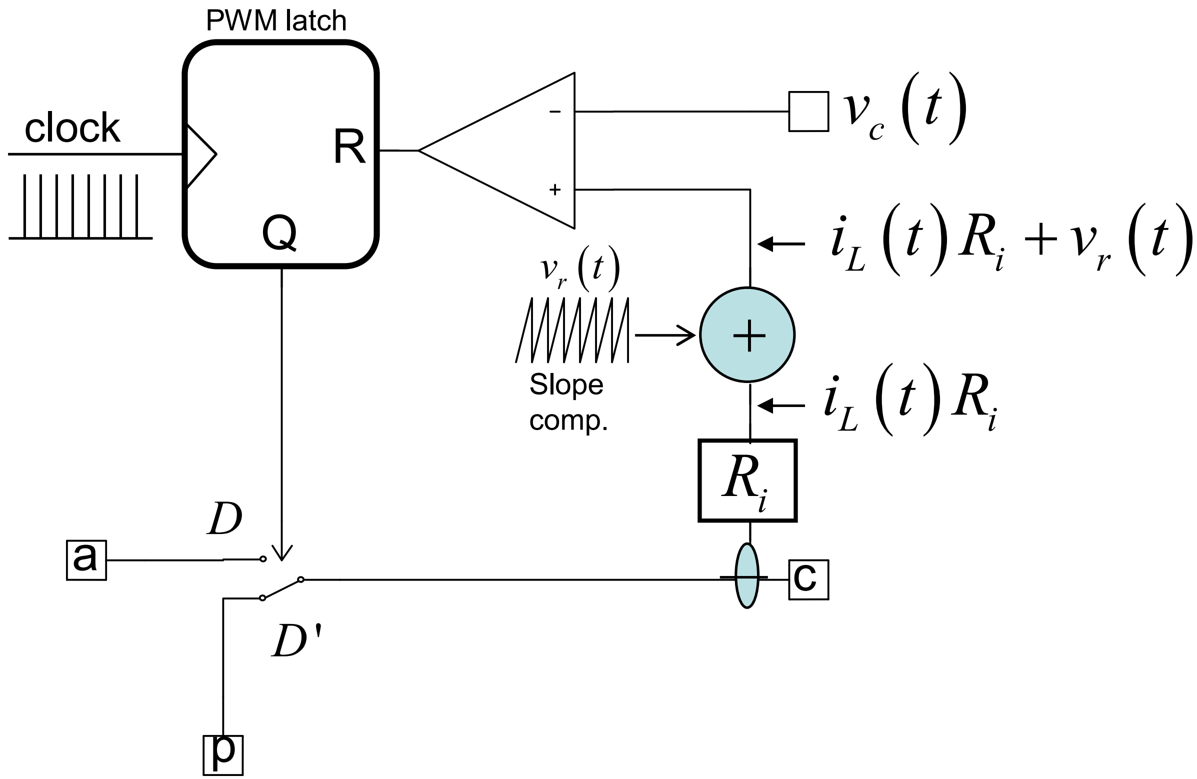
ABM: Analog Behavioral Model

Course Agenda

- ❑ Introducing the PWM Switch Model
- ❑ CCM, DCM and BCM in Voltage Mode
- ❑ Pulse Width Modulator Gain
- ❑ **The PWM Switch Model in Current Mode**
- ❑ PWM Switch at Work in a Buck Converter
- ❑ A Simplified Approach to Modeling a DCM Boost
- ❑ Transfer Function of a BCM Boost in Current Mode
- ❑ Small-Signal Model of The Active Clamp Forward

Peak Current Mode Control

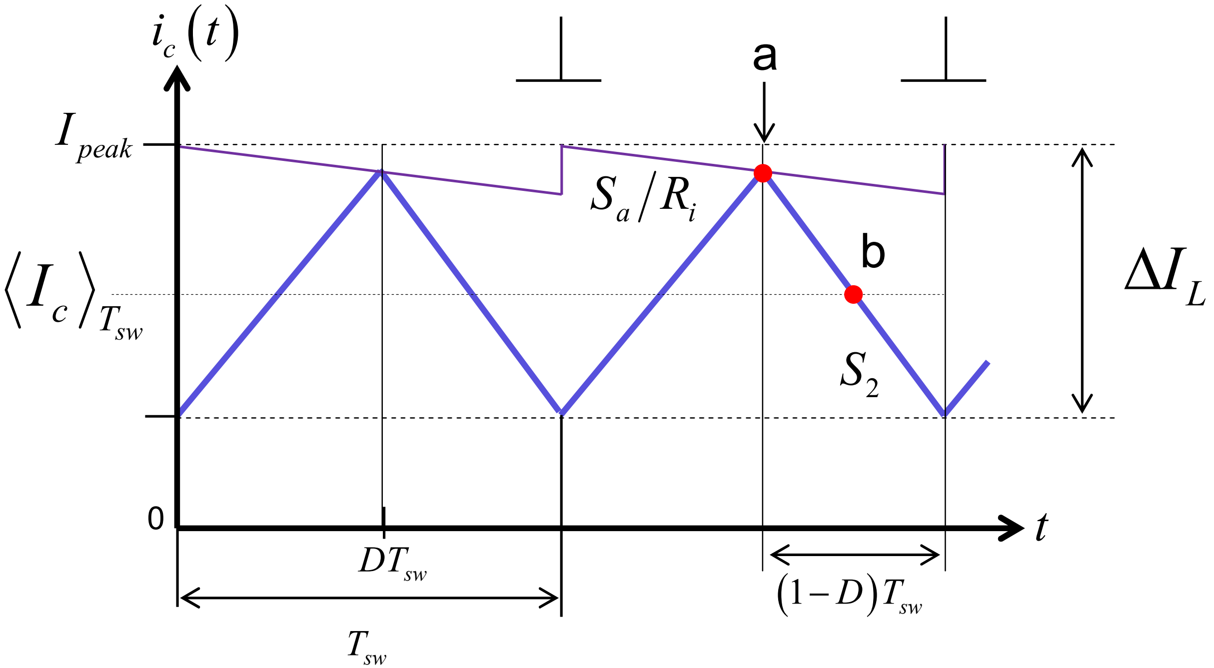
- In voltage-mode, the loop controls the duty ratio
- In current-mode, the inductor peak current is controlled



- An artificial ramp is added for stabilization purposes

We Want the Average Current Definition

- The value I_c is the inductor current at half the ripple



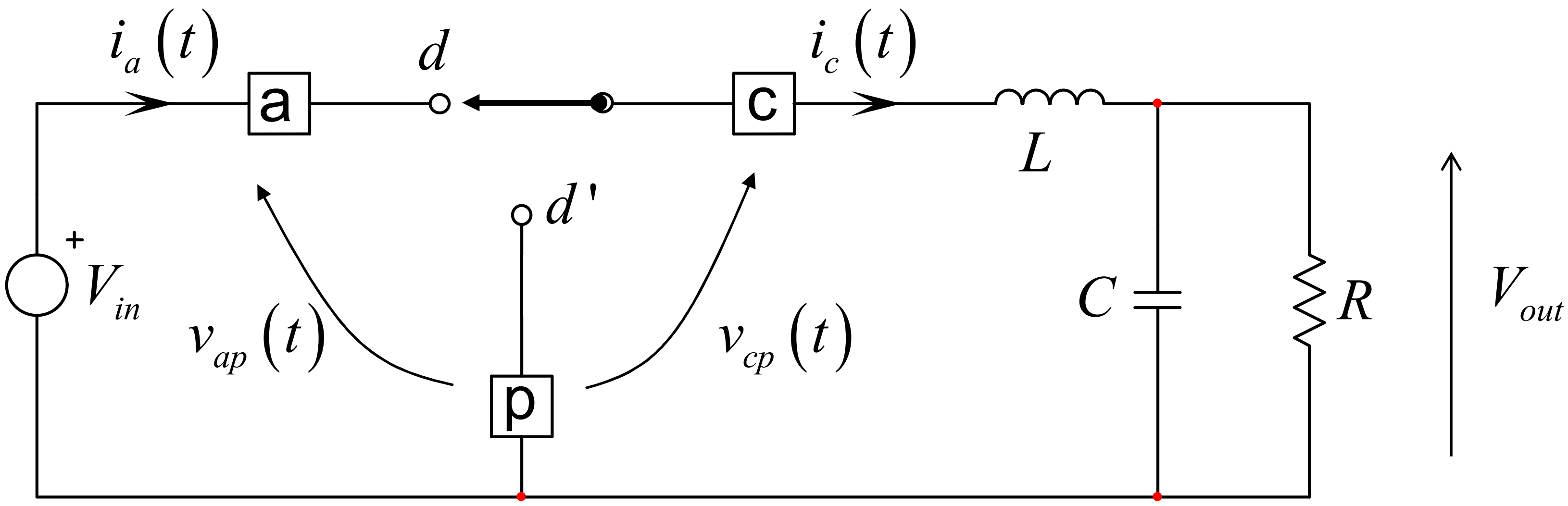
- Current at point **b** is that of **a** minus half the inductor ripple

$$\langle I_c \rangle_{T_{sw}} = \frac{V_c}{R_i} - \frac{S_a}{R_i} DT_{sw} - \frac{S_2 D' T_{sw}}{2}$$

CCM CM

Define the Converter off-Slope

- Use a buck configuration to see voltages at play



- The downslope depends on the output voltage V_{out}:

$$S_2 = -\frac{V_{out}}{L}$$

- The inductor average voltage is 0 at steady-state

$$V_{cp} = V_{out} \longrightarrow S_2 = -\frac{V_{cp}}{L}$$

CCM CM

A Current Mode Generator

- Update the previous equation to obtain final definition

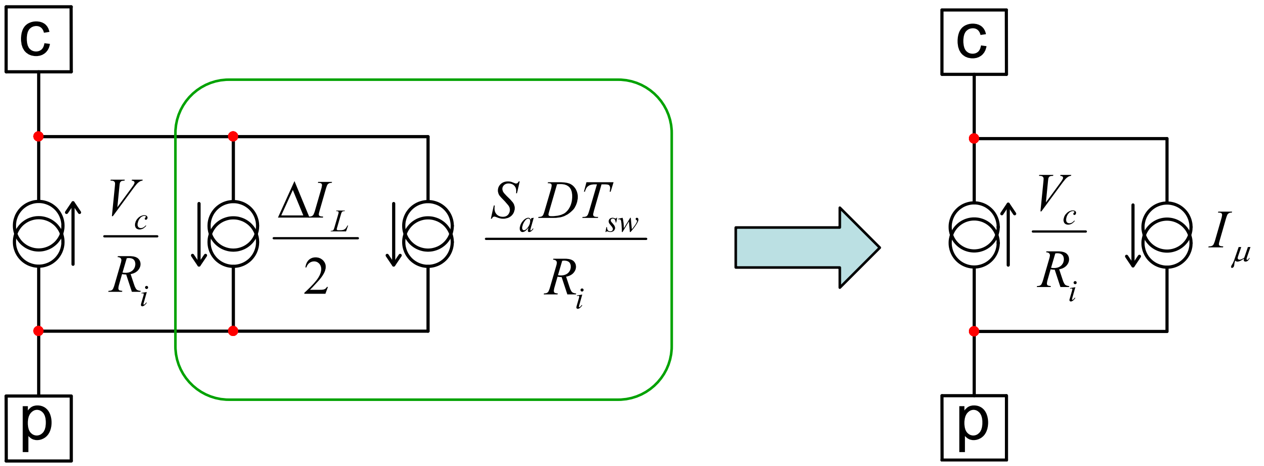
$$I_c = \frac{V_c}{R_i} - V_{cp}(1-D)\frac{T_{sw}}{2L} - \frac{S_a}{R_i}DT_{sw}$$

Peak current setpoint — Half inductor ripple — Compensation ramp

Group 2[nd] and 3[rd] terms →

$$I_\mu = V_{cp}(1-D)\frac{T_{sw}}{2L} + \frac{S_a}{R_i}DT_{sw}$$

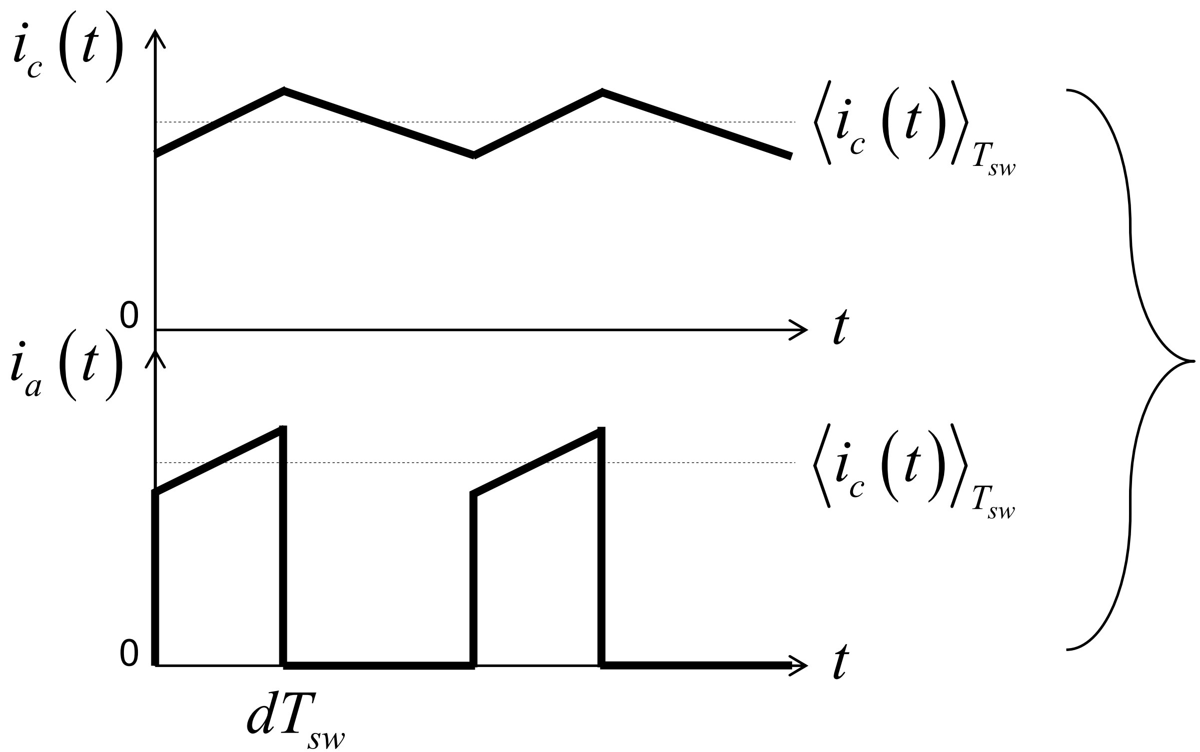
- Inductor ripple and compensation ramp alter peak value



CCM CM

CM or VM Lead to Similar Input Currents

- ❑ Average the current waveforms across the PWM switch



$$I_a = DI_c$$

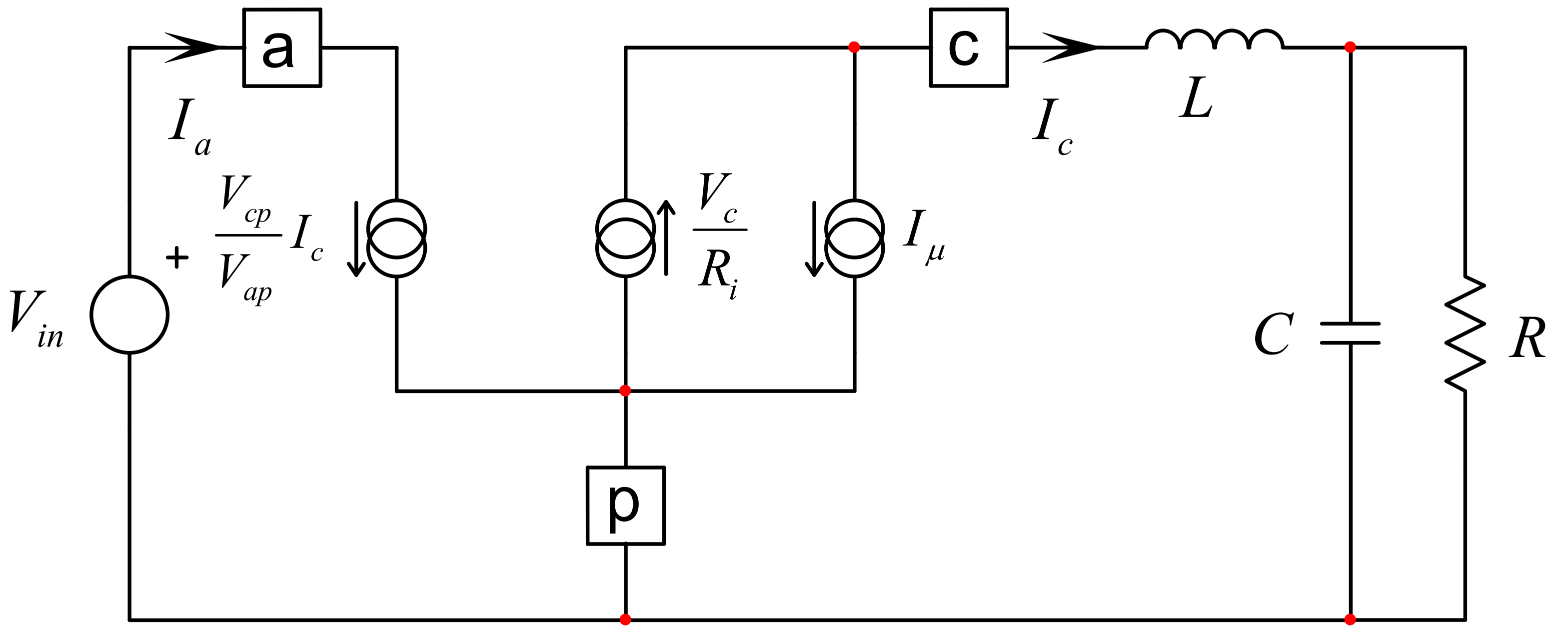
$$I_a = \frac{V_{cp}}{V_{ac}} I_c$$

$$\langle i_a(t)\rangle_{T_{sw}} = I_a = \frac{1}{T_{sw}} \int_0^{dT_{sw}} i_a(t)\,dt = D\langle i_c(t)\rangle_{T_{sw}} = DI_c \qquad D = \frac{V_{cp}}{V_{ap}}$$

CCM CM

The PWM Switch Model in Current Mode

- The final model associates three current sources

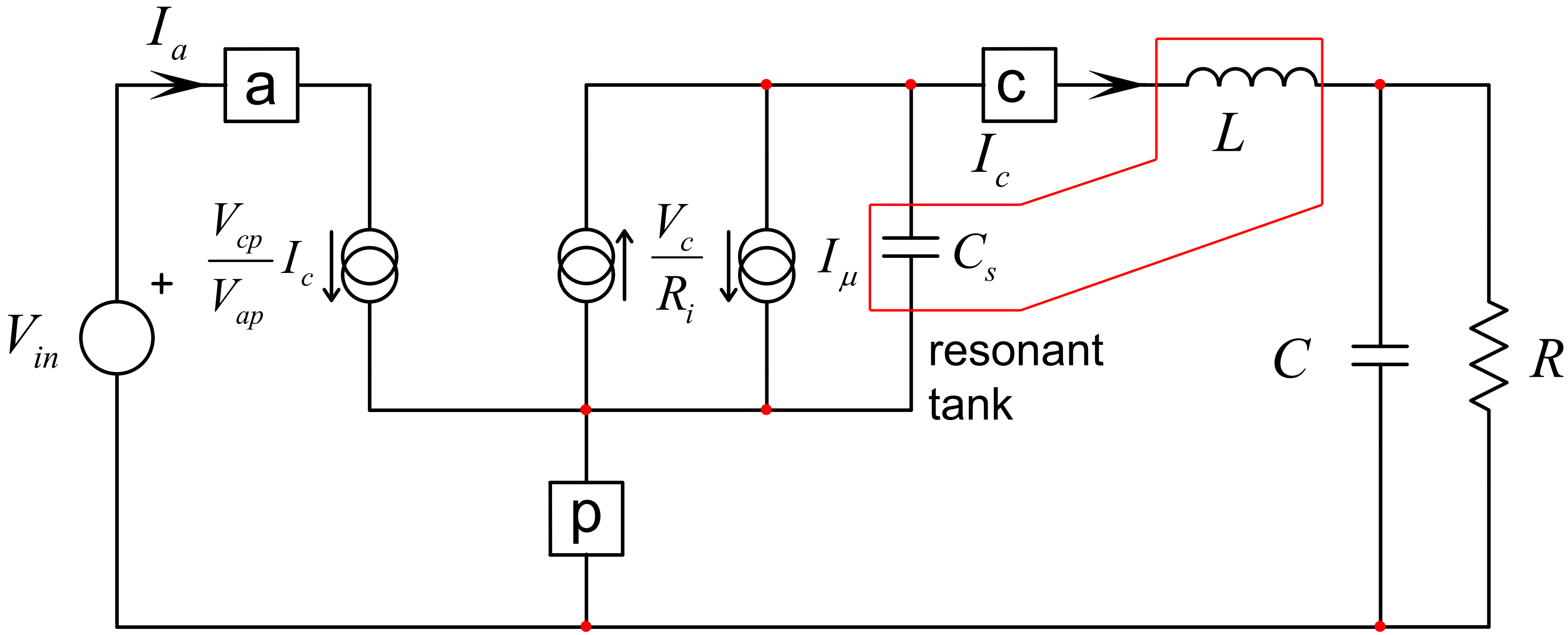


- This is the large-signal current-mode PWM switch model

V. Vorpérian,"Analysis of Current-Controlled PWM Converters using the Model of the PWM Switch", PCIM Conference ,1990

Final Model Includes Subharmonic Effects

- A simple capacitor is enough to mimic instability



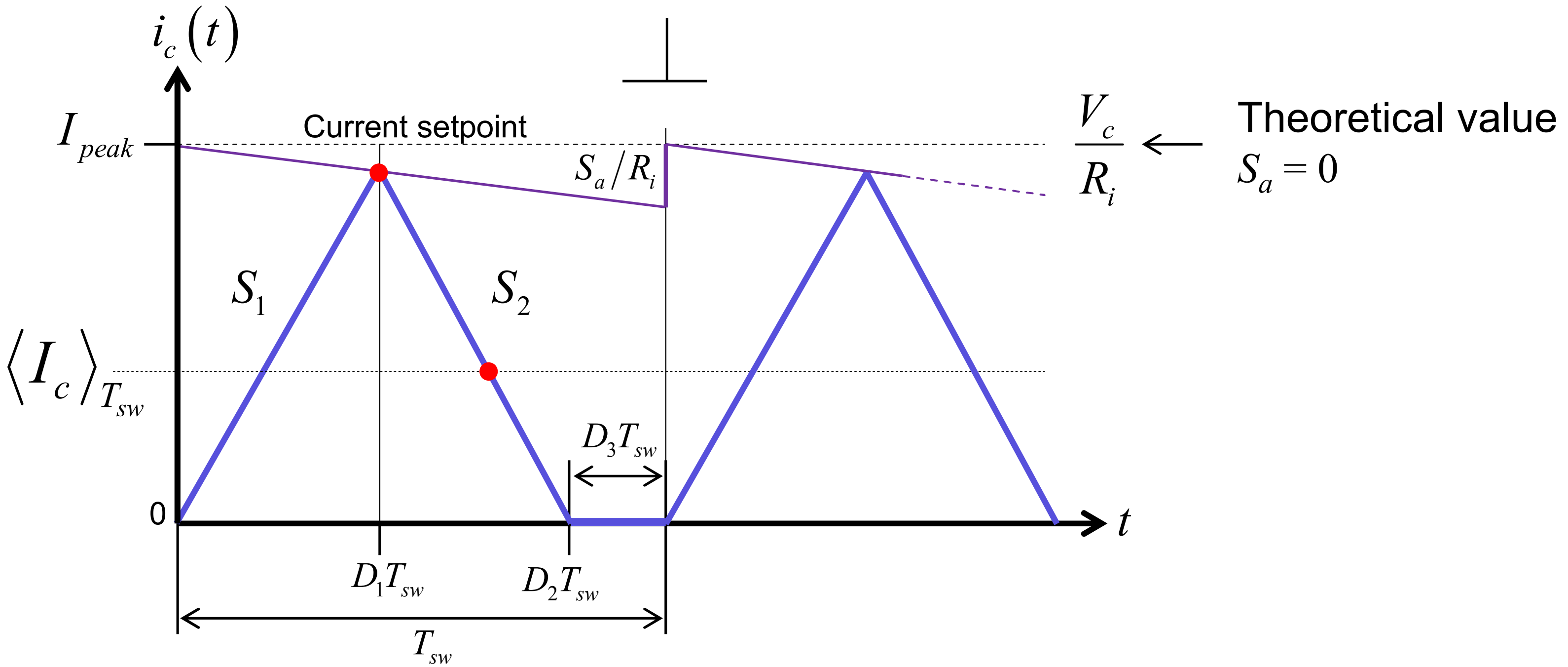
- As the instability is placed at half the switching frequency:

$$\frac{F_{sw}}{2} = \frac{1}{2\pi\sqrt{LC_s}} \quad \Rightarrow \quad C_s = \frac{1}{L(F_{sw}\pi)^2}$$

CCM CM

The PWM Switch in DCM

- The PWM CM can work in discontinuous mode



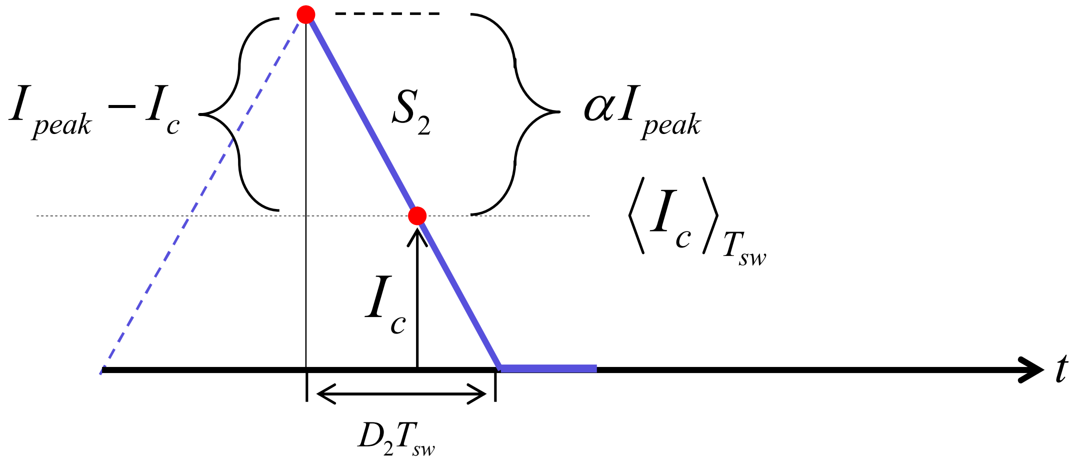
- The average current I_c is somewhere in the downslope S_2

$$I_{peak} = \frac{V_c - D_1T_{sw}S_a}{R_i} \longrightarrow I_c = \frac{V_c - D_1T_{sw}S_a}{R_i} - \alpha D_2T_{sw}S_2$$

DCM CM

Derive the Inductor Average Current

- We must now obtain the value of α to get I_c



- I_c is the area under the triangle divided by the switching period

$$I_c = \frac{I_{peak} D_1}{2} + \frac{I_{peak} D_2}{2} = I_{peak} \frac{D_1 + D_2}{2}$$

$$\alpha I_{peak} = I_{peak} - I_{peak} \frac{D_1 + D_2}{2}$$

$$\alpha = 1 - \frac{D_1 + D_2}{2}$$

DCM CM

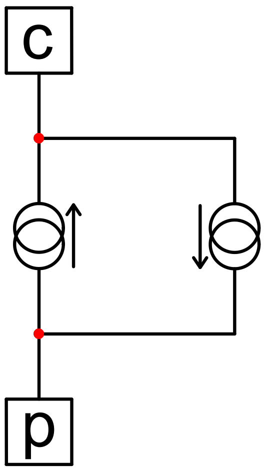
Adopt the CCM Structure for DCM

- Substitute and rearrange to get the inductor current

$$I_c = \frac{V_c}{R_i} - \frac{D_1 T_{sw} S_a}{R_i} - D_2 T_{sw} \frac{V_{cp}}{L}\left(1 - \frac{D_1 + D_2}{2}\right)$$

- If we stick to the original CCM architecture

$$I_c = \frac{V_c}{R_i} - I_\mu \quad \text{with} \quad I_\mu = \frac{D_1 T_{sw} S_a}{R_i} + D_2 T_{sw} \frac{V_{cp}}{L}\left(1 - \frac{D_1 + D_2}{2}\right)$$



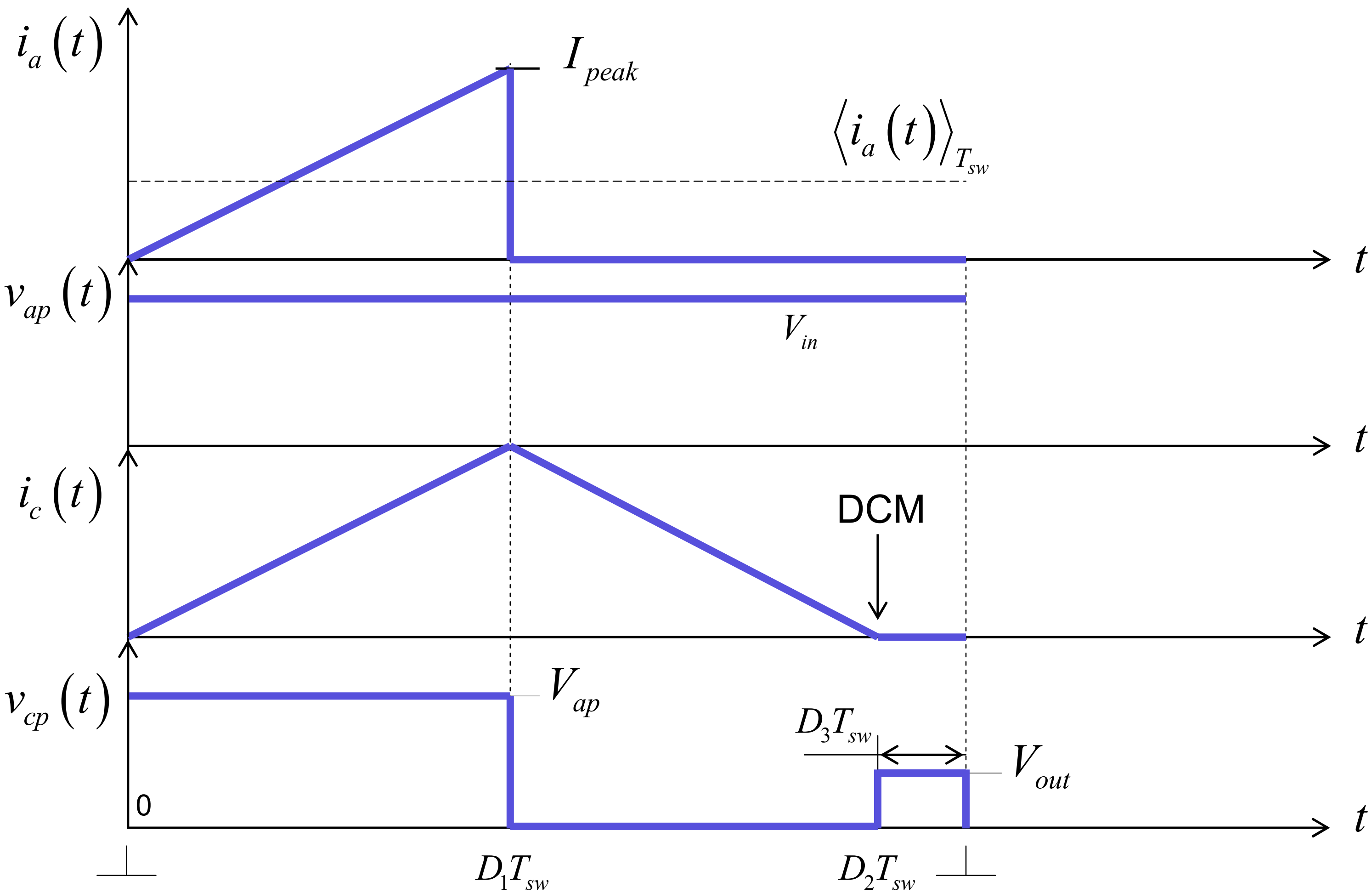
$$\frac{V_c}{R_i}$$

$$\frac{D_1 T_{sw} S_a}{R_i} + D_2 T_{sw} \frac{V_{cp}}{L}\left(1 - \frac{D_1 + D_2}{2}\right)$$

DCM CM

Discontinuous Waveforms

❑ Let's have a look at the PWM switch voltages in DCM



DCM CM

Derive the Duty Ratios

- From the DCM voltage-mode PWM switch we have:

$$V_{cp} = V_{ap}\frac{D_1}{D_1 + D_2} \longrightarrow D_1 = \frac{D_2 V_{cp}}{V_{ap} - V_{cp}}$$

- From the operating DCM waveforms

$$I_a = \frac{I_{peak} D_1}{2} \longrightarrow I_{peak} = \frac{2I_a}{D_1} \longrightarrow I_c = I_{peak}\frac{D_1 + D_2}{2} \Rightarrow I_a = I_c \frac{D_1}{D_1 + D_2}$$

- Almost there, just need to express D_2

$$I_{peak} = \frac{V_{ac}}{L} D_1 T_{sw} \quad \text{and} \quad I_{peak} = \frac{2I_c}{D_1 + D_2}$$

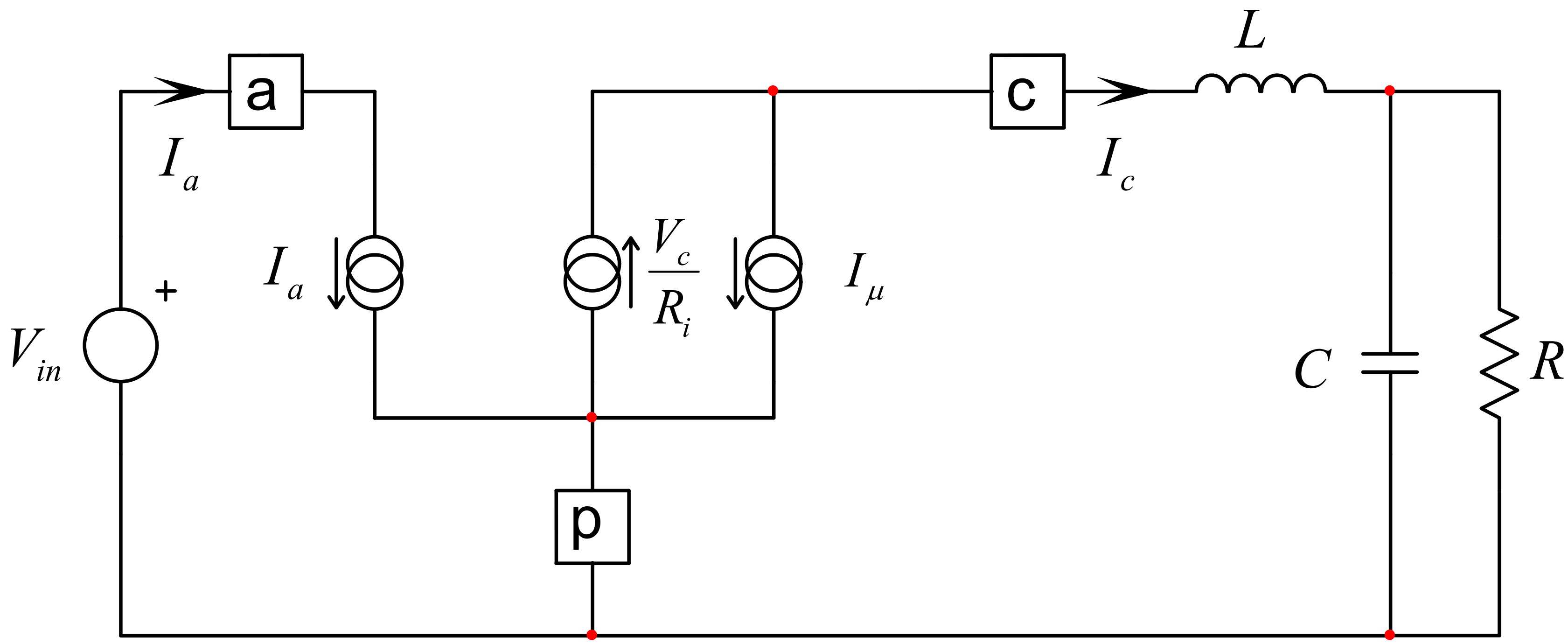
$$\frac{V_{ac}}{L} D_1 T_{sw} = \frac{2I_c}{D_1 + D_2}$$

$$D_2 = \frac{2LF_{sw}}{D_1}\frac{I_c}{V_{ac}} - D_1$$

DCM CM

The DCM Model is Complete!

- We can use this model for DCM simulations



$$I_a = I_c \frac{D_1}{D_1 + D_2}$$

$$I_\mu = \frac{D_1 T_{sw} S_a}{R_i} + D_2 T_{sw} \frac{V_{cp}}{L}\left(1 - \frac{D_1 + D_2}{2}\right)$$

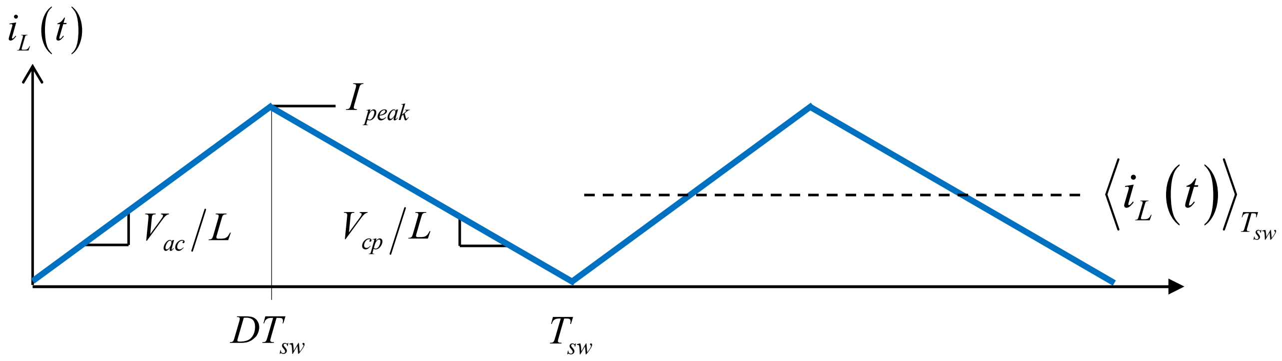
DCM CM

Current Mode Borderline

- In CM, the error voltage sets the peak current

$$I_{peak} = \frac{V_c}{R_i}$$

- The peak current also depends on duty ratio D



$$I_{peak} = \frac{V_{ac}}{L} DT_{sw} \quad \Rightarrow \quad D = \frac{V_c}{R_i}\frac{L}{V_{ac}T_{sw}} \quad \Rightarrow \quad t_{on} = \frac{V_c}{R_i}\frac{L}{V_{ac}}$$

BCM CM

Operating Points of BCM Current Mode

- The off-time duration depends on I_{peak} too

$$I_{peak} = \frac{V_{cp}}{L}(1-D)T_{sw} \Rightarrow t_{off} = \frac{V_{err}}{R_i}\frac{L}{V_{cp}}$$

- Switching frequency comes easily

$$T_{sw} = t_{on} + t_{off} = \frac{V_c L}{R_i}\left(\frac{1}{V_{ap}} + \frac{1}{V_{cp}}\right)$$

- The average inductor current I_c is straigthforward

$$I_c = \frac{I_{peak}}{2} = \frac{V_c}{2R_i}$$

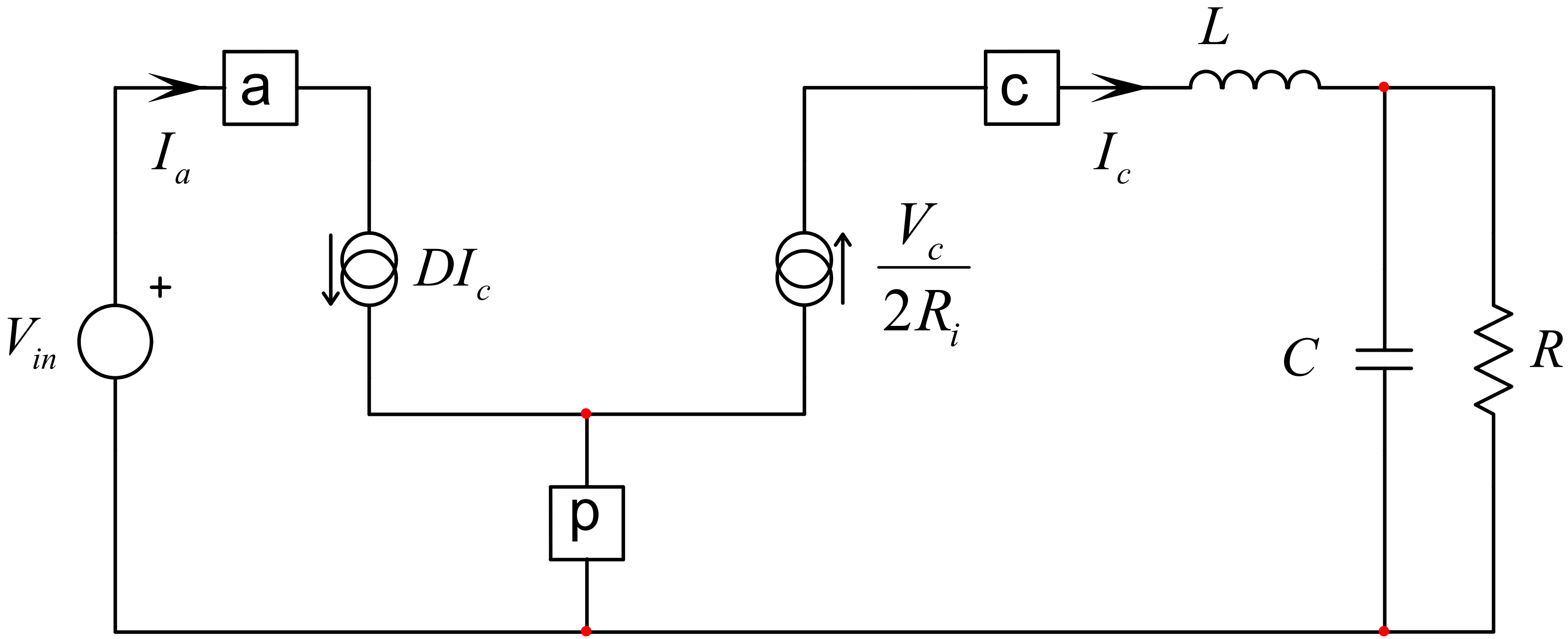
- The relationship between I_a and I_c is always the same

$$I_a = DI_c$$

BCM CM

This Completes the BCM CM Model

- The model is really simple, two current sources



- Other ABM sources will compute D and T_{sw}

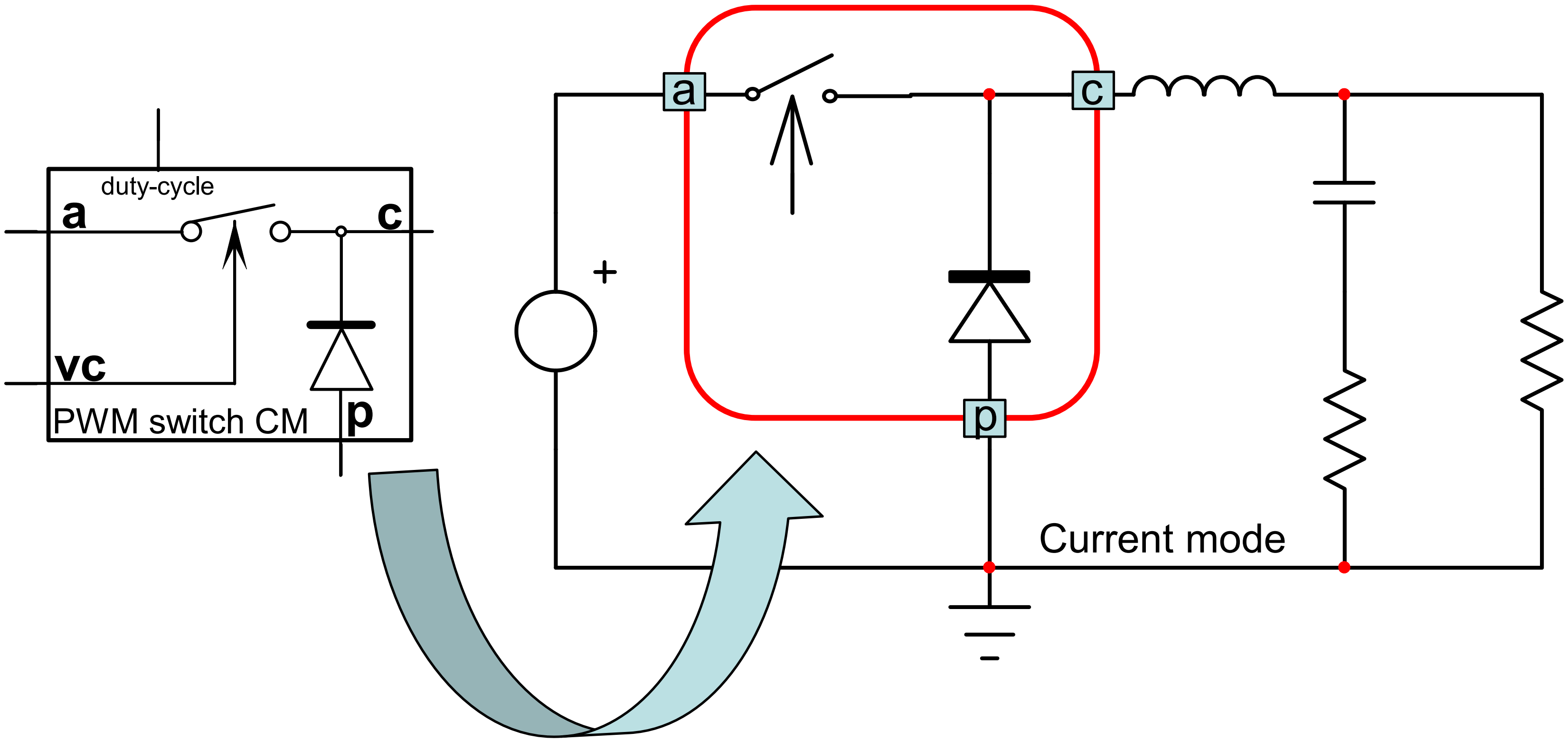
BCM CM

Course Agenda

- ❑ Introducing the PWM Switch Model
- ❑ CCM, DCM and BCM in Voltage Mode
- ❑ Pulse Width Modulator Gain
- ❑ The PWM Switch Model in Current Mode
- ❑ **PWM Switch at Work in a Buck Converter**
- ❑ A Simplified Approach to Modeling a DCM Boost
- ❑ Transfer Function of a BCM Boost in Current Mode
- ❑ Small-Signal Model of The Active Clamp Forward

A Buck Converter in Current Mode

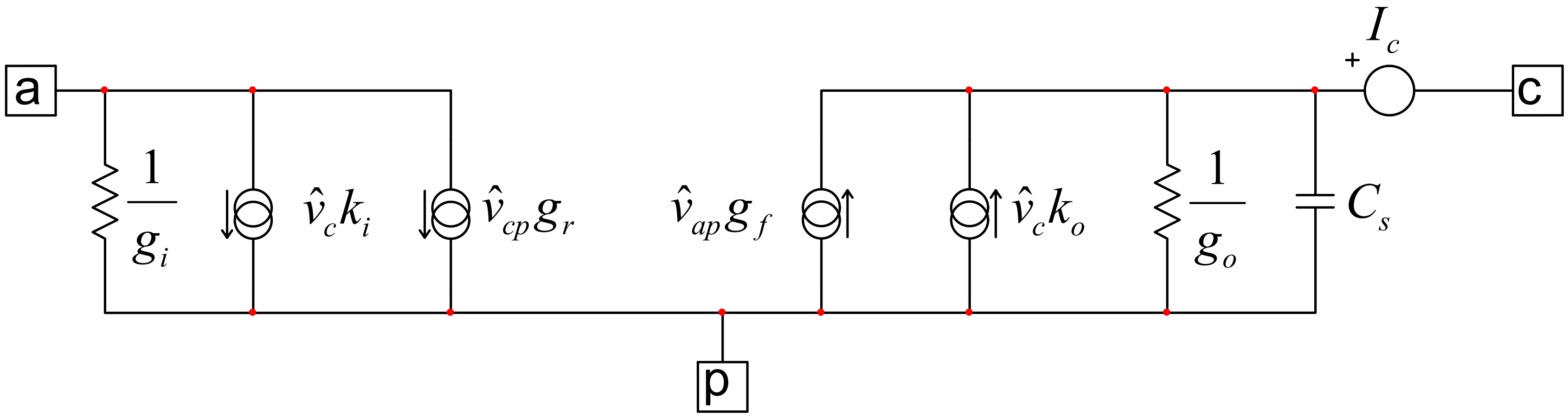
❑ Identify the diode and switch position in a buck CM



❑ Replace switches by the small-signal PWM switch model

A Small Signal Model

- The model includes current sources and conductances



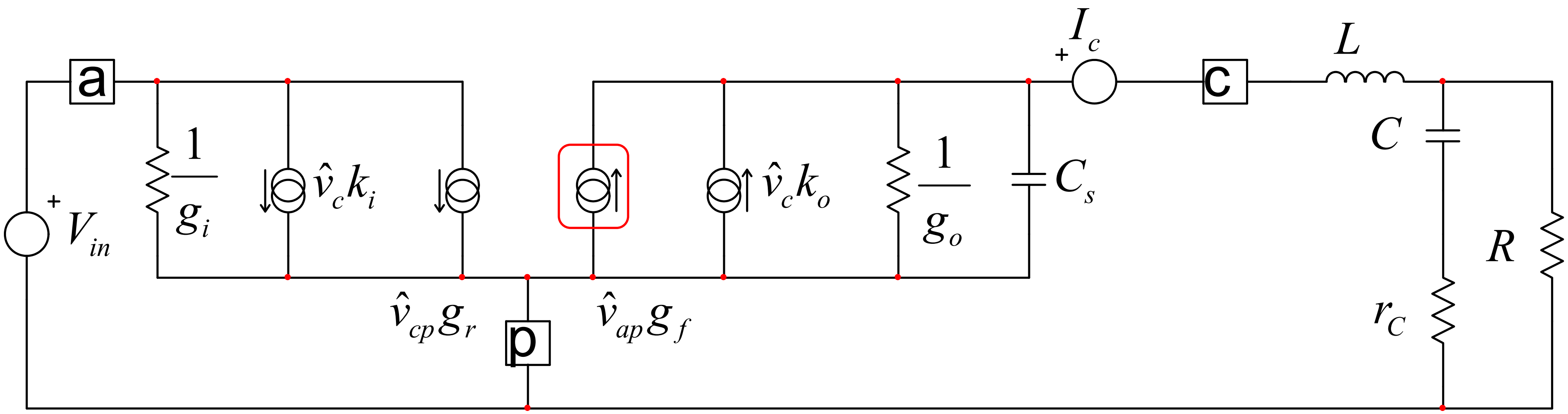
$$k_o = \frac{1}{R_i} \qquad g_f = Dg_o - \frac{DD'T_{sw}}{2L} \qquad g_o = \frac{T_{sw}}{L}\left(D'\frac{S_a}{S_n} + \frac{1}{2} - D\right)$$

$$k_i = \frac{D}{R_i} \qquad g_i = D\left(g_f - \frac{I_c}{V_{ap}}\right) \qquad g_r = \frac{I_c}{V_{ap}} - g_o D$$

V. Vorpérian,"Analysis of Current-Controlled PWM Converters using the Model of the PWM Switch", PCIM Conference ,1990

Plug the Model and Simplify

- Plug, simplify and rearrange: test in between!



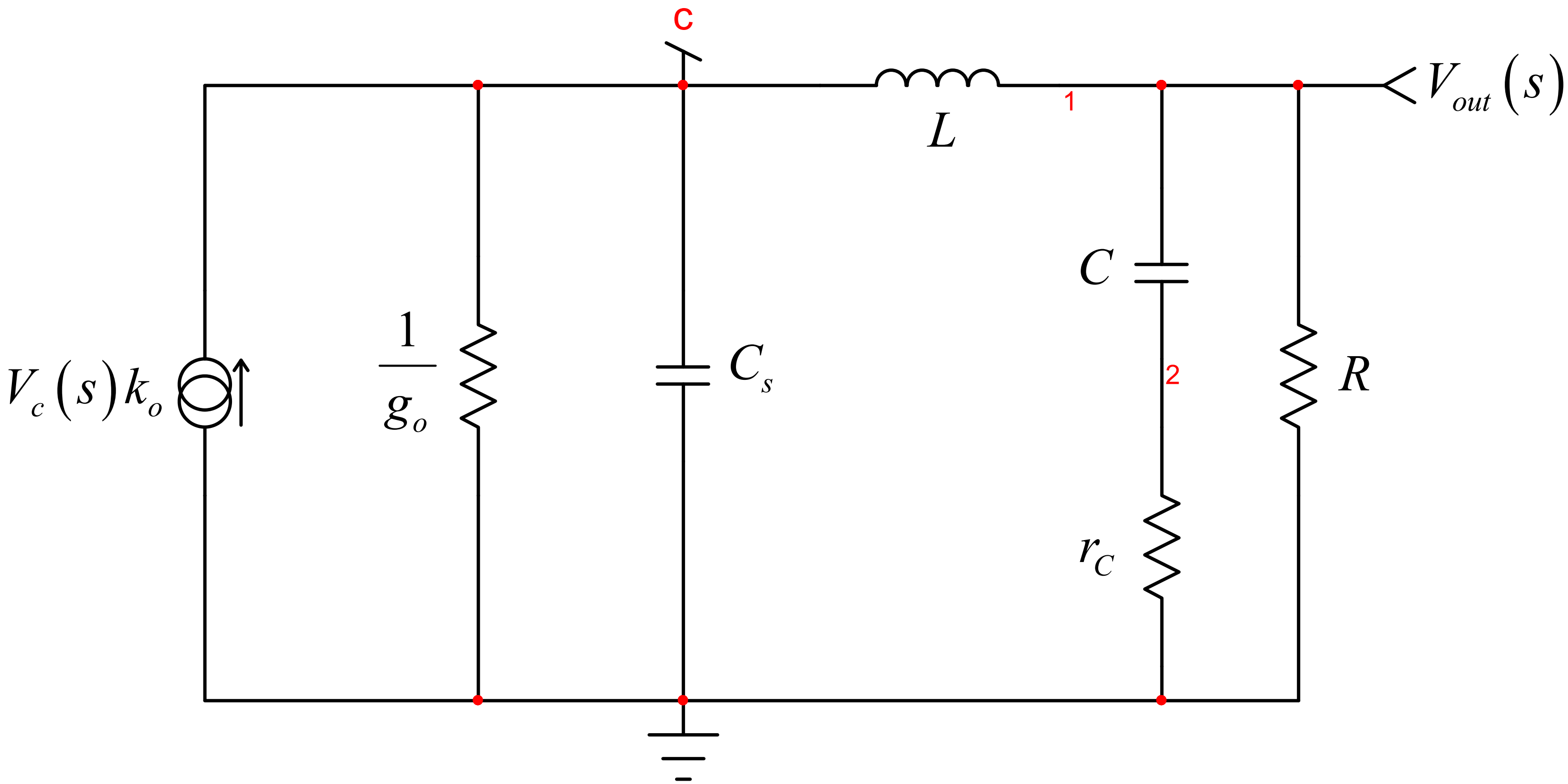
- We want the control-to-output function, remove input stimulus

$$\hat{v}_{in} = 0 \longrightarrow \hat{v}_{ap} g_f = 0$$

- The input contribution can also disappear, no interest in Z_{in}

Time to Call FACTS!

❑ End-up with a simpler and less ugly sketch

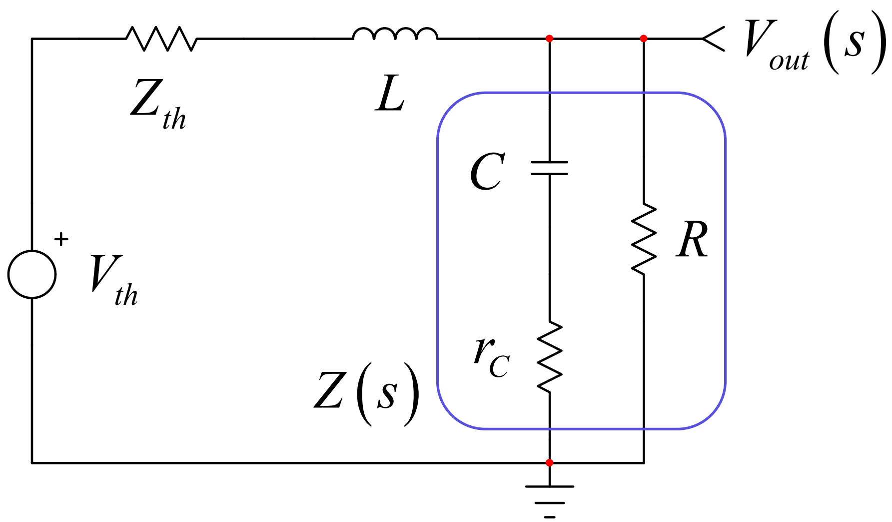


❑ There are 3 storage elements: 3rd-order system

FACTS: Fast Analytical Circuit TechniqueS

Don't Use Brute-Force Algebra!

- ❑ You can use brute-force analysis…



$$V_{th} = V_c(s) k_0 \left(\frac{1}{g_o} \middle\| \frac{1}{sC_s} \right)$$

$$Z_{th} = \left(\frac{1}{g_o} \middle\| \frac{1}{sC_s} \right)$$

- ❑ …or consider FACTS to write a 3rd-order system TF

$$H(s) = H_0 \frac{N(s)}{D(s)} = H_0 \frac{N(s)}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$

Start with the Dc Gain

- Consider dc and high-frequency states for L and C

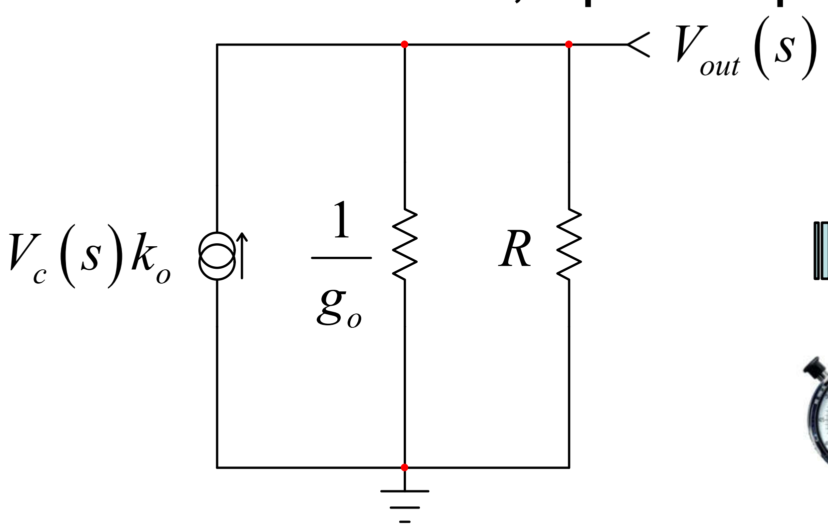
C —impedance→ $Z_C = \frac{1}{sC}$

Dc state	$Z_C = \infty$	Cap. is an open circuit
HF state	$Z_C = 0$	Cap. is a short circuit

L —impedance→ $Z_L = sL$

Dc state	$Z_L = 0$	Inductor is a short circuit
HF state	$Z_L = \infty$	Inductor is an open circuit

- In the circuit, open capacitors and short inductors

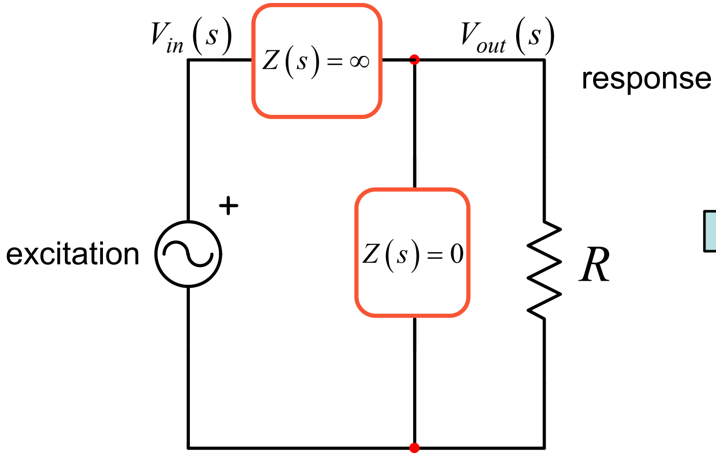


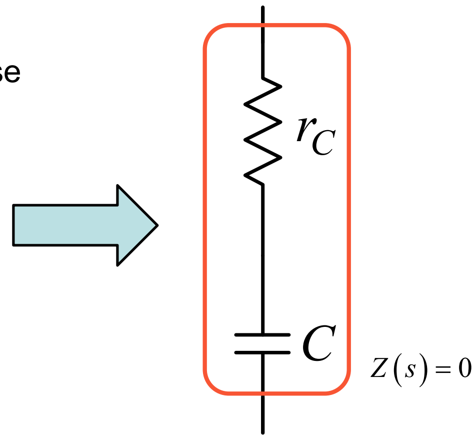
$$H_0 = k_0\left(R \,\Big\|\, \frac{1}{g_0}\right)$$

10 s!

Identify The Zeros

- Zeros prevent the excitation from reaching the output





$$r_C+\frac{1}{sC}=\frac{sr_CC+1}{sC}$$

$$sr_CC+1=0$$

$$\omega_{z_1}=\frac{1}{r_CC}$$

- We have the zero expression, half of the work is done

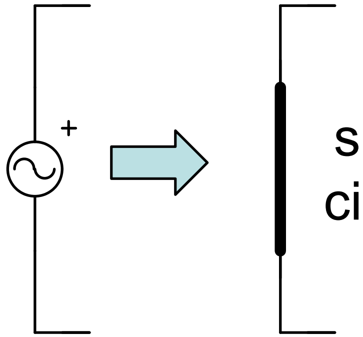
$$H(s)=H_0\frac{1+\frac{s}{s_{z_1}}}{1+a_1s+a_2s^2+a_3s^3}$$

Finding the Poles

- ❑ The poles are linked to the time constants of the system
- ❑ These time constants solely depend on the structure

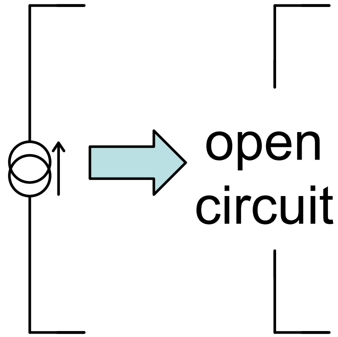
→ Remove the excitation signal to isolate the structure

voltage source



short circuit

current source

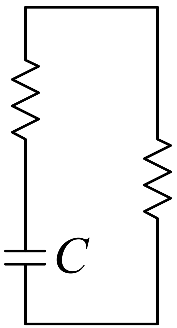


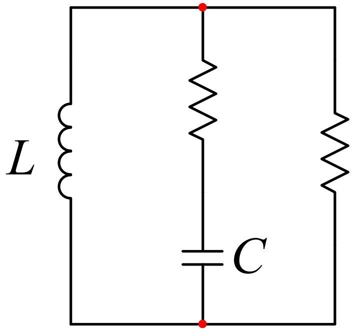
open circuit

- ❑ The denominator order depends on the storage elements

1 storage element

1st-order





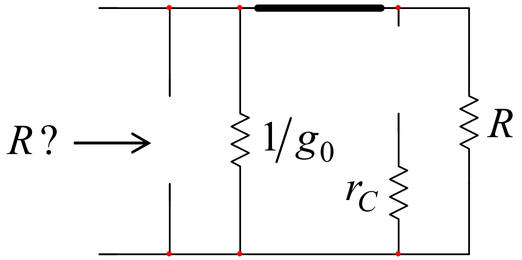
2 storage elements

2nd-order

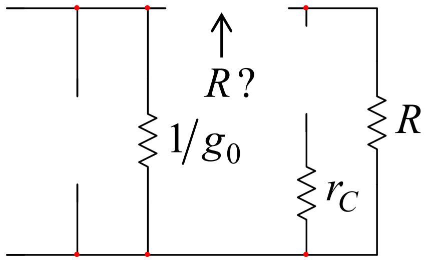
Not always! Consider individual state variables.

Start by Identifying the Time Constants

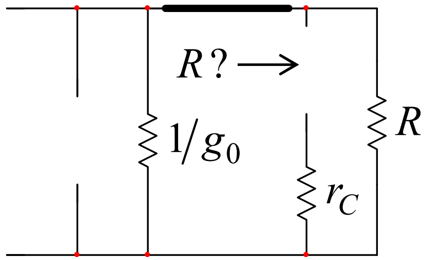
- The excitation is zero, elements are in their dc states
- 3 storage elements, 3 time constants, 3 drawings



$$\tau_1 = f(C_s) \longrightarrow \tau_1 = C_s\left(\frac{1}{g_o}\|R\right)$$



$$\tau_2 = f(L) \longrightarrow \tau_2 = \frac{L}{\frac{1}{g_0}+R}$$



$$\tau_3 = f(C) \longrightarrow \tau_3 = C\left(r_C + \left(\frac{1}{g_0}\|R\right)\right)$$

First Coefficients a_1 and a_2

- FACTs tell us that a_1 sums up all time constants

$$a_1 = \tau_1 + \tau_2 + \tau_3 \longrightarrow$$ Dimension is time

- For a_2, we multiply combined-time constants

$$a_2 = \tau_1\tau_2^1 + \tau_1\tau_3^1 + \tau_2\tau_3^2 \longrightarrow$$ Dimension is time²

- What is this new time constants definition, τ_2^1 ?

$$\tau_2^1 \left| \begin{array}{l} C_s \text{ (HF)} \\ \\ L \;\; C \text{ (dc)} \end{array} \right. \qquad \tau_3^1 \left| \begin{array}{l} C_s \text{ (HF)} \\ \\ L \text{ (dc)} \;\; C \end{array} \right. \qquad \tau_3^2 \left| \begin{array}{l} L \text{ (HF)} \\ \\ C \;\; C_s \text{ (dc)} \end{array} \right.$$

$$\uparrow \qquad\qquad \uparrow \qquad\qquad \uparrow$$

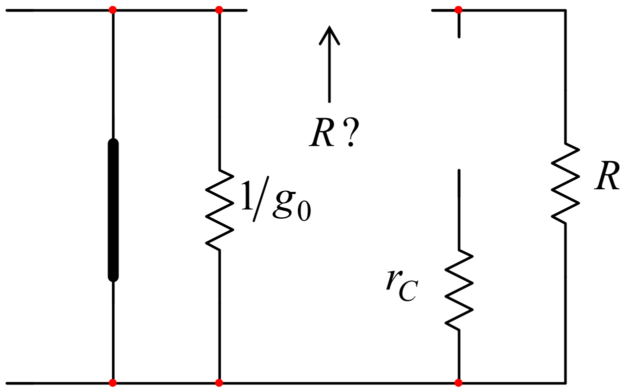
$$R\,? \qquad\qquad R\,? \qquad\qquad R\,?$$

V. Vorpérian, "Fast Analytical Techniques for Electrical and Electronic Circuits", Cambridge Press, 2002

Coefficient a_2 Mixes Time Constants

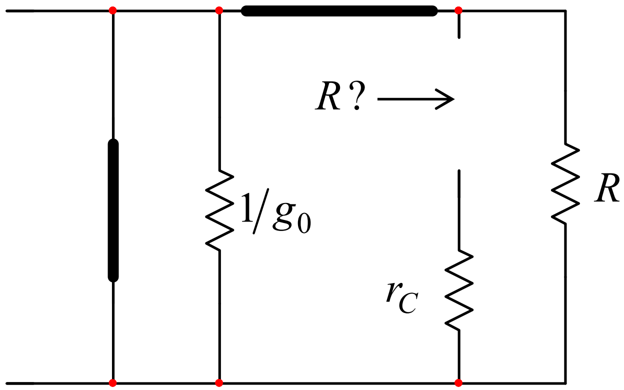
- Drawings are key to avoid mistakes

τ_2^1 | C_s (HF) / L C (dc) — R?



$$\tau_2^1 = \frac{L}{R}$$

τ_3^1 | C_s (HF) / L (dc) C — R?

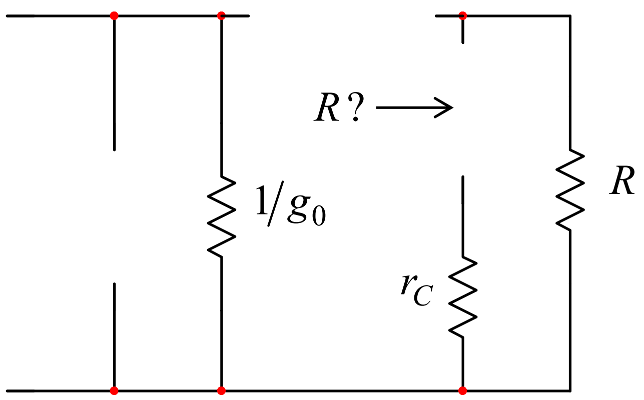


$$\tau_3^1 = r_C C$$

(Carefully) Mixing Time Constants

- The last drawing completes the a_2 expression

τ_3^2 | L (HF) / C C_s(dc) ↑ R?



$$\tau_3^2 = (r_C + R)C$$

- a_2 coefficient is there!

$$a_2 = C_s\left(\frac{1}{g_o}\|R\right)\frac{L}{R} + C_s\left(\frac{1}{g_o}\|R\right)r_C C + \frac{L}{\frac{1}{g_0}+R}(r_C + R)C$$

…And a_3 is ?

- For a_3, we multiply by a third time-constant

$$a_3 = \tau_1 \tau_2^1 \tau_3^{1,2} \longrightarrow$$ Dimension is time³

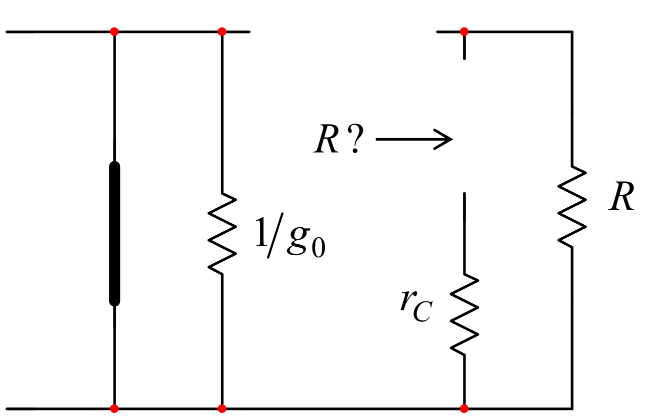
- What is this new time constant definition?

$\tau_3^{1,2}$ | C_s (HF) L (HF)

C (dc)

↑

$R\,?$



$$\tau_3^{1,2} = \left(r_C + R\right)C$$

- The final coefficient has been identified

$$a_3 = C_s \left(\frac{1}{g_o} \| R\right) \frac{L}{R} \left(r_C + R\right) C$$

Time to Check Results

- A Mathcad® sheet can be built to verify these calculations

$$H(s) = G_0 \frac{1+\frac{s}{s_{z_1}}}{1+a_1 s + a_2 s^2 + a_3 s^3} \qquad G_0 = k_0\left(R \left\| \frac{1}{g_0}\right.\right)$$

$$\omega_{z_1} = \frac{1}{r_C C}$$

$$a_1 = C_s\left(\frac{1}{g_o}\|R\right) + \frac{L}{\frac{1}{g_0}+R} + C\left(r_C + \left(\frac{1}{g_0}\|R\right)\right)$$

$$a_2 = C_s\left(\frac{1}{g_o}\|R\right)\frac{L}{R} + C_s\left(\frac{1}{g_o}\|R\right)r_C C + \frac{L}{\frac{1}{g_0}+R}(r_C+R)C$$

$$a_3 = C_s\left(\frac{1}{g_o}\|R\right)\frac{L}{R}(r_C+R)C$$

5 V/1 A buck

$V_{in} = 10\,\text{V}, F_{sw} = 100\,\text{kHz}, R_i = 0.25\,\Omega, S_e = 2.5\,\text{kV/s}$

$C = 100\,\mu\text{F}, r_C = 0.1\,\Omega, L = 100\,\mu\text{H}, C_s = 101\,\text{nF}, V_c = 1.28\,\text{V}$

$I_c = 4.94\,\text{A}$

$k_i = 2\,\Omega^{-1} \quad k_0 = 4\,\Omega^{-1}$

$g_0 = 0.01\,\Omega^{-1}$

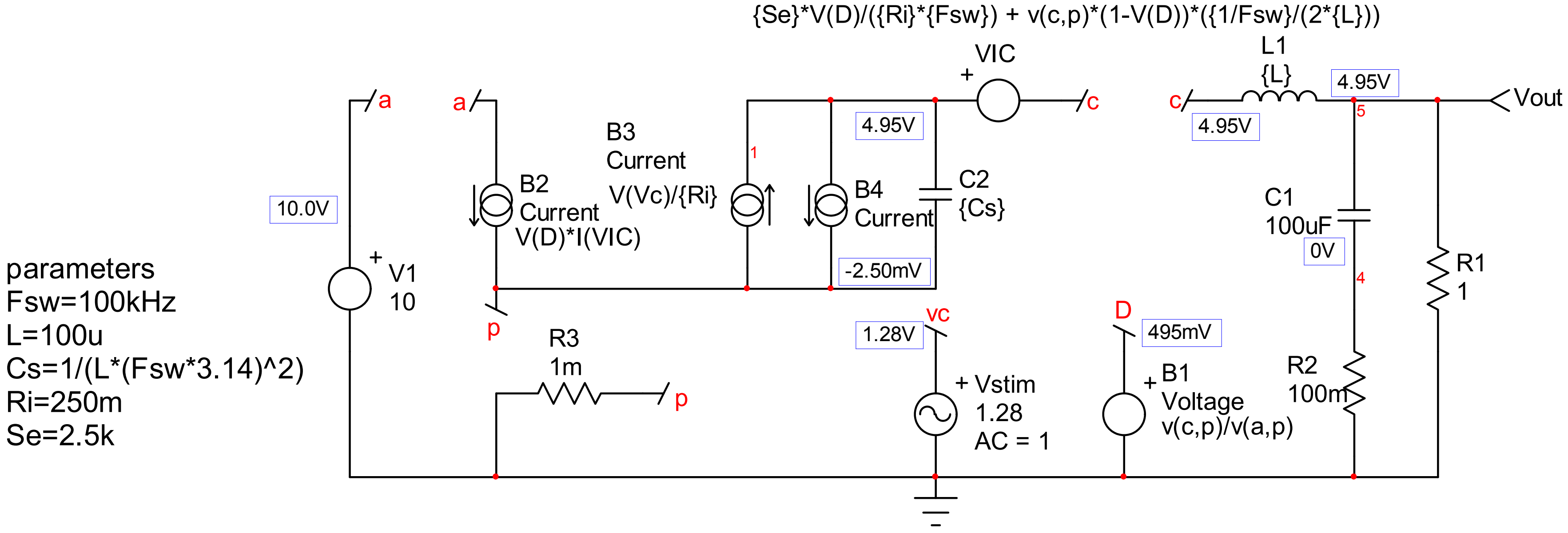
$g_f = -7.5\,\text{m}\Omega^{-1} \quad g_r = 0.49\,\Omega^{-1}$

$g_i = -250\,\text{m}\Omega^{-1}$

$G_0 = 12\,\text{dB} \qquad f_{z_1} = 15.9\,\text{kHz}$

See What SPICE is Saying

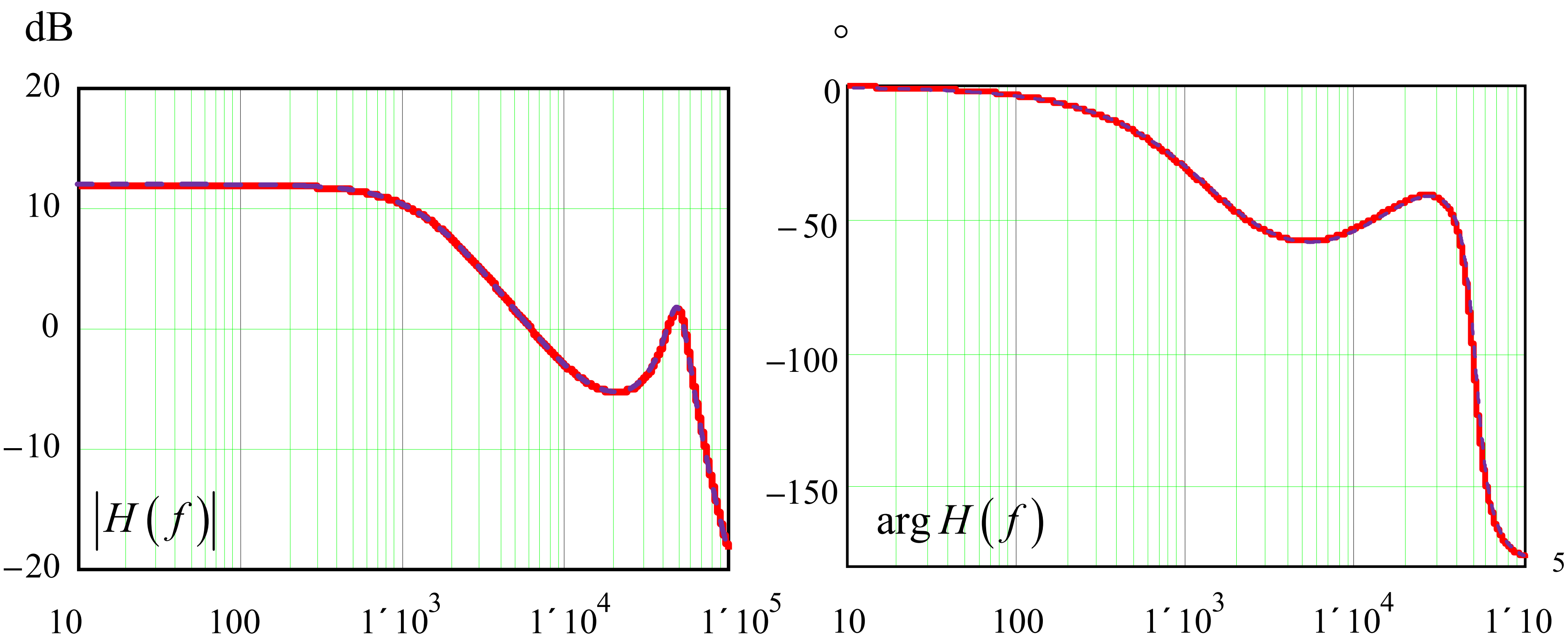
- ❑ Use the large-signal model, SPICE linearizes it for you



- ❑ Then compare results with those of Mathcad®

Excellent Agreement!

- Superimposed curves mean transfer functions are identical

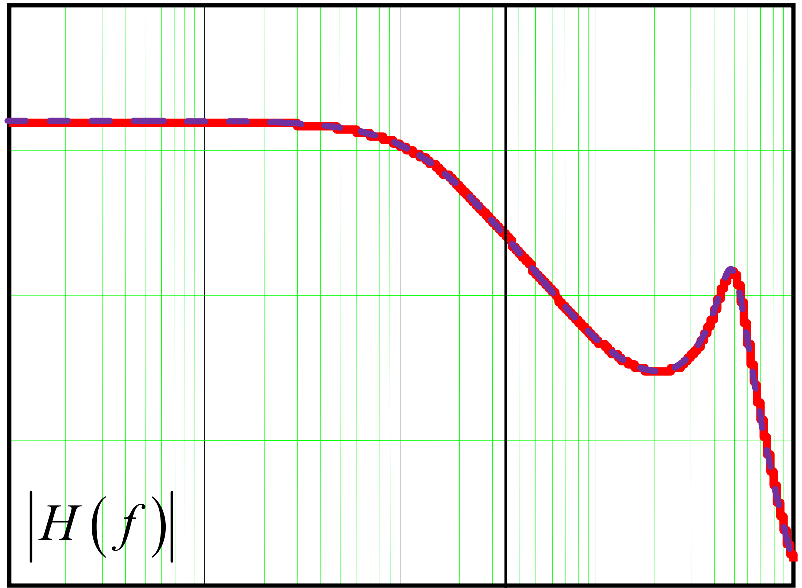


Rearranging Expressions

- The denominator is not really in a low-entropy form

$$D(s) = 1 + a_1 s + a_2 s^2 + a_3 s^3$$

- This is a third-order polynomial form that can be factored



$$D(s) \approx 1 + a_1 s$$

Low frequency ⟵ ⟶ High frequency

$$D(s) \approx 1 + \frac{a_2}{a_1} s + \frac{a_3}{a_1} s^2$$

Final Lap!

❑ The transfer function can now unveil peaking and damping

$$H(s) \approx H_0 \frac{1+\frac{s}{\omega_{z_1}}}{1+\frac{s}{\omega_{p_1}}} \frac{1}{1+\frac{s}{\omega_n Q}+\left(\frac{s}{\omega_n}\right)^2}$$

$$H_0 = \frac{R}{R_i} \frac{1}{1+\frac{RT_{sw}}{L}\left[m_c(1-D)-0.5\right]}$$

$$\omega_{p_1} = \frac{1}{RC}+\frac{T_{sw}}{LC}\left[m_c(1-D)-0.5\right]$$

$$\omega_{z_1} = \frac{1}{r_C C}$$

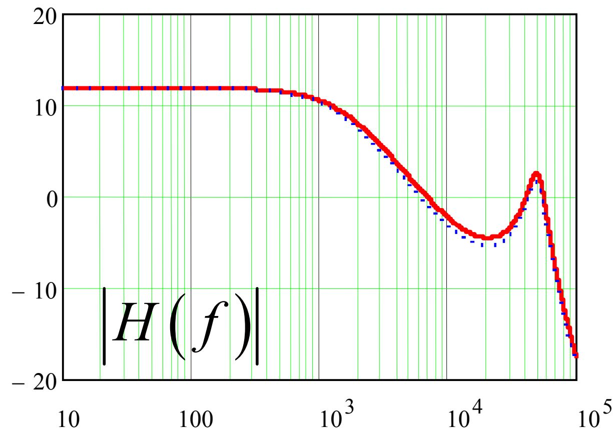
$$\omega_n = \frac{\pi}{T_{sw}}$$

$$Q = \frac{1}{\pi\left[m_c(1-D)-0.5\right]}$$

$$m_c = 1+\frac{S_e}{S_n}$$

S_e: Artificial ramp

S_n: Inductor on slope



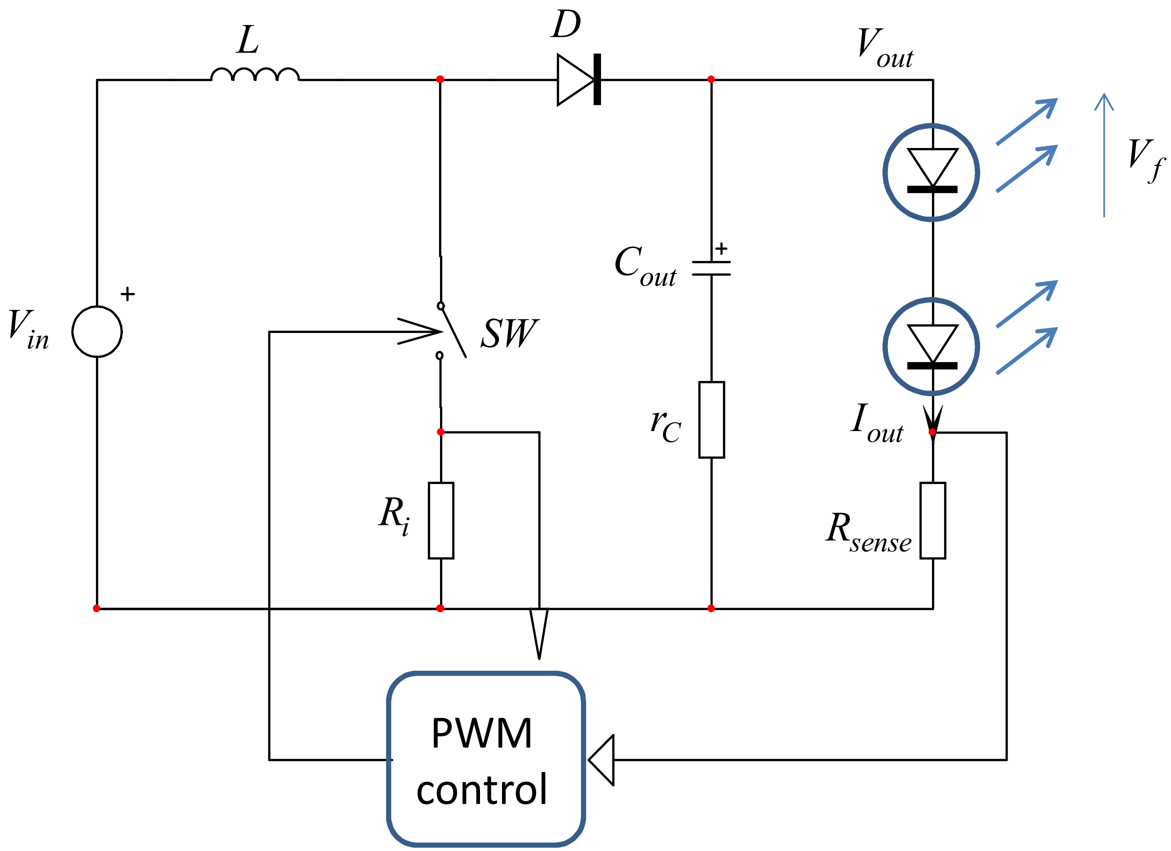
R. B. Ridley, “A new Continuous-Time Model for CM Control”, IEEE Transactions of Power Electronics, Vol. 6, April 1991

Course Agenda

- ❑ Introducing the PWM Switch Model
- ❑ CCM, DCM and BCM in Voltage Mode
- ❑ Pulse Width Modulator Gain
- ❑ The PWM Switch Model in Current Mode
- ❑ PWM Switch at Work in a Buck Converter
- ❑ **A Simplified Approach to Modeling a DCM Boost**
- ❑ Transfer Function of a BCM Boost in Current Mode
- ❑ Small-Signal Model of The Active Clamp Forward

A DCM Boost Converter

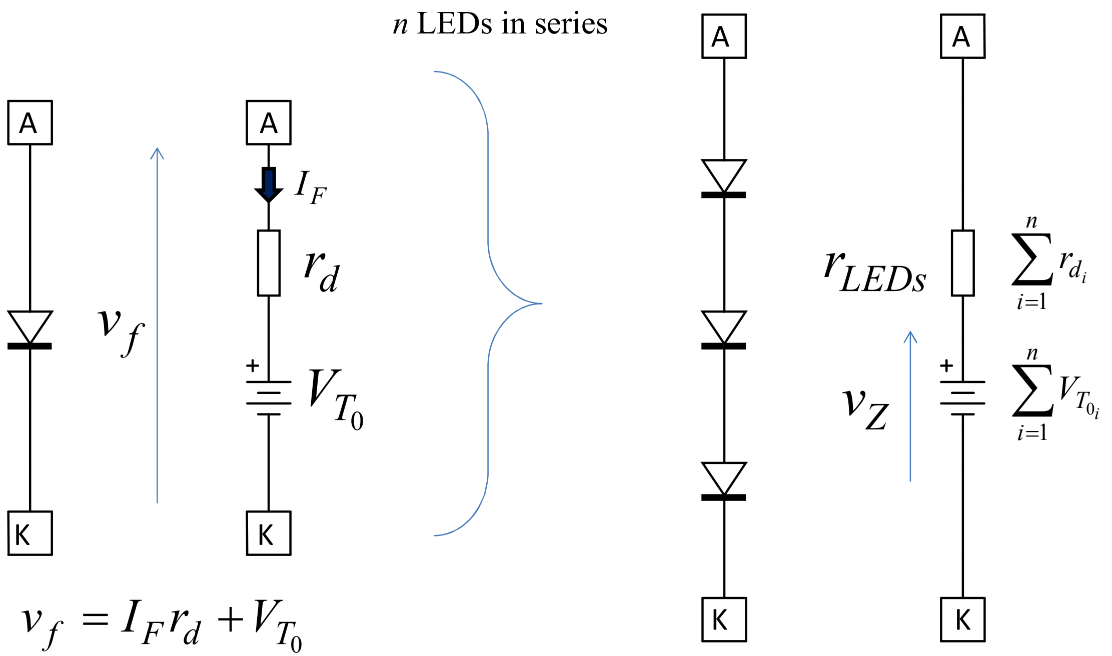
- The goal is to regulate the LED string current



- The LED current is sensed via a shunt element

Characterize the LED String

- ❑ Evaluate forward drop and dynamic resistance



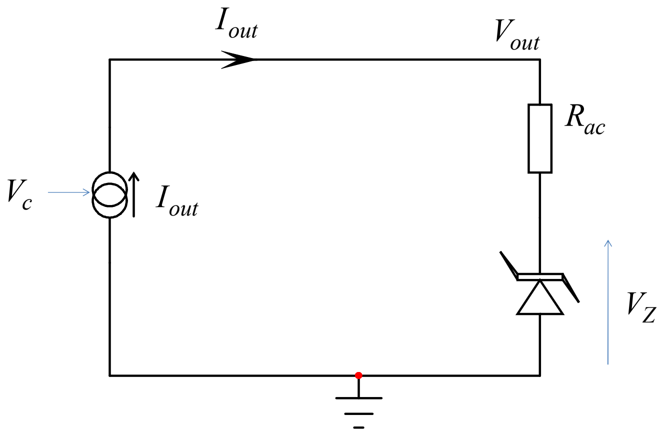
$$v_f = I_F r_d + V_{T_0}$$

- ❑ Lab. measurements require simple V and A-meters

$$r_{LEDs} = \frac{V_{f_1} - V_{f_2}}{I_{F_1} - I_{F_2}} = \frac{27.5 - 26.4}{0.1 - 0.08} = 55\,\Omega \qquad V_Z \approx V_{f_1} - R_{LEDs} I_{F_1} = 27.5 - 0.1 \times 55 = 22\,\text{V}$$

A Simplified Approach

- The LED string is driven by a current source



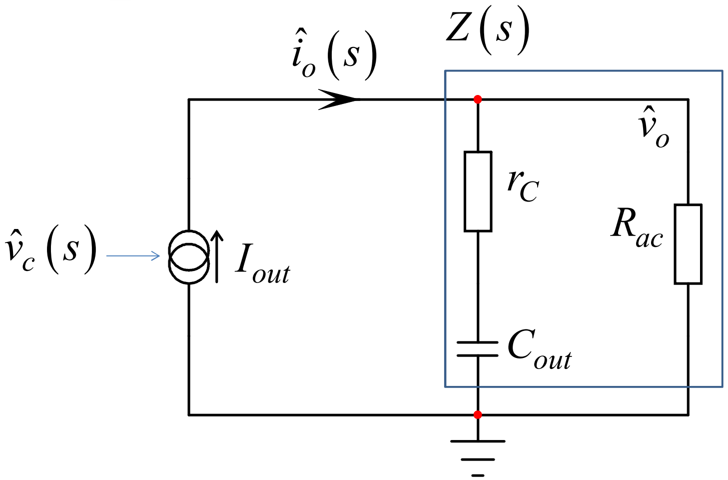
$$R_{ac} = r_{LEDs} + R_{sense}$$

$$V_{out} = R_{ac} I_{out} + V_z$$

⇩

$$V_{out}(s) = R_{ac} I_{out}(s)$$

- V_Z sets the operating point, R_{ac} sets the ac response



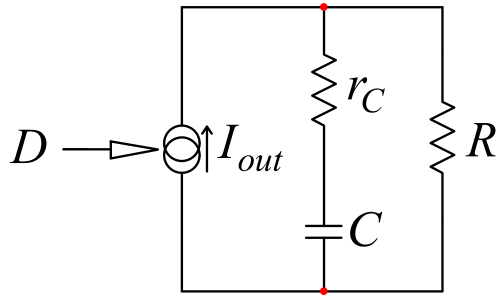
Current Source Model

- It is convenient and fast to consider 1st-order models
- Use the converter transfer function in DCM

$$\frac{V_{out}}{V_{in}} = \frac{1+\sqrt{1+\frac{2T_{sw}D^2R_{dc}}{L}}}{2}$$

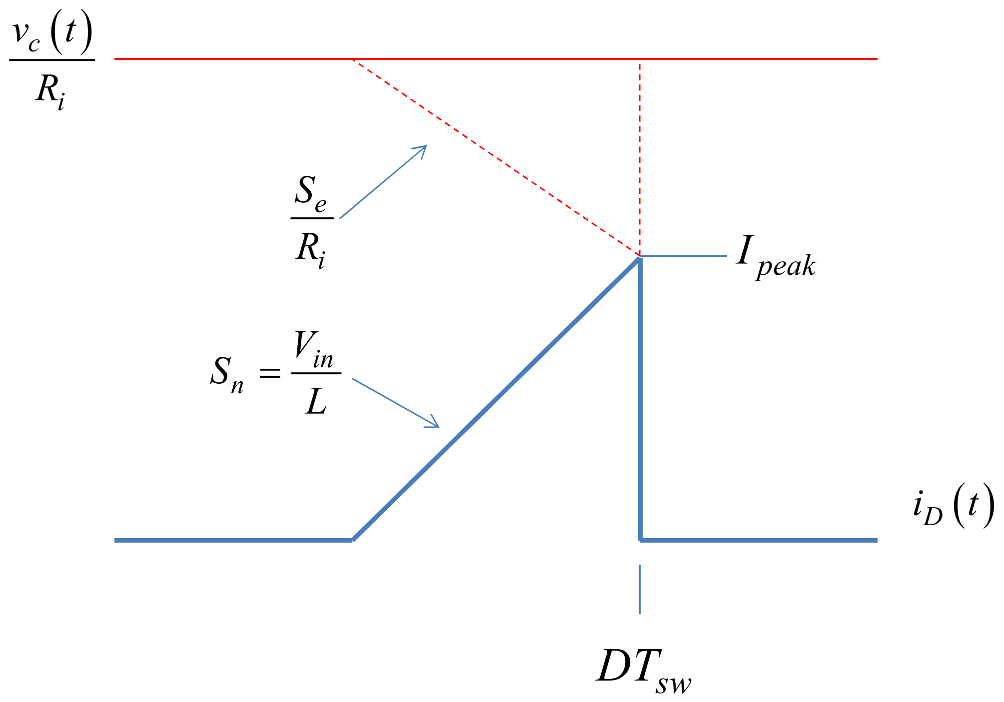
- We purposely consider an instantaneous power response
  - if D changes, P_{out} immediately translates
  - this is not true for boost or buck-boost converters: RHPZ
  - high-frequency phenomena are also lost

1st-order



Define the Duty Ratio

- In the previous expression, D is unknown



$$\left.\begin{aligned} I_{peak} &= \frac{V_c}{R_i} - \frac{S_e}{R_i} DT_{sw} \\ I_{peak} &= \frac{DT_{sw} V_{in}}{L} \end{aligned}\right\} \quad D = \frac{V_c L}{S_e T_{sw} L + R_i T_{sw} V_{in}}$$

Update the Output Current Equation

- Massage the dc transfer equation to unveil I_{out}

$$\frac{V_{out}}{V_{in}}=\frac{1+\sqrt{1+\dfrac{2T_{sw}D^2\dfrac{V_{out}}{I_{out}}}{L}}}{2}$$

- Inject the duty ratio definition, solve for I_{out}

$$I_{out}=\frac{2V_{out}LV_c^2}{T_{sw}}\frac{1}{\left[\left(\dfrac{2V_{out}}{V_{in}}-1\right)^2-1\right]\left(S_eL+R_iV_{in}\right)^2}$$

- There are two modulated variables, V_c and V_{out}

$$\left.\frac{\partial I_{out}}{\partial V_c}\right|_{\hat{v}_{out}}\hat{v}_c$$

$$\left.\frac{\partial I_{out}}{\partial V_{out}}\right|_{\hat{v}_c}\hat{v}_o$$

$\longrightarrow \hat{v}_{in}=0$

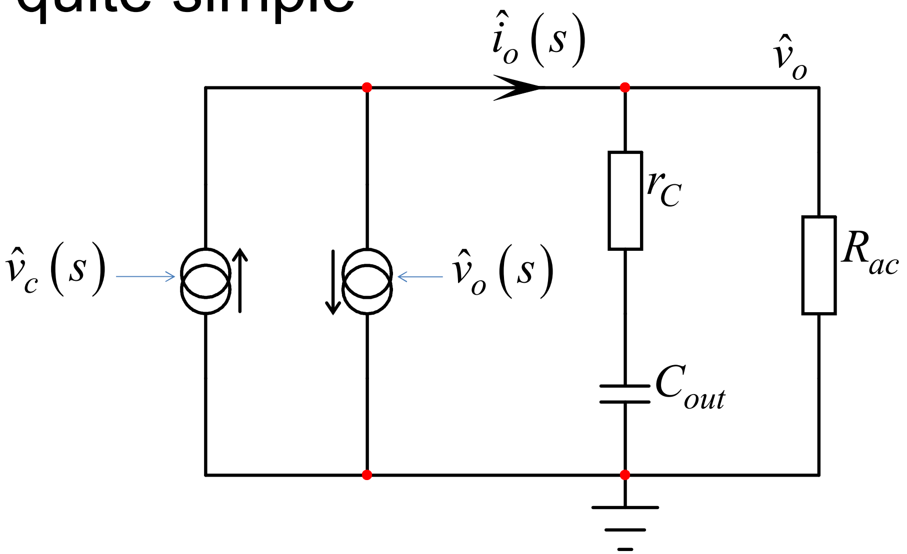
A Simple Model

- You obtain two small-signal sources: $\hat{i}_{out} = g_1\hat{v}_c + g_2\hat{v}_{out}$

$$g_1 = \frac{\partial I_{out}(V_c, V_{out})}{\partial V_c} = \frac{V_{in}^2 V_c L}{T_{sw}(V_{out} - V_{in})(S_e L + R_i V_{in})^2}$$

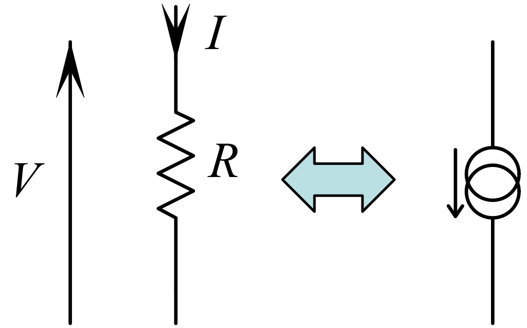
$$g_2 = \frac{\partial I_{out}(V_c, V_{out})}{\partial V_{out}} = -\frac{V_{in}^2 V_c^2 L}{2T_{sw}(V_{in} - V_{out})^2 (S_e L + R_i V_{in})^2}$$

- The circuit is quite simple



A Current Driving an Impedance

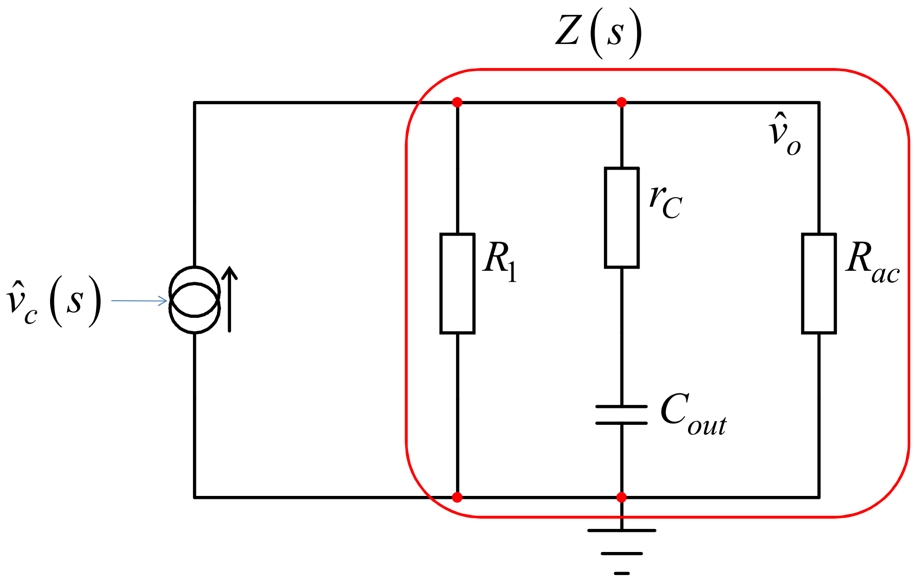
- The second term is a simple resistance



$$I = \frac{V}{R}$$

$$R_1 = \frac{2T_{sw}\left(V_{in} - V_{out}\right)^2\left(S_e L + R_i V_{in}\right)^2}{V_c^{\,2} V_{in}^{\,2} L}$$

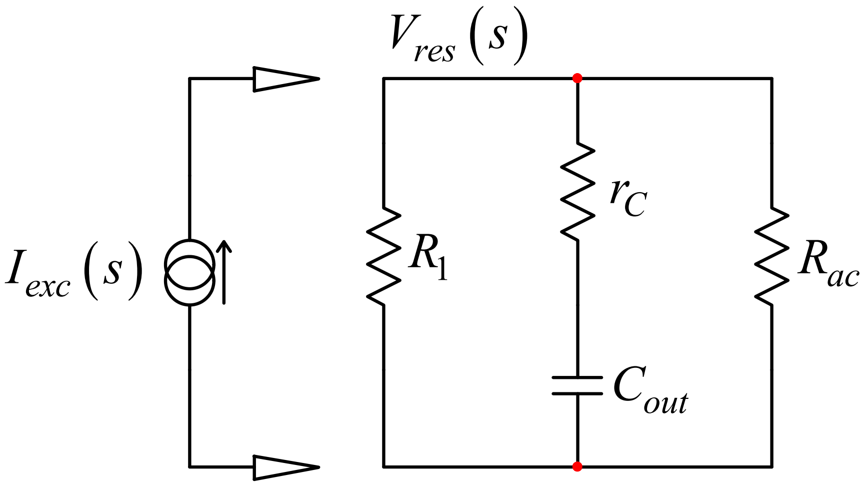
- Update the final model



$$V_{out}(s) = V_c(s)\, g_1 Z(s)$$

Use FACTS to Get the Impedance

- Obtain the impedance (transfer function) in a snapshot

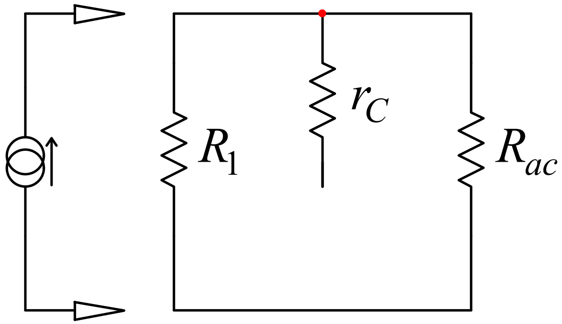


Response — Ω

$$Z(s) = \frac{V_{res}(s)}{I_{exc}(s)} = Z_0 \frac{1 + \frac{s}{s_{z_1}}}{1 + \frac{s}{s_{p_1}}}$$

Excitation

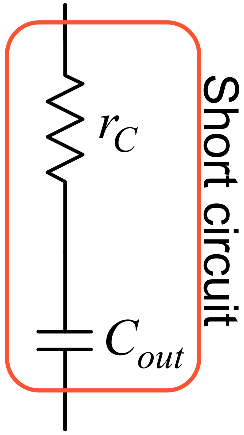
- For dc, open the capacitor



$$Z_0 = R_1 \| R_{ac}$$

No Algebra to Get the Result!

- At the zero frequency, the response disappears

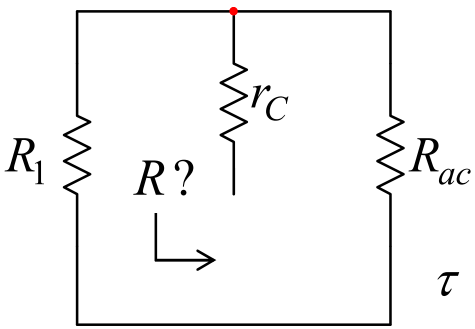


$$r_C + \frac{1}{sC_{out}} = \frac{sr_C C_{out} + 1}{sC_{out}}$$

$$sr_C C_{out} + 1 = 0$$

$$\omega_{z_1} = \frac{1}{r_C C_{out}}$$

- Remove the excitation and look at the resistance driving C_{out}



$$Z(s) = R_1 \| R_{ac} \frac{1 + sr_C C_{out}}{1 + sC_{out}\left(r_C + R_1 \| R_{ac}\right)}$$

$$\tau = \left[r_C + \left(R_1 \| R_{ac}\right)\right] C_{out}$$

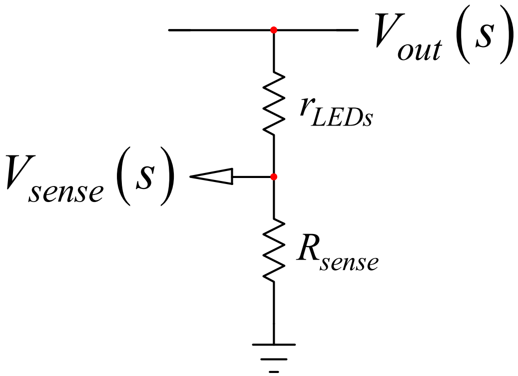
Final Expression

- Associate the impedance expression with g_1

$$\frac{V_{out}(s)}{V_c(s)} = H_0 \frac{1+\frac{s}{\omega_{z_1}}}{1+\frac{s}{\omega_{p_1}}} \qquad H_0 = \frac{V_{in}{}^2 V_c L}{T_{sw}\left(V_{out}-V_{in}\right)\left(S_e L + R_i V_{in}\right)^2}\left(R_{ac} \| R_1\right)$$

$$\omega_{z_1} = \frac{1}{r_C C_{out}} \qquad \omega_{p_1} = \frac{1}{C_{out}\left(r_C + R_1 \| R_{ac}\right)}$$

- However, we want the control-to-output current expression



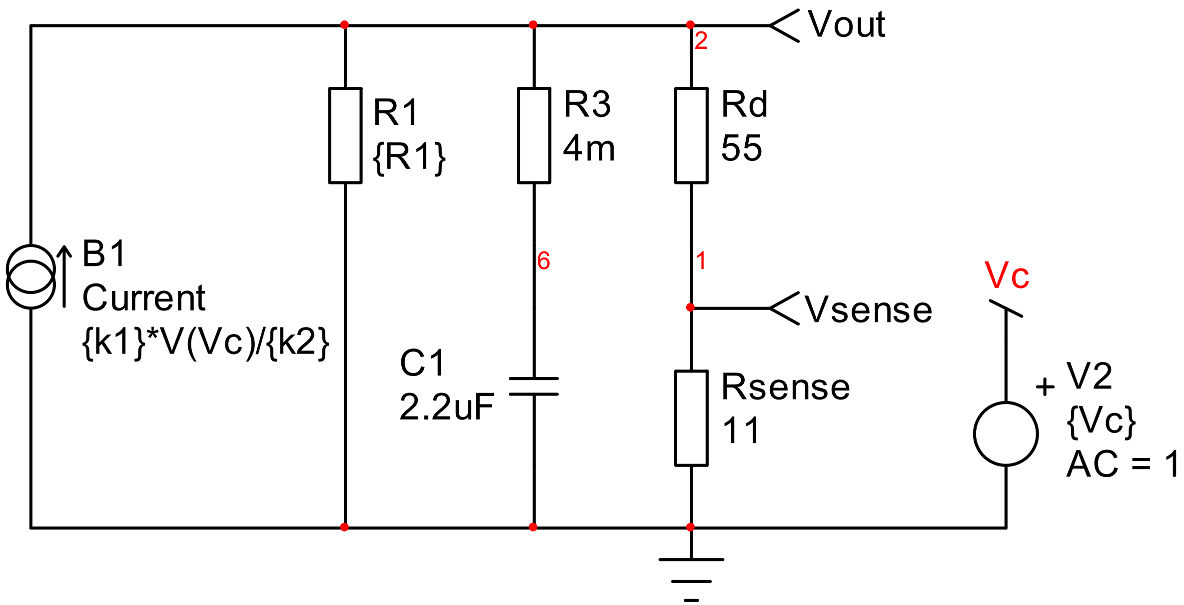
$$\frac{V_{sense}(s)}{V_c(s)} = \frac{R_{sense}}{R_{sense}+r_{LEDs}} H_0 \frac{1+\frac{s}{\omega_{z_1}}}{1+\frac{s}{\omega_{p_1}}}$$

Checking our Model Response

- ❑ Ac simulation of the current source-based approach

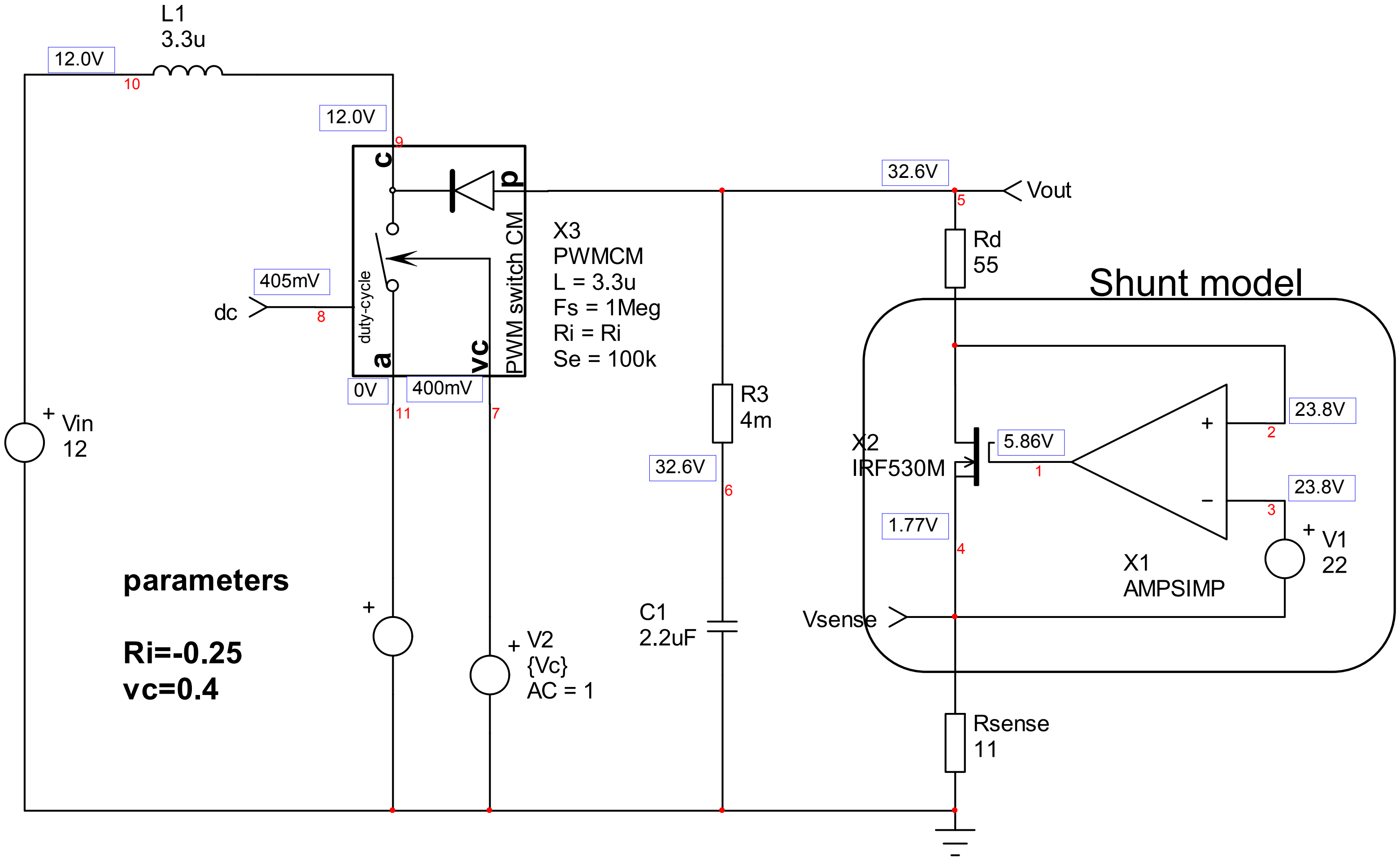
parameters

Ri=0.25
vc=0.4
Se=100k
Fsw=1Meg
L=3.3u
Tsw=1/Fsw
Vin=12
Vout=32.85
k1=Vin^2*vc*L
k2=Tsw*(Vout-Vin)*(Se*L+Ri*Vin)^2
R1=2*Tsw*(Vin-Vout)^2*(Se*L+Ri*Vin)^2/(Vc^2*(Vin^2)*L)



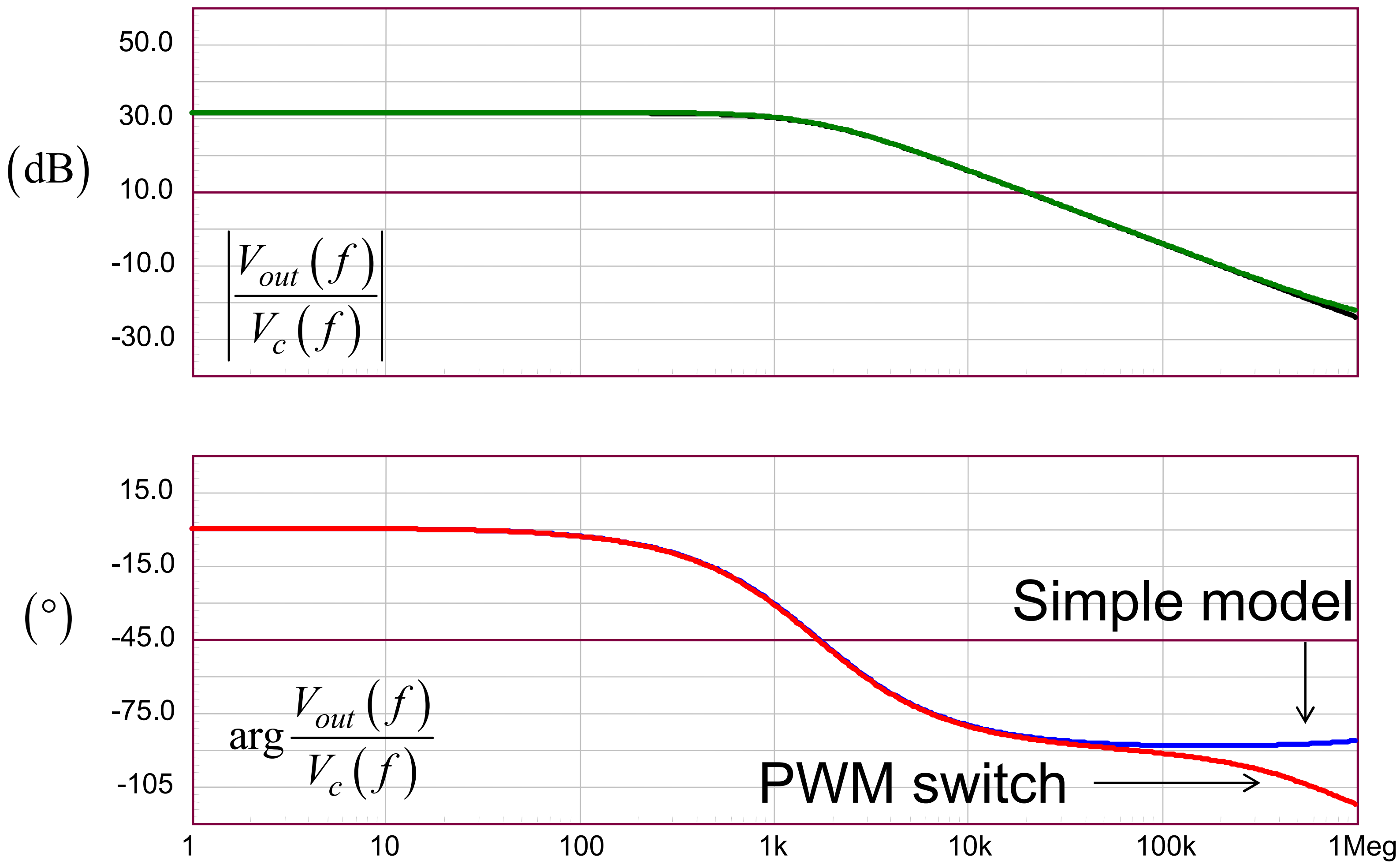
Simplified Approach vs PWM Switch

- The comprehensive model with the PWM switch



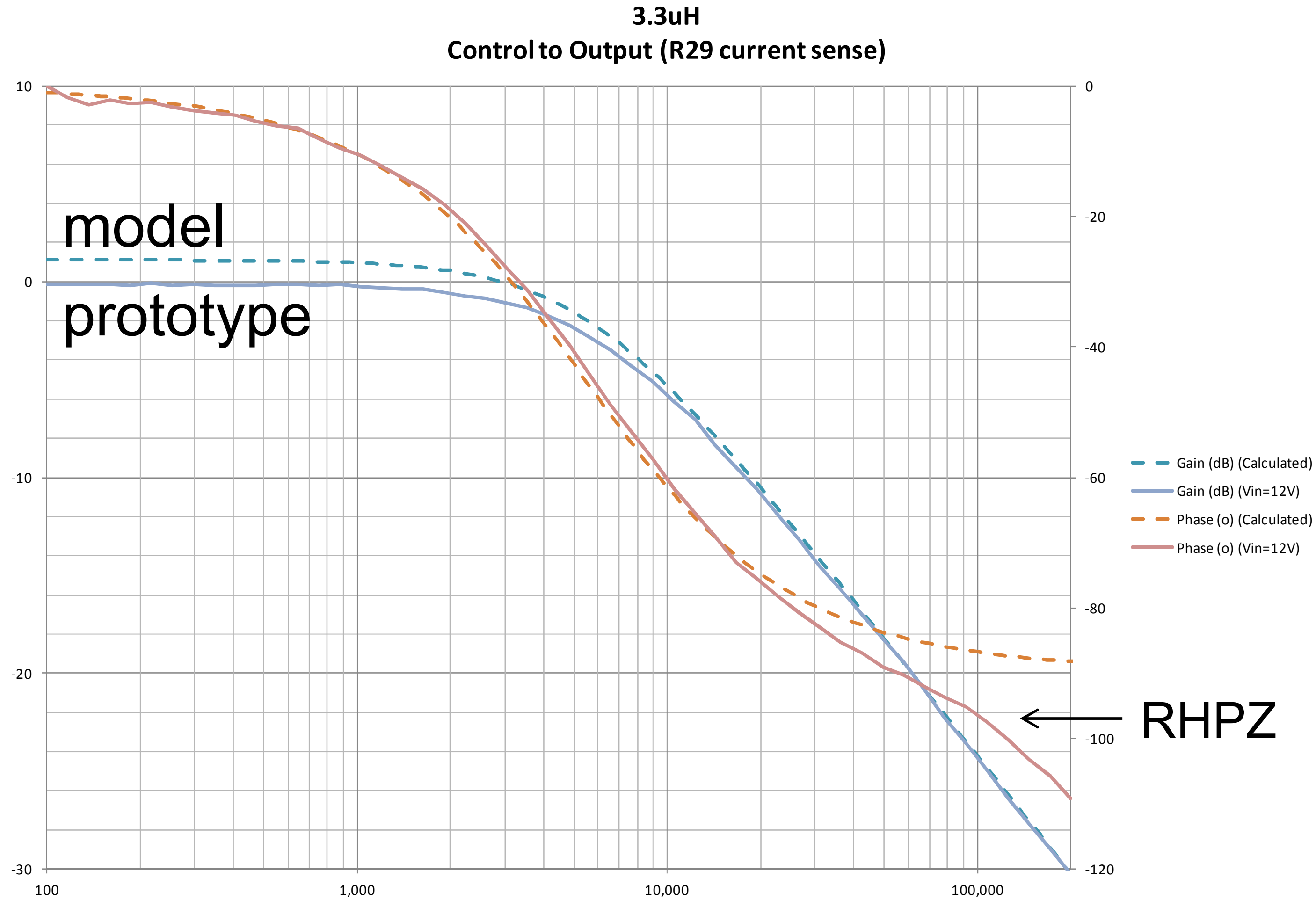
Final Results

- The RHPZ effect is not modeled in the simplified approach



Real Measurement vs Model

- Deviation occurs, as expected, because of the RHPZ



C. Basso, A. Laprade, "Simplified Analysis of a DCM Boost Converter Driving an LED String", www.how2power.com

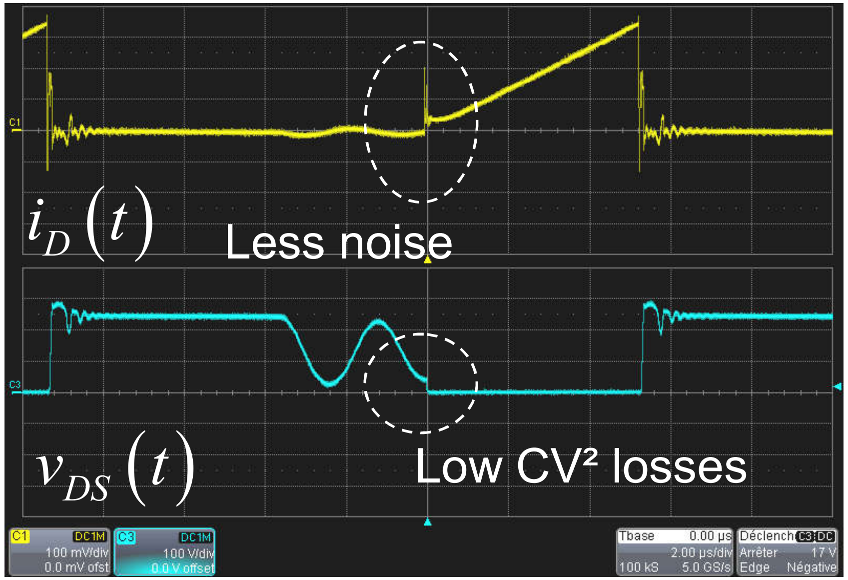
ON Semiconductor®

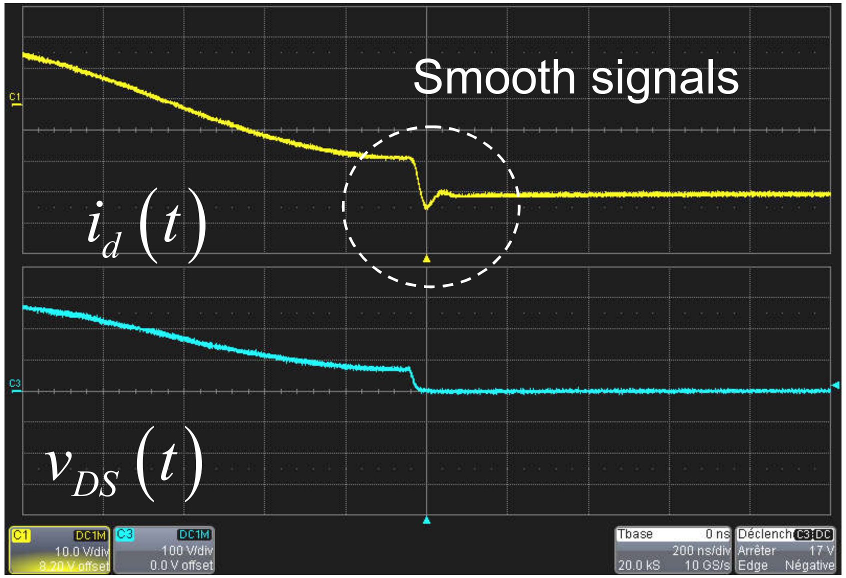
Course Agenda

- ❑ Introducing the PWM Switch Model
- ❑ CCM, DCM and BCM in Voltage Mode
- ❑ Pulse Width Modulator Gain
- ❑ The PWM Switch Model in Current Mode
- ❑ PWM Switch at Work in a Buck Converter
- ❑ A Simplified Approach to Modeling a DCM Boost
- ❑ **Transfer Function of a BCM Boost in Current Mode**
- ❑ Small-Signal Model of The Active Clamp Forward

Why Bordeline Operation?

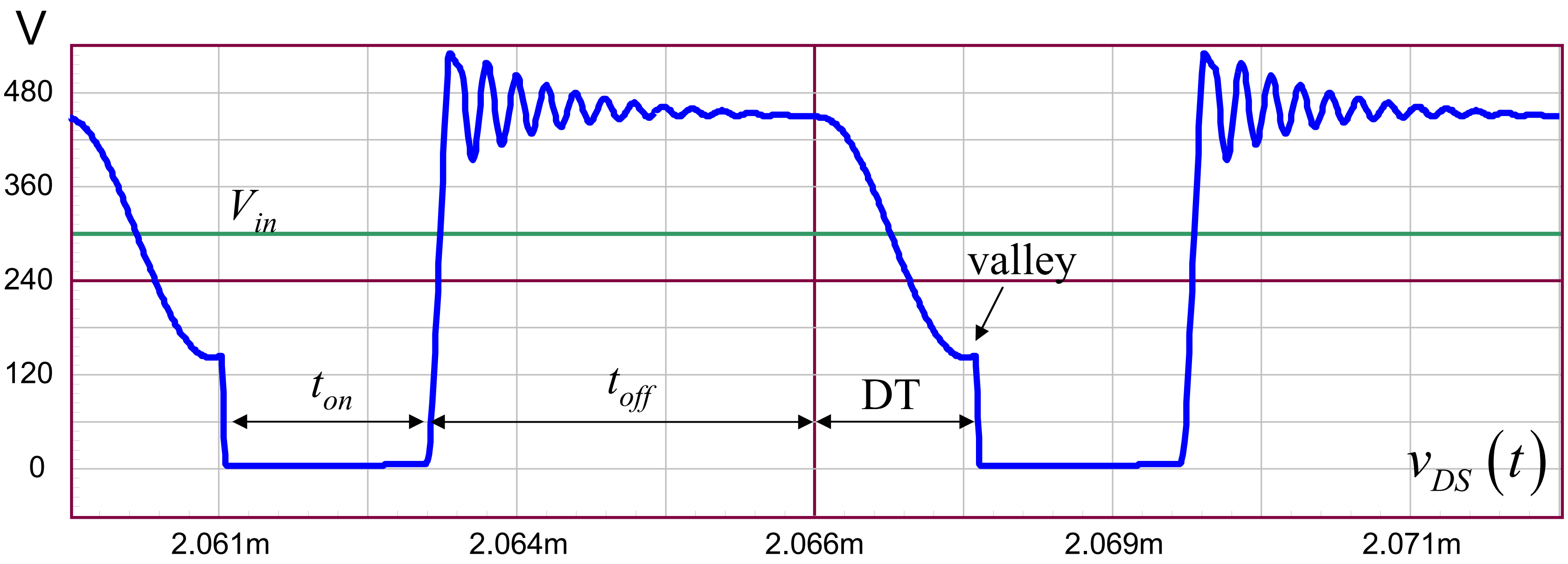
- ❑ More converters are using variable-frequency operation
- ❑ This is known as Quasi-Square Wave Resonant mode: QR
  - ➢ Valley switching ensures extremely low capacitive losses
  - ➢ DCM operation saves losses in the secondary-side diode
  - ➢ Easier synchronous rectification
  - ➢ The Right Half-Plane Zero is pushed to high frequencies





What is the Principle of Operation?

- ❑ The drain-source signal is made of peaks and valleys
- ❑ A valley presence means:
  - ➢ The drain is at a minimum level, capacitors are naturally discharged
  - ➢ The converter is operating in the discontinuous conduction mode

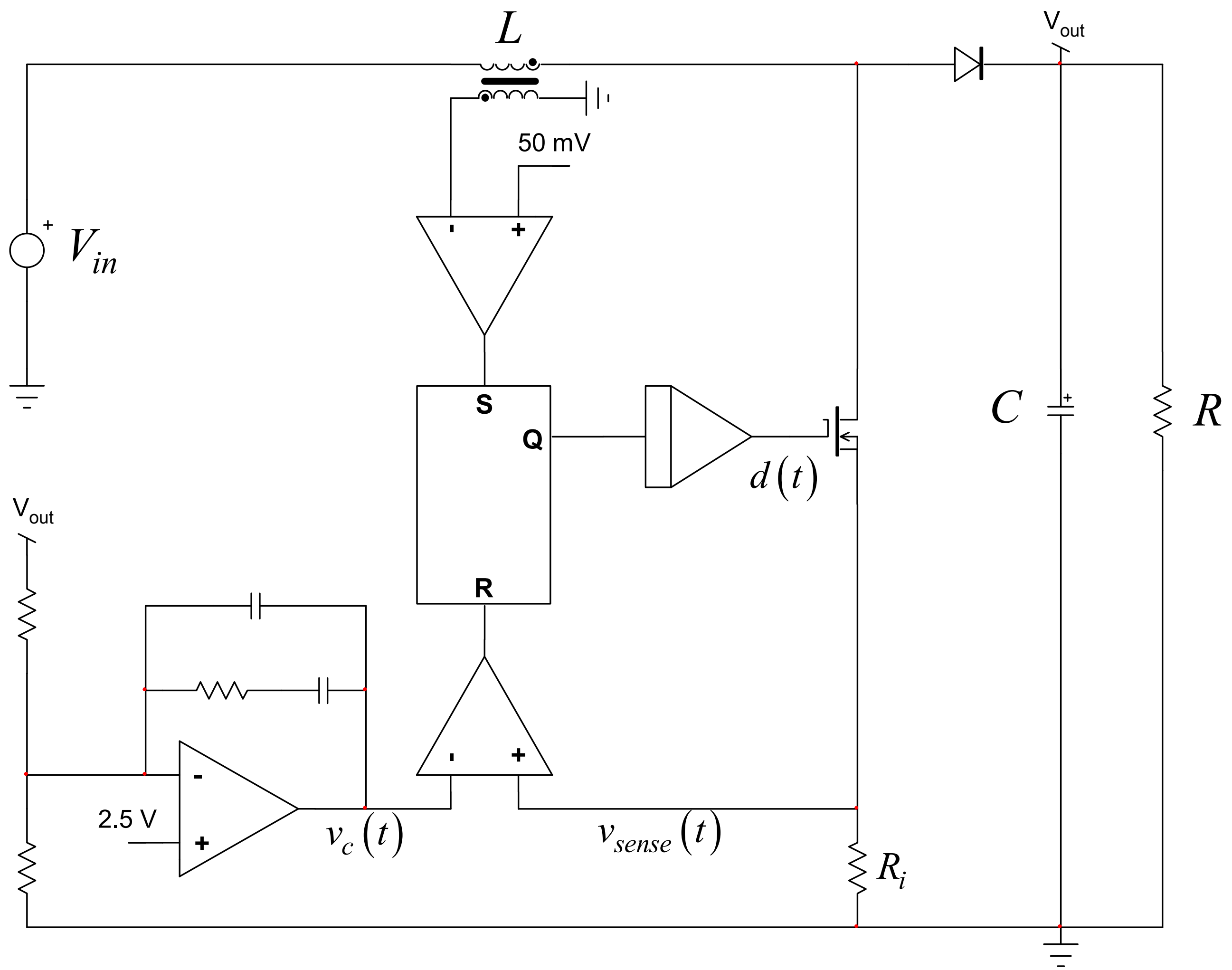


BCM = Borderline or Boundary Conduction Mode

Flyback structure

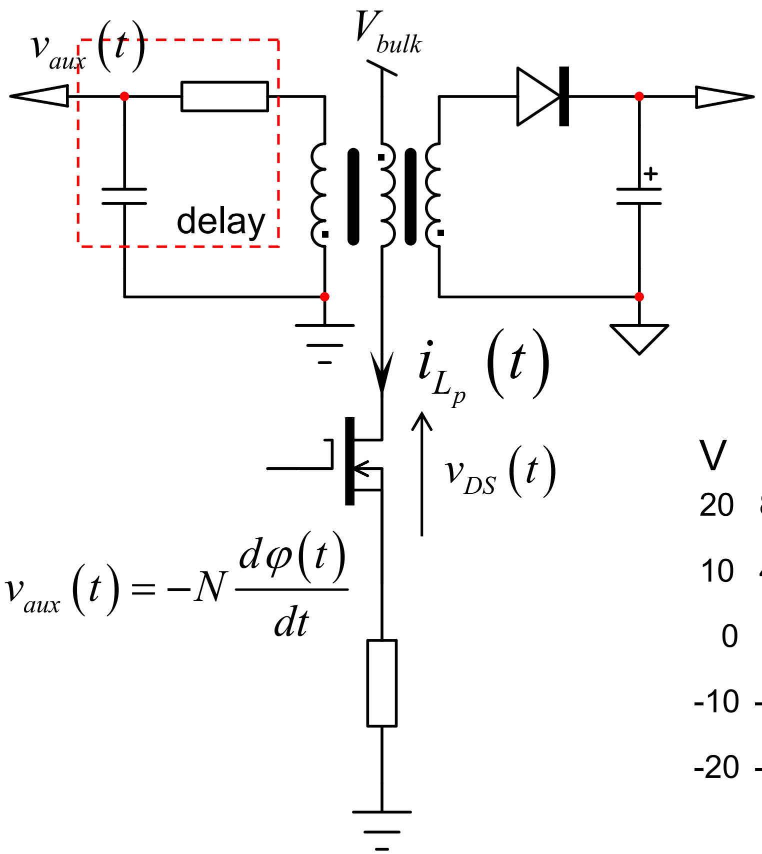
A QR Circuit Does not Need a Clock

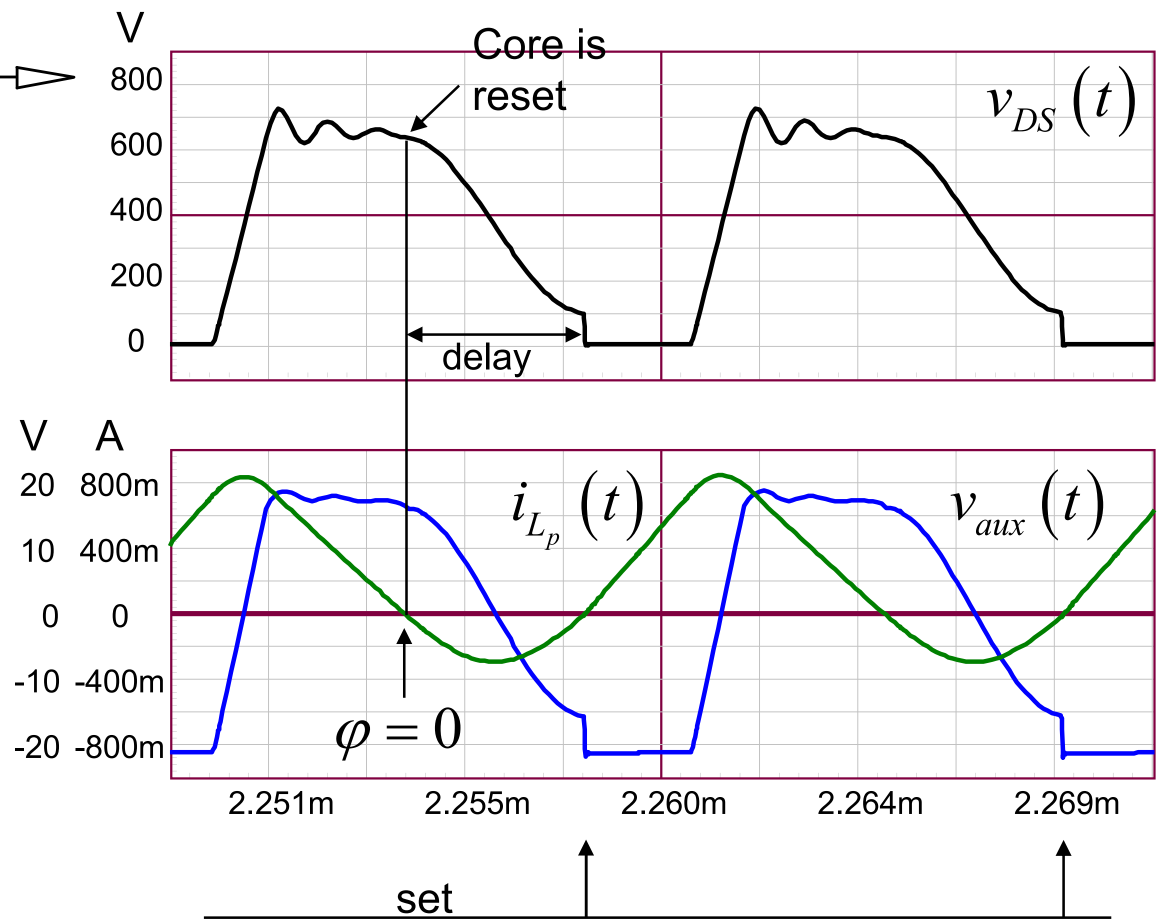
- The system is a self-oscillating current-mode converter



A Winding is Used to Detect Core Reset

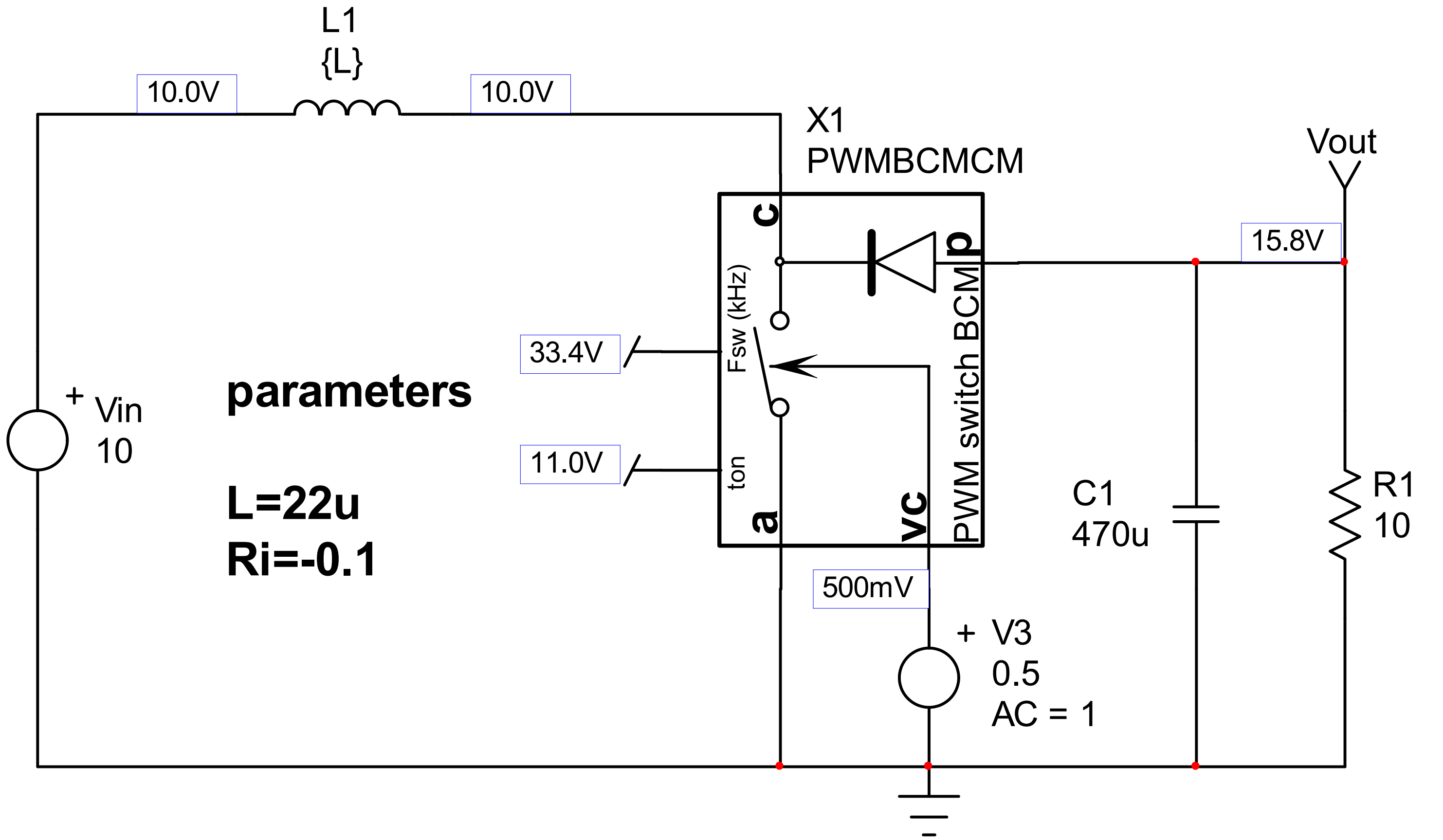
- When the flux returns to zero, the aux. voltage drops
- Discontinuous mode is always maintained





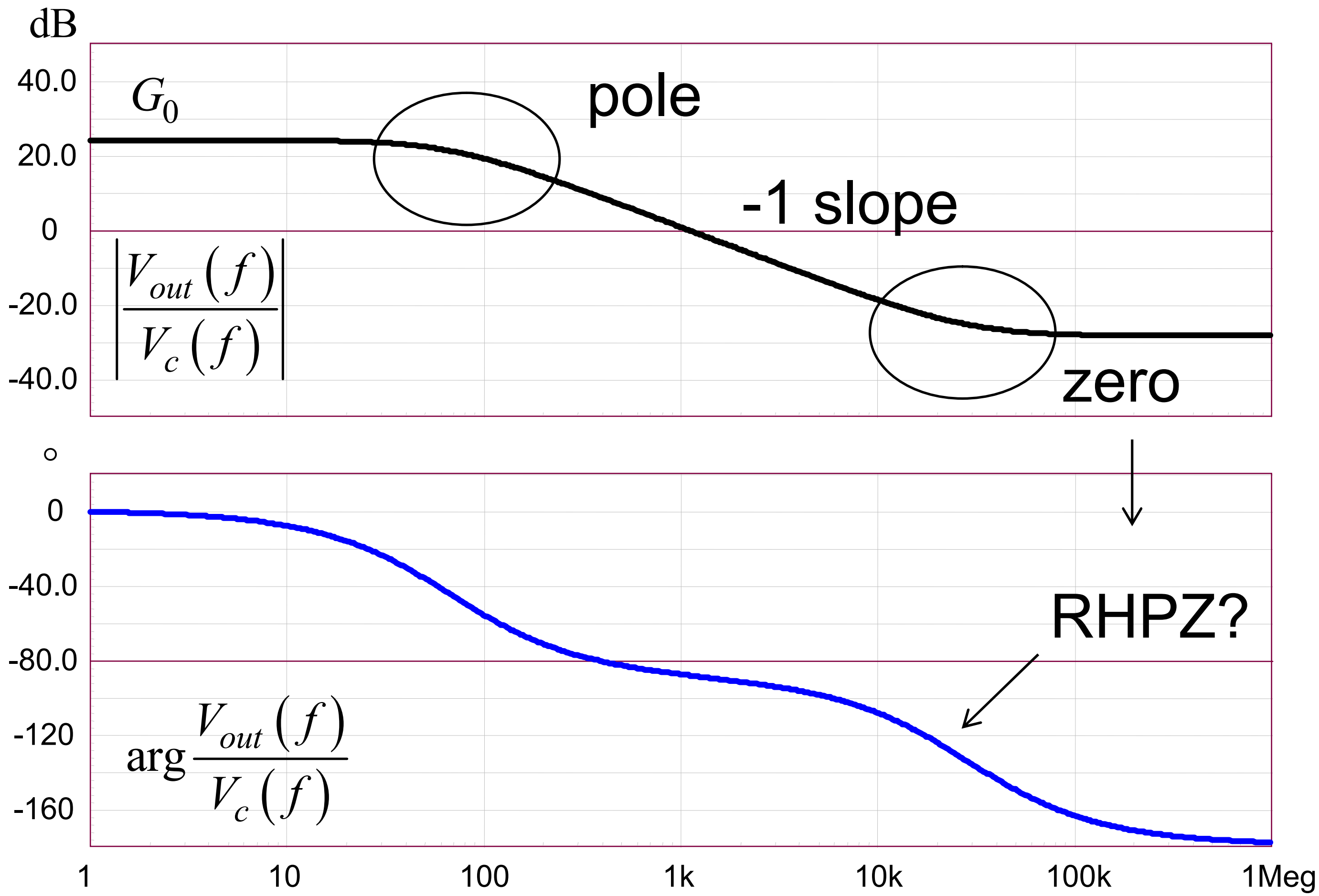
Test the Large-Signal Model Response

- ❑ Insert the PWM BCM CM model in the boost converter



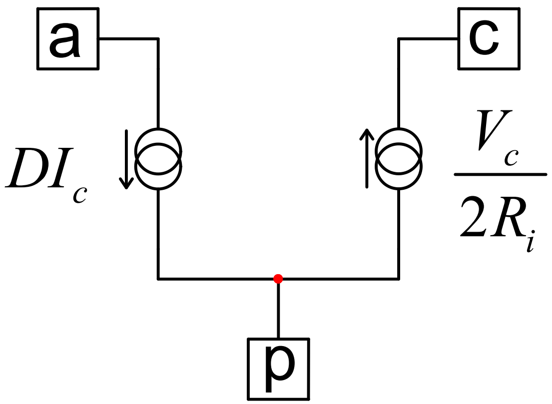
SPICE Gives Us the Response

- SPICE linearizes the model for us around the bias point



Derive a Small-Signal Model

- A Quasi Resonant model is built with the PWM switch model



$$D = \frac{V_c}{R_i}\frac{L}{V_{ac}T_{sw}} \qquad T_{sw} = \frac{V_c L}{R_i}\left(\frac{1}{V_{ac}} + \frac{1}{V_{cp}}\right)$$

$$I_a = \frac{V_c}{R_i}\frac{L}{V_{ac}T_{sw}} I_c = \frac{I_c}{V_{ac}\left(\frac{1}{V_{ac}} + \frac{1}{V_{cp}}\right)}$$ 3 variables

$$I_c = \frac{V_c}{2R_i}$$ 1 variable

- These are large-signal equations that need linearization

$$I_c = f\left(V_c\right) \longrightarrow \hat{i}_c = \frac{\partial I_c\left(V_c\right)}{\partial V_c}\hat{v}_c \qquad \hat{i}_c = \hat{v}_c\left(\frac{1}{2R_i}\right) = \hat{v}_c k_c \qquad k_c = \frac{1}{2R_i}$$

$$\hat{i}_a = \left.\frac{\partial I_a\left(V_{cp}, V_{ac}, I_c\right)}{\partial V_{cp}}\right|_{I_c, V_{ac}} \hat{v}_{cp} + \left.\frac{\partial I_a\left(V_{cp}, V_{ac}, I_c\right)}{\partial I_c}\right|_{V_{ap}, V_{ac}} \hat{i}_c + \left.\frac{\partial I_a\left(V_{cp}, V_{ac}, I_c\right)}{\partial V_{ac}}\right|_{I_c, V_{cp}} \hat{v}_{ac}$$

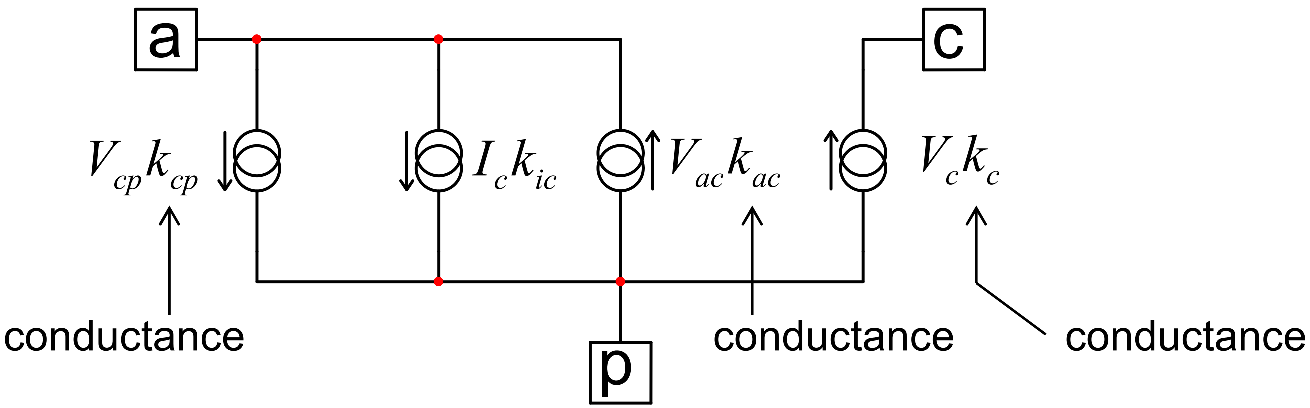
Large to Small-Signal

- Final steps before the small-signal model

$$\hat{i}_a = \hat{v}_{cp} \frac{I_{c0} V_{ac0}}{\left(V_{ac0} + V_{cp0}\right)^2} + \frac{V_{cp0}}{V_{cp0} + V_{ac0}} \hat{i}_c - \hat{v}_{ac} \frac{V_{cp0} I_{c0}}{\left(V_{ac0} + V_{cp0}\right)^2}$$

$$k_{cp} = \frac{I_{c0} V_{ac0}}{\left(V_{ac0} + V_{cp0}\right)^2} \qquad k_{ic} = \frac{V_{cp0}}{V_{cp0} + V_{ac0}} \qquad k_{ac} = \frac{V_{cp0} I_{c0}}{\left(V_{ac0} + V_{cp0}\right)^2}$$

- A small-signal model can now be assembled



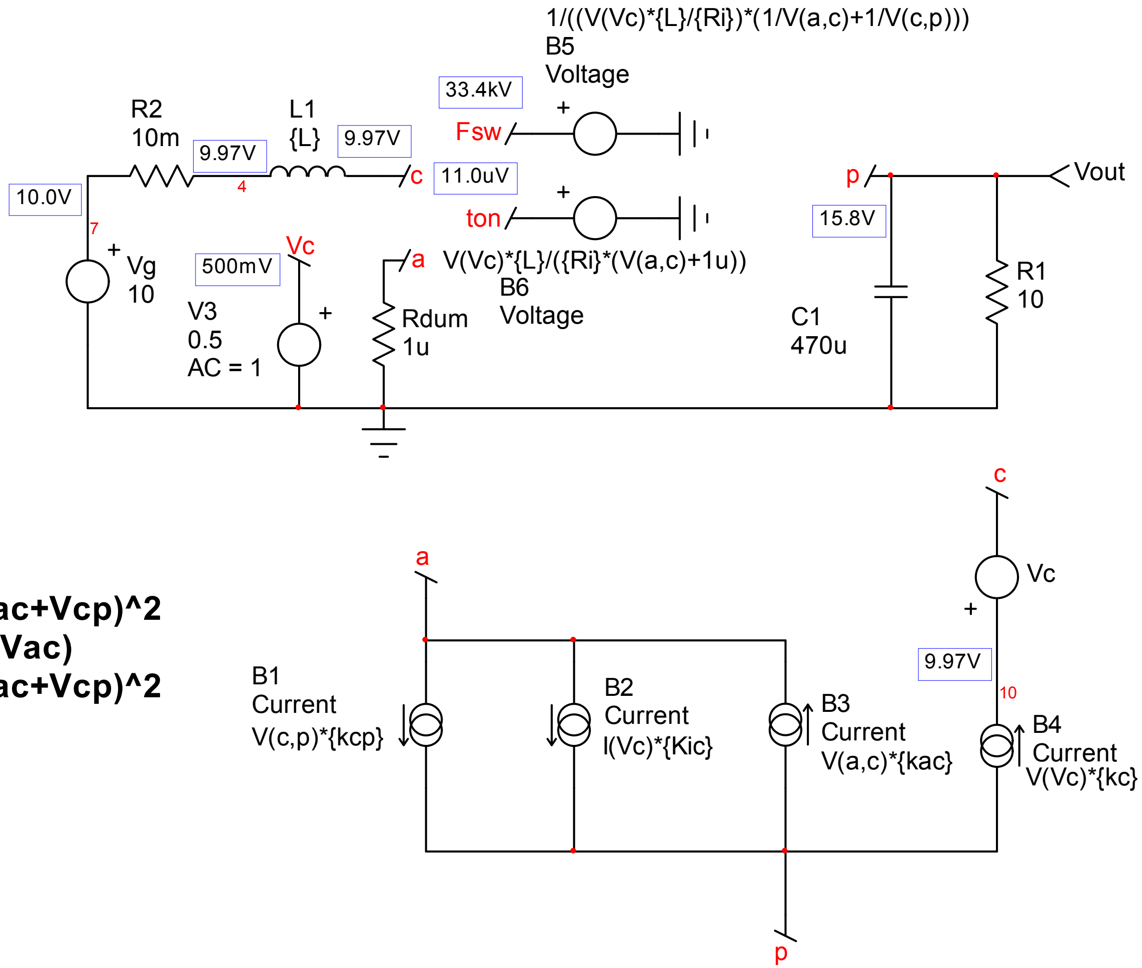
Always Check the Small-Signal Response

- Always verify if coefficients are well derived!

parameters

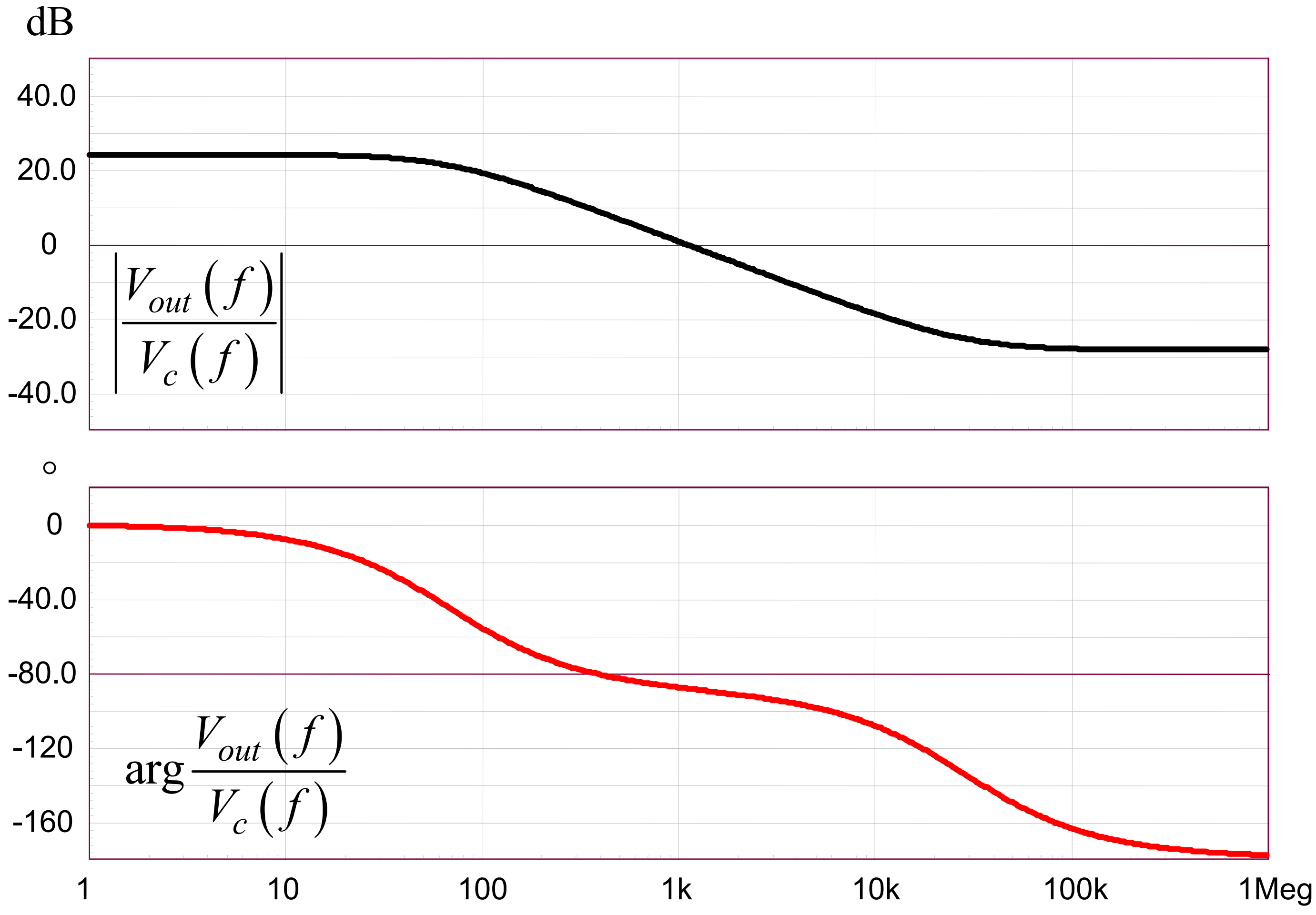
L=22u
Ri=-0.1

Vin=10
Vout=15.6
M=Vout/Vin
Ic=-2.5
Vac=-10
Vap=-Vout
Vcp=Vin-Vout
kcp=Ic*Vac/(Vac+Vcp)^2
kic=Vcp/(Vcp+Vac)
kac=Vcp*Ic/(Vac+Vcp)^2
kc=1/(2*Ri)



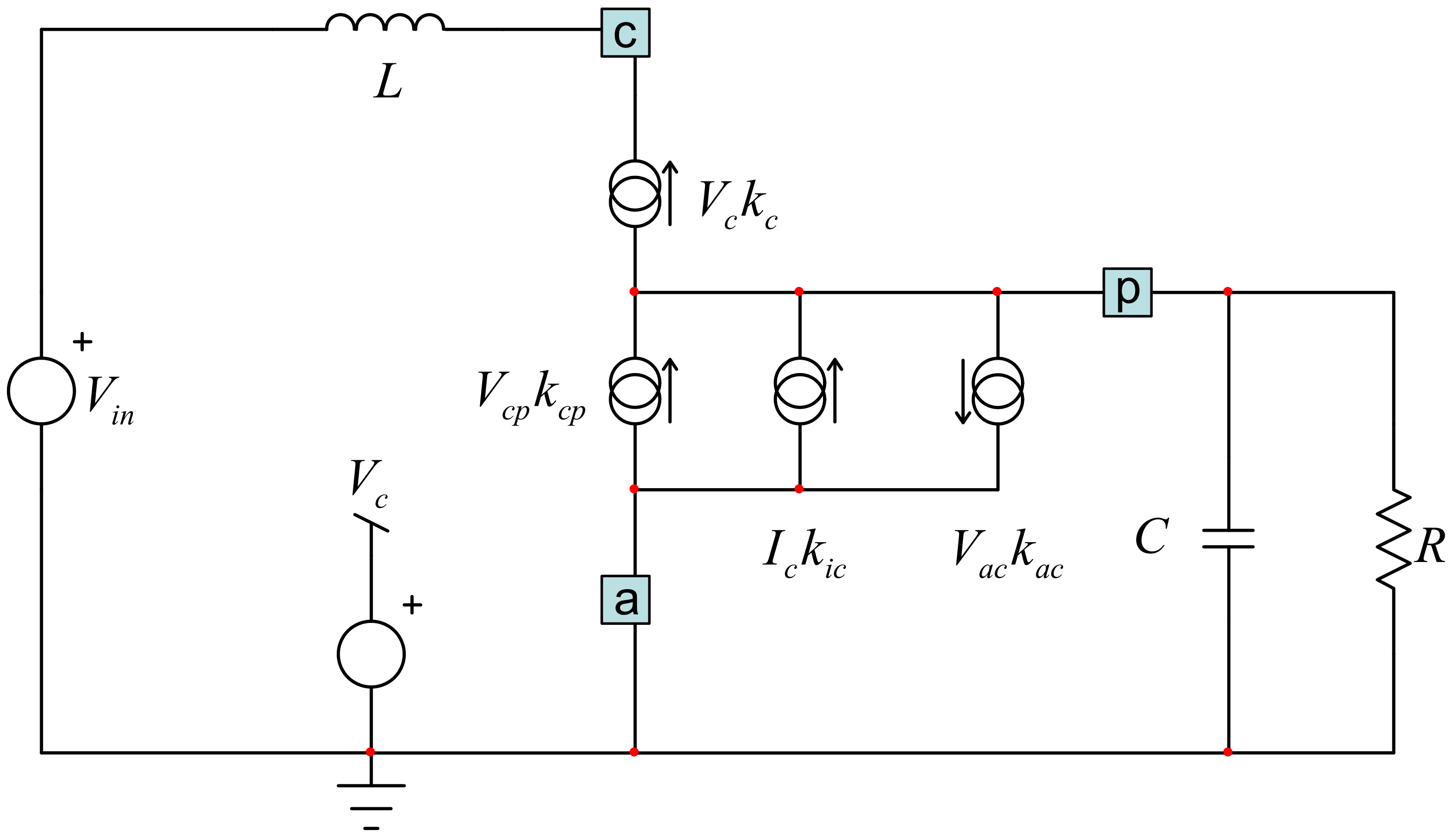
A Simple Intermediate Sanity Check

- Same response as with the non-linear model: good to go



The Model at Work in the BCM Boost

- Replace the switch/diode in the boost configuration



- In the original model, I_c leaves node c. It enters it in a boost.

Identify Static Parameters in the Coefficients

- Depending on configurations, update variables in coefficients

$$V_{cp0} = V_{in} - V_{out}$$

$$k_{cp} = \frac{I_{out}V_{out}}{V_{in}} \frac{V_{in}}{\left(-V_{in} + V_{in} - V_{out}\right)^2}$$

$$I_{c0} = -\frac{I_{out}V_{out}}{V_{in}}$$

$$k_{ac} = -\frac{I_{out}V_{out}}{V_{in}} \frac{\left(V_{in} - V_{out}\right)}{\left(-V_{in} + V_{in} - V_{out}\right)^2}$$

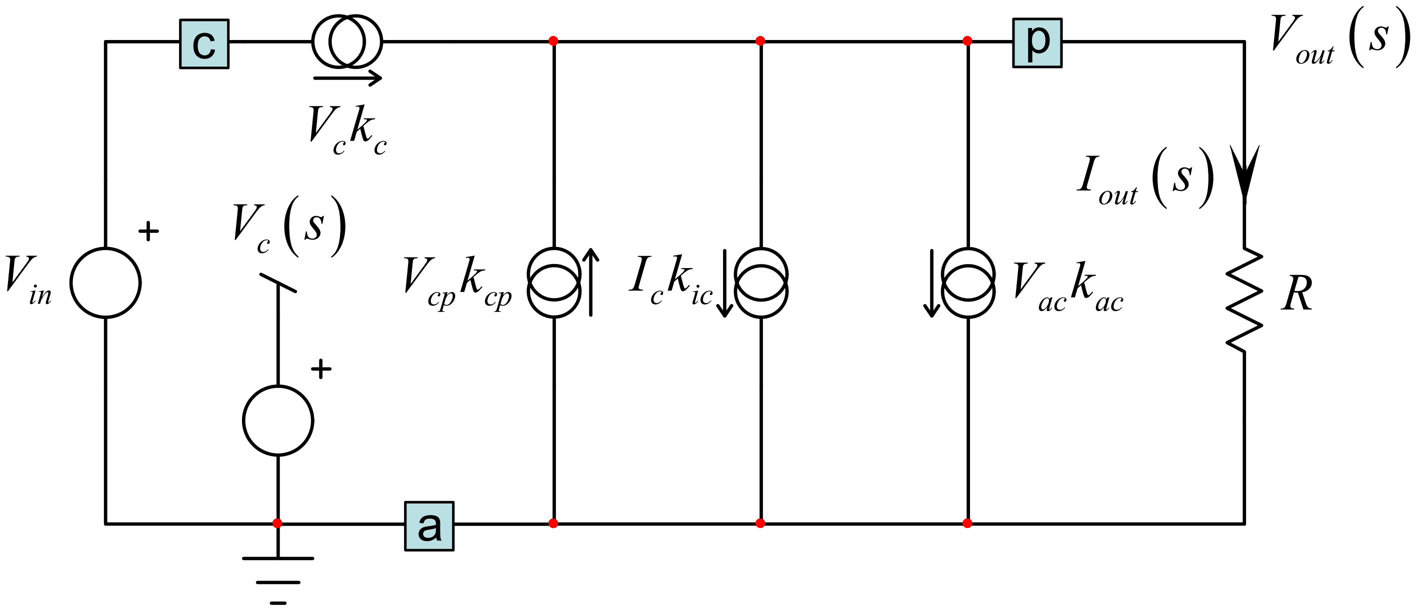
$$V_{ac0} = -V_{in}$$

$$k_{ic} = \frac{V_{in} - V_{out}}{V_{in} - V_{out} - V_{in}}$$

$$k_{cp} = \frac{1}{R} \qquad k_{ac} = \frac{M}{R} - \frac{1}{R} \qquad k_{ic} = 1 - \frac{1}{M} \qquad k_c = \frac{1}{2R_i}$$

First Step is Dc Gain

- Open the output capacitor, short the inductor



- Write output voltage expression

$$I_{out}(s) = V_c(s)k_c + V_{cp}(s)k_{cp} - I_c(s)k_{ic} - V_{ac}(s)k_{ac}$$

$$\longrightarrow \quad I_{out}(s) = V_c(s)k_c + \left(\cancel{V_{(c)}}(s) - V_{(p)}(s)\right)k_{cp} - V_c(s)k_c k_{ic} - \left(\cancel{V_{(a)}}(s) - \cancel{V_{(c)}}(s)\right)k_{ac}$$

Dc Gain Derivation

- There is no contribution from the source, $V_{in}(s) = 0$

$$I_{out}(s) = V_c(s)k_c - V_{out}(s)k_{cp} - V_c(s)k_c k_{ic} = V_c(s)k_c(1-k_{ic}) - V_{out}(s)k_{cp}$$

$$I_{out}(s) = \frac{V_{out}(s)}{R} \longrightarrow V_c(s)k_c(1-k_{ic}) = V_{out}(s)\left(k_{cp} + \frac{1}{R}\right)$$

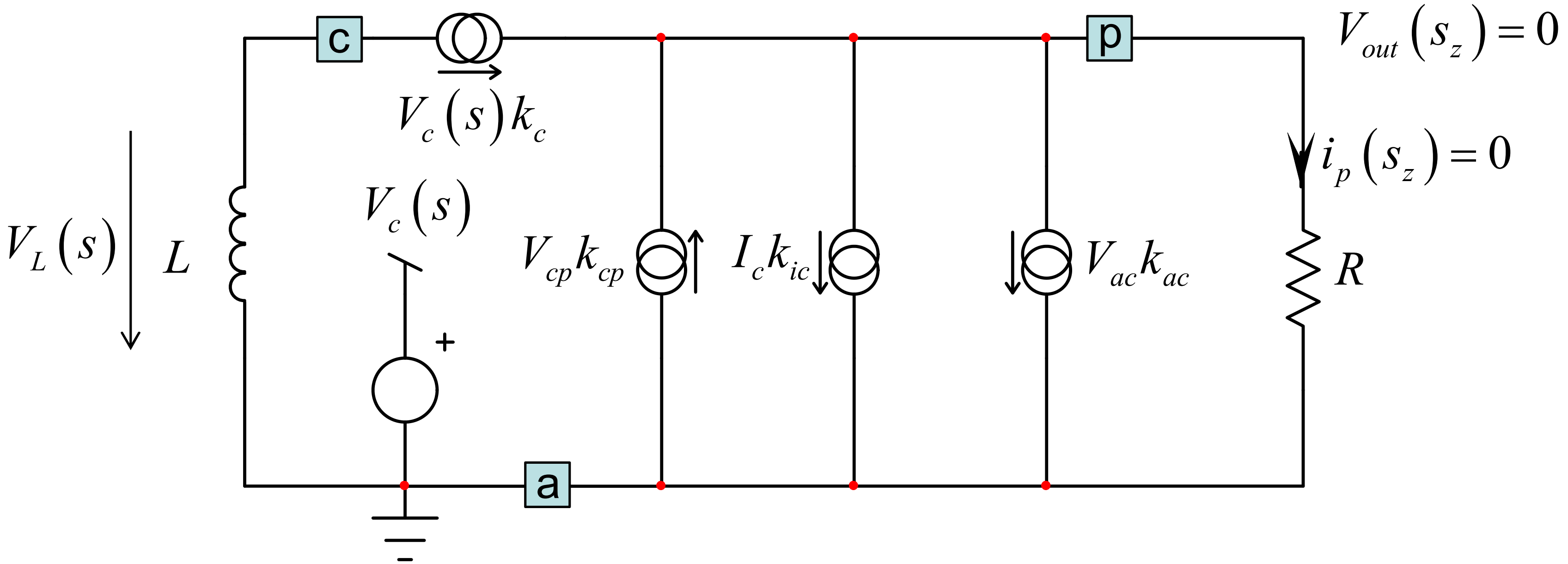
$$\frac{V_{out}(s)}{V_c(s)} = \frac{k_c(1-k_{ic})}{k_{cp} + \frac{1}{R}} \Rightarrow H_0 = \frac{R}{4MR_i}$$

- We have the first term of our transfer function

$$\frac{V_{out}(s)}{V_c(s)} = H_0 \frac{1 + \frac{s}{s_{z_1}}}{1 + \frac{s}{s_{p_1}}}$$

Deriving the Zero Position

- ❑ For the zero, the stimulus does not reach the output
  - ➢ current in the resistance is 0, node p voltage is also 0



- ❑ All the inductor ac current is absorbed before reaching R

$$V_c(s)k_c + \left[V_{(c)}(s) - V_{(p)}(s)\right]k_{cp} = V_c(s)k_c k_{ic} + \left[V_{(a)}(s) - V_{(c)}(s)\right]k_{ac}$$

What is the Root to $N(s)$ = 0?

- The voltage at node c depends on the inductance

$$V_{(c)}(s) = -V_L(s) = -I_c(s)sL = -V_c(s)k_c sL$$

- Substitute in the previous equations and simplify

$$V_c(s)k_c - V_c(s)k_c sLk_{cp} = V_c(s)k_c k_{ic} + V_c(s)k_c sLk_{ac}$$

- Solve for s, this is the zero position

$$1 - k_{ic} = sL\left(k_{ac} + k_{cp}\right) \longrightarrow s_{z_1} = \frac{1 - k_{ic}}{L\left(k_{ac} + k_{cp}\right)}$$

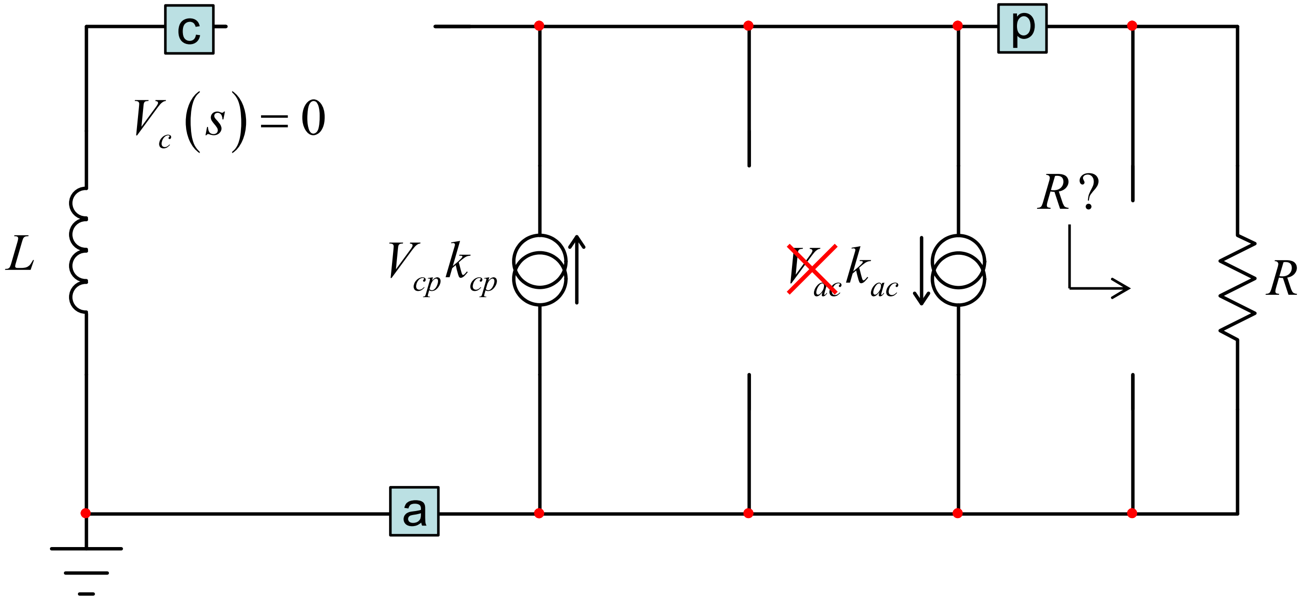
Substitute k_{ic}, k_{ac}, k_{cp}

- The root is positive, this is a Right Half Plane Zero

$$s_{z_1} = \frac{R}{LM^2}$$

For the Pole, Reduce Excitation to 0

- Excitation is V_c: all current sources $f(V_c)$ are open



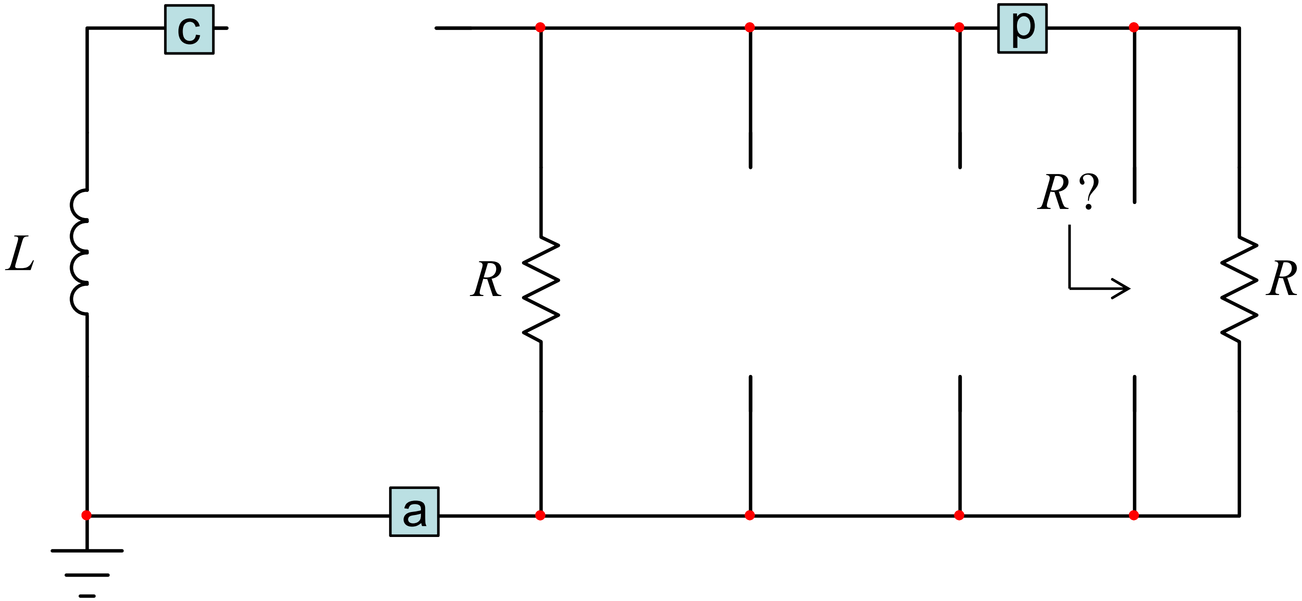
- Look for the resistance “seen” by the capacitor C
- The source $V_{cp}k_{cp}$ can be reworked

$$V_{cp}(s)k_{cp} = -V_p(s)k_{cp} = -\frac{V_{out}(s)}{R} \longrightarrow$$ Replace by a resistance R

$\hat{v}_c = 0$

A Really Simple Circuit

- Difficult to beat in terms of problem solving



- The pole due to the capacitor comes immediately

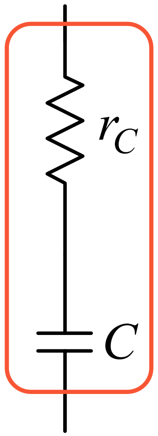
$$R? = R \,||\, R = \frac{R}{2} \longrightarrow \tau = \frac{R}{2}C \longrightarrow s_{p_1} = \frac{2}{RC}$$

The Complete Expression is Ready

- With a few steps, we have our transfer function

$$\frac{V_{out}(s)}{V_c(s)} = H_0 \frac{1-\dfrac{s}{\omega_{z_1}}}{1+\dfrac{s}{\omega_{p_1}}} \qquad H_0 = \frac{R}{4MR_i} \qquad \omega_{p_1} = \frac{2}{RC} \qquad \omega_{z_1} = \frac{R}{LM^2}$$

- We can easily add the capacitor ESR contribution



$Z(s)=0$

$$r_C + \frac{1}{sC} = \frac{sr_C C + 1}{sC}$$

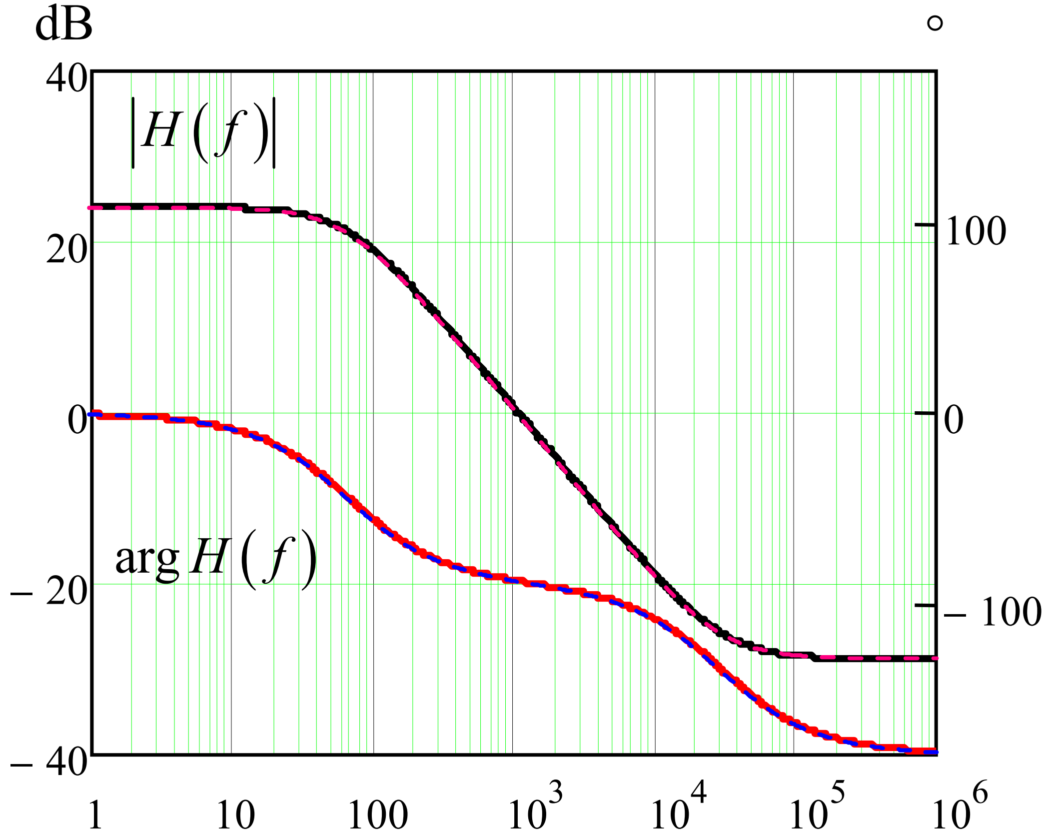
$$sr_C C + 1 = 0$$

$$\omega_{z_2} = \frac{1}{r_C C}$$

$$\frac{V_{out}(s)}{V_c(s)} = H_0 \frac{\left(1-\dfrac{s}{\omega_{z_1}}\right)\left(1+\dfrac{s}{\omega_{z_2}}\right)}{1+\dfrac{s}{\omega_{p_1}}}$$

Time to Confront Mathcad® with SPICE!

- If equations are correct, curves must superimpose perfectly



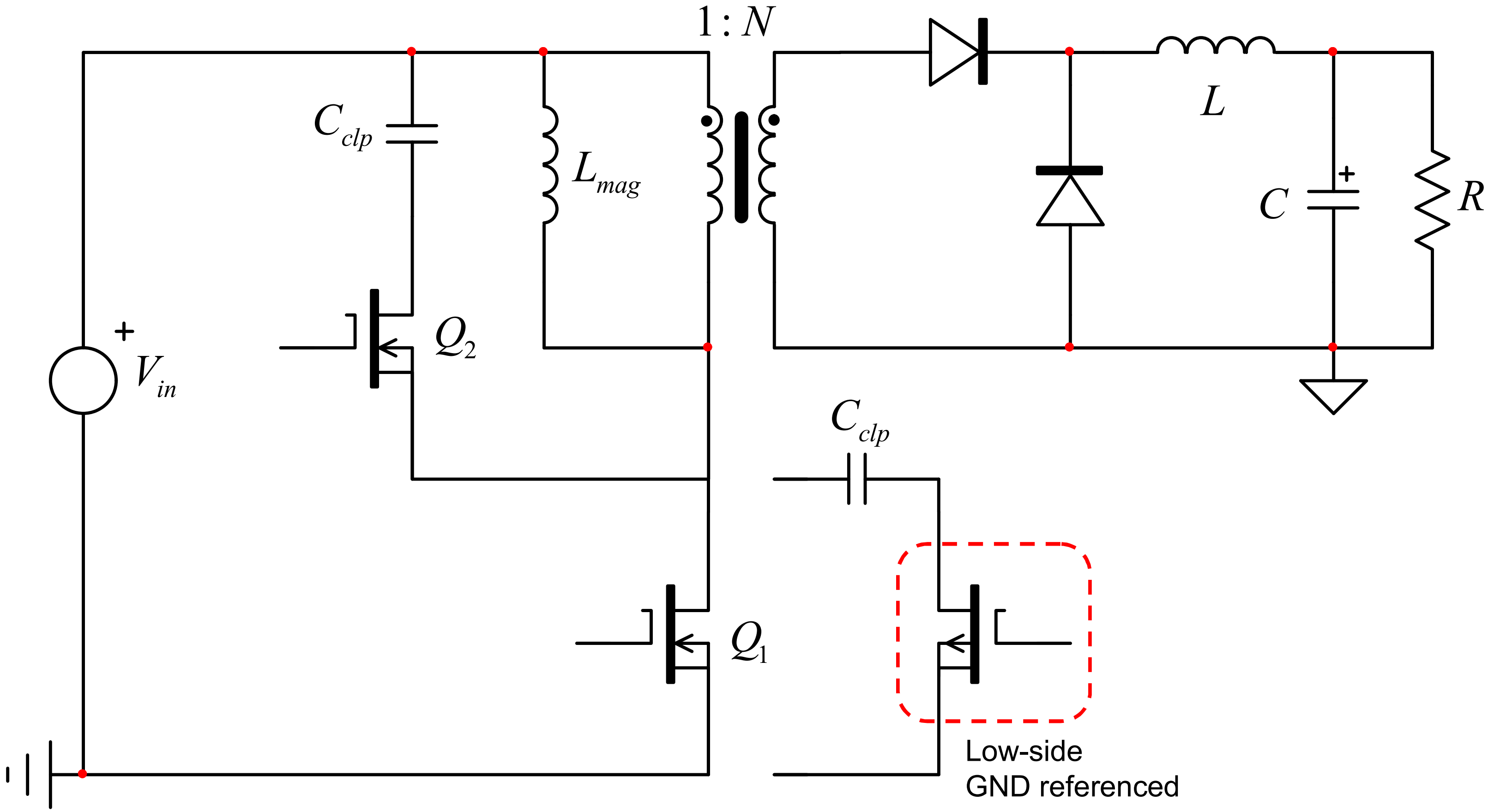
- The BCM boost response in CM is first-order with RHPZ

Course Agenda

- ❑ Introducing the PWM Switch Model
- ❑ CCM, DCM and BCM in Voltage Mode
- ❑ Pulse Width Modulator Gain
- ❑ The PWM Switch Model in Current Mode
- ❑ PWM Switch at Work in a Buck Converter
- ❑ A Simplified Approach to Modeling a DCM Boost
- ❑ Transfer Function of a BCM Boost in Current Mode
- ❑ **Small-Signal Model of The Active Clamp Forward**

Active Clamp Forward in Voltage Mode

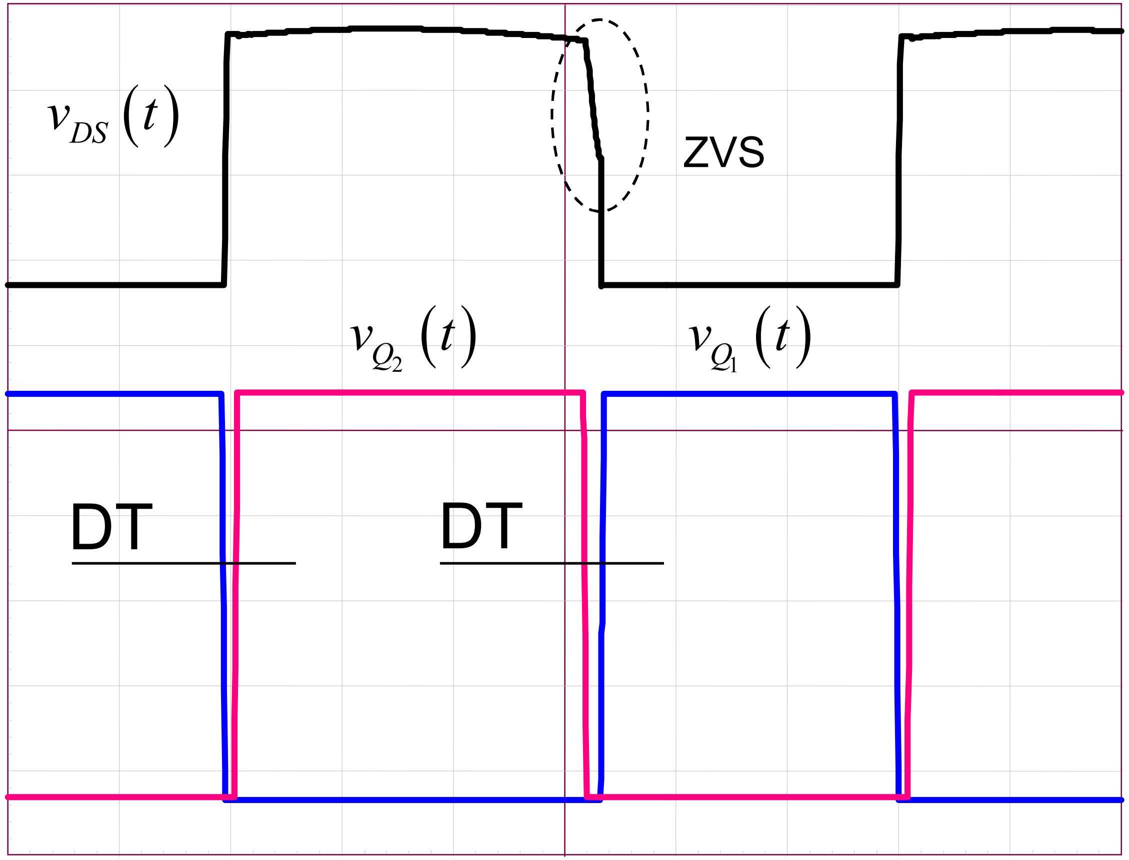
❑ An Active Clamp Forward (ACF) is a forward converter…



❑ …featuring a controlled-upper-side switch for ZVS operations

Control Strategy

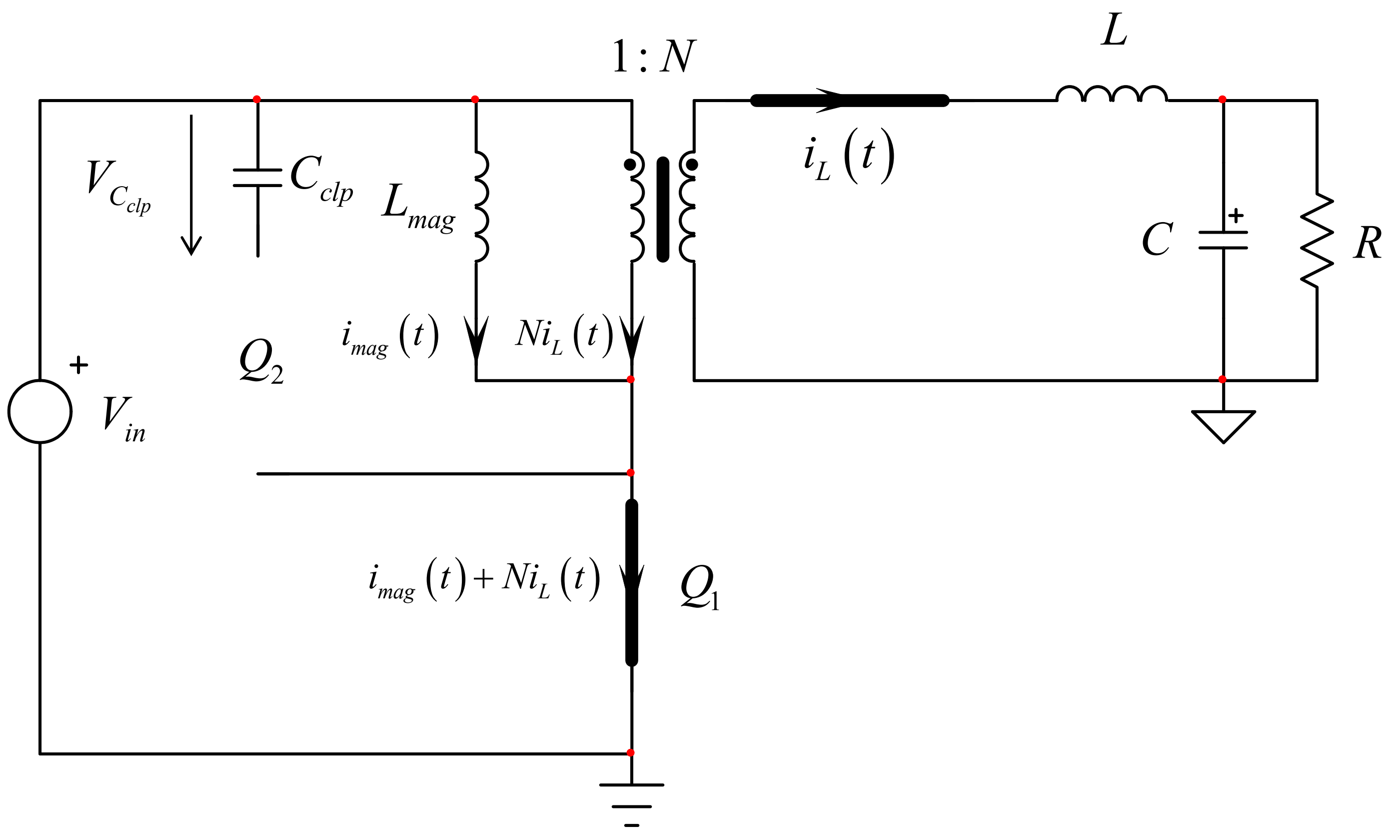
- A deadtime is inserted to let V_{DS} swing towards grounds



- Quasi-ZVS can be implemented on the drain voltage

Active Clamp Forward in Voltage Mode

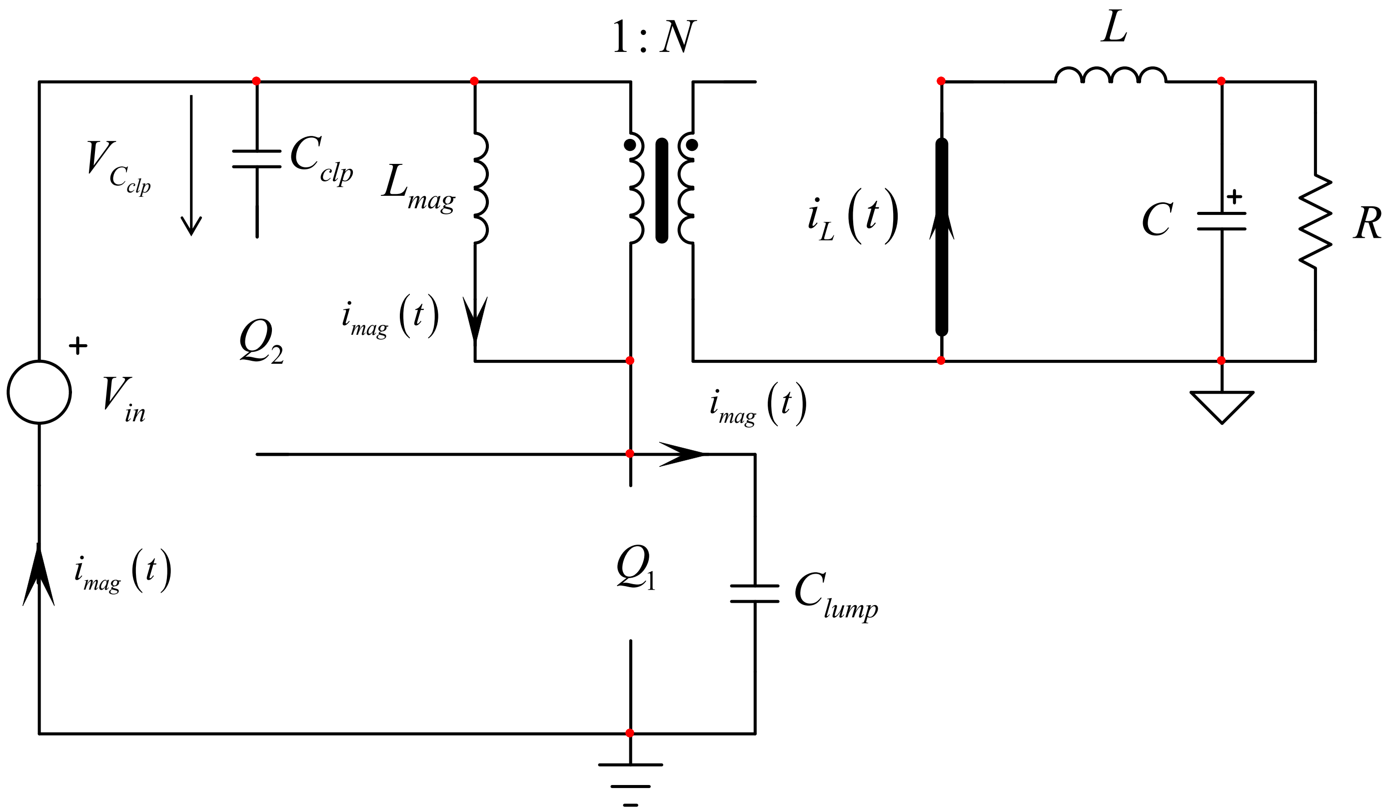
- Energy is stored in the magnetizing inductance at turn on



- We need to offer a path at turn off to demagnetize the core

All Parasitic Capacitors are Charged

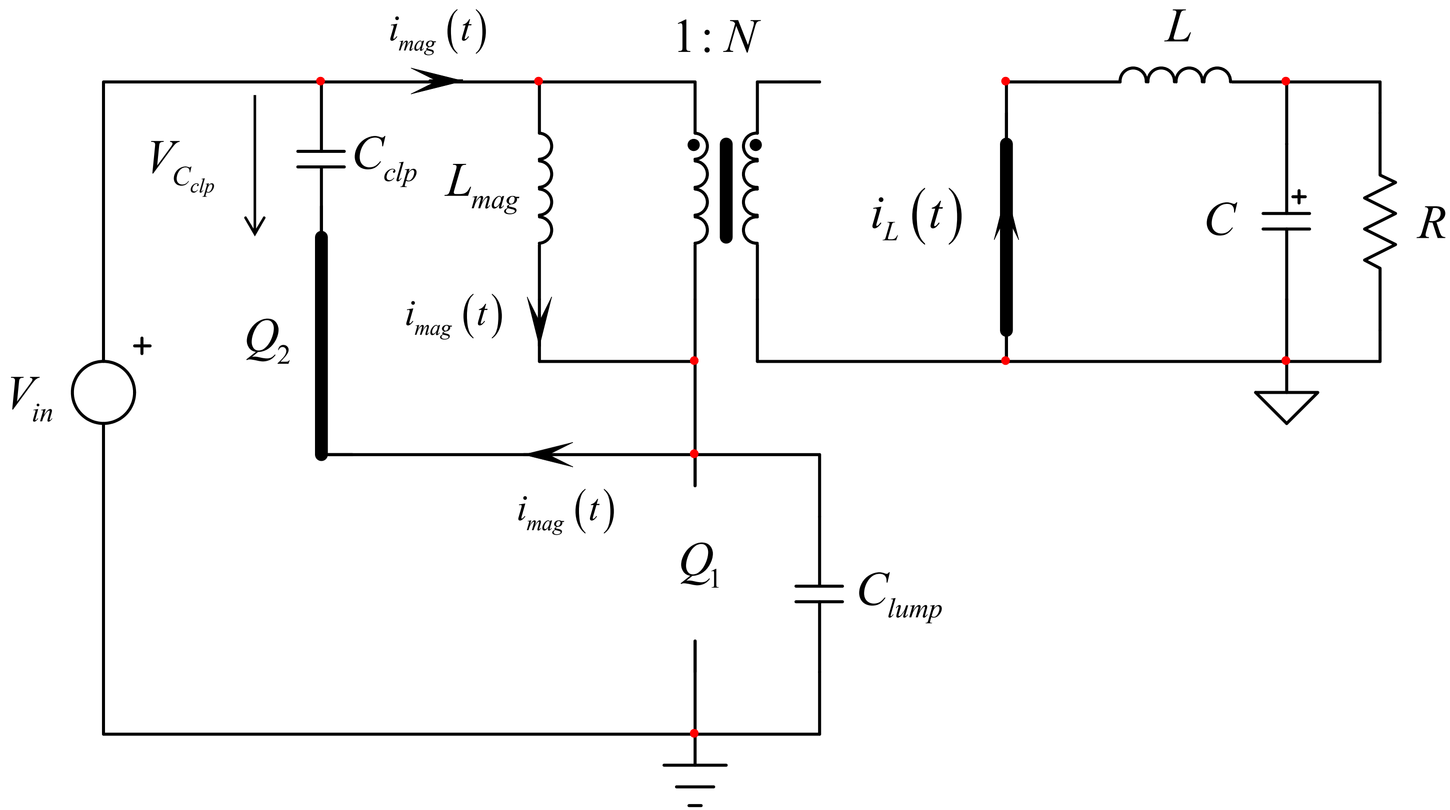
- At turn off, the current charges the lump capacitor



- The drain voltage increases until it touches V_{clamp}

Magnetizing Current Resonates

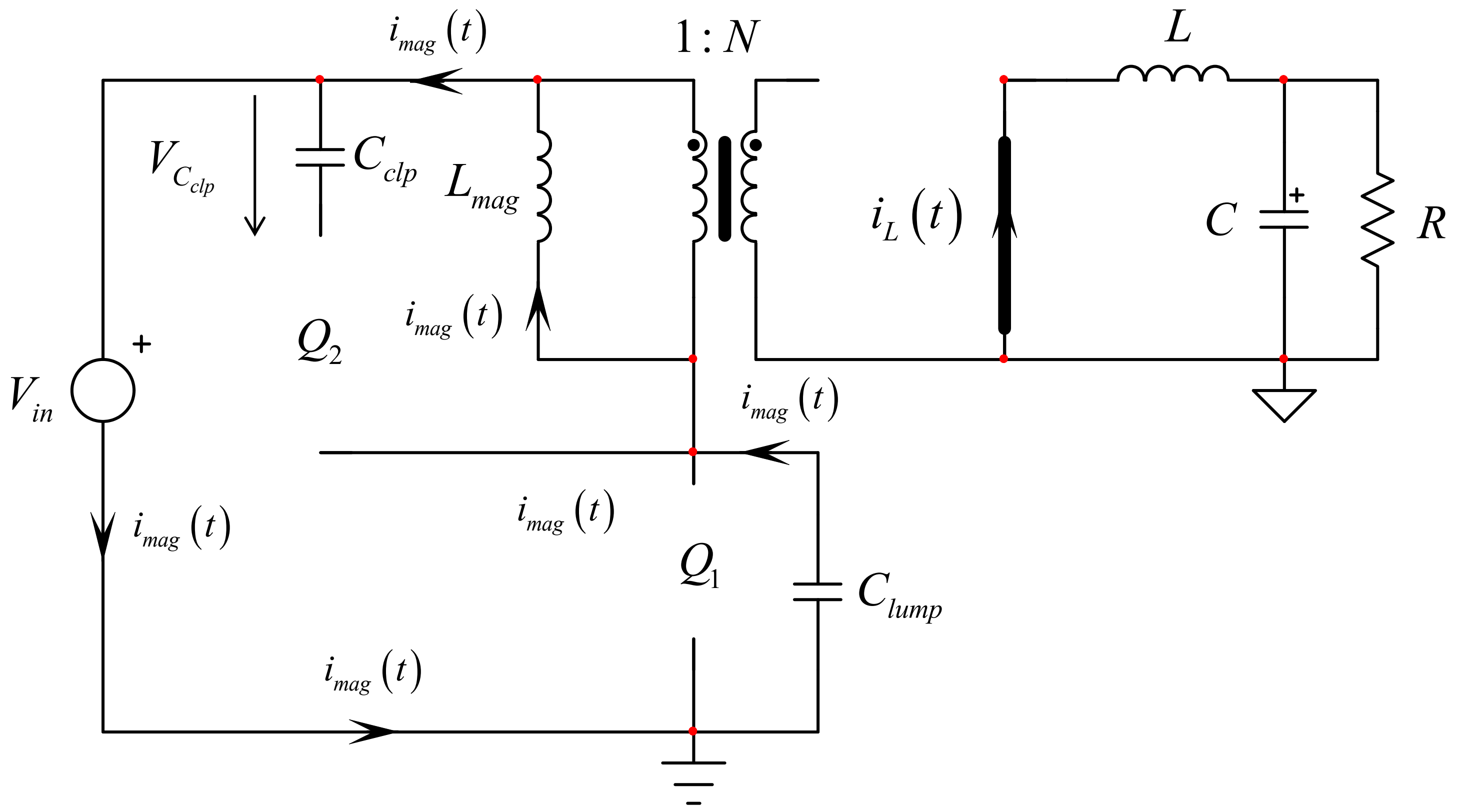
❑ The upper-side MOSFET switches on with delay: ZVS



❑ Magnetizing current decreases to 0 then reverses

Deadtime Gives Quasi-ZVS Operation

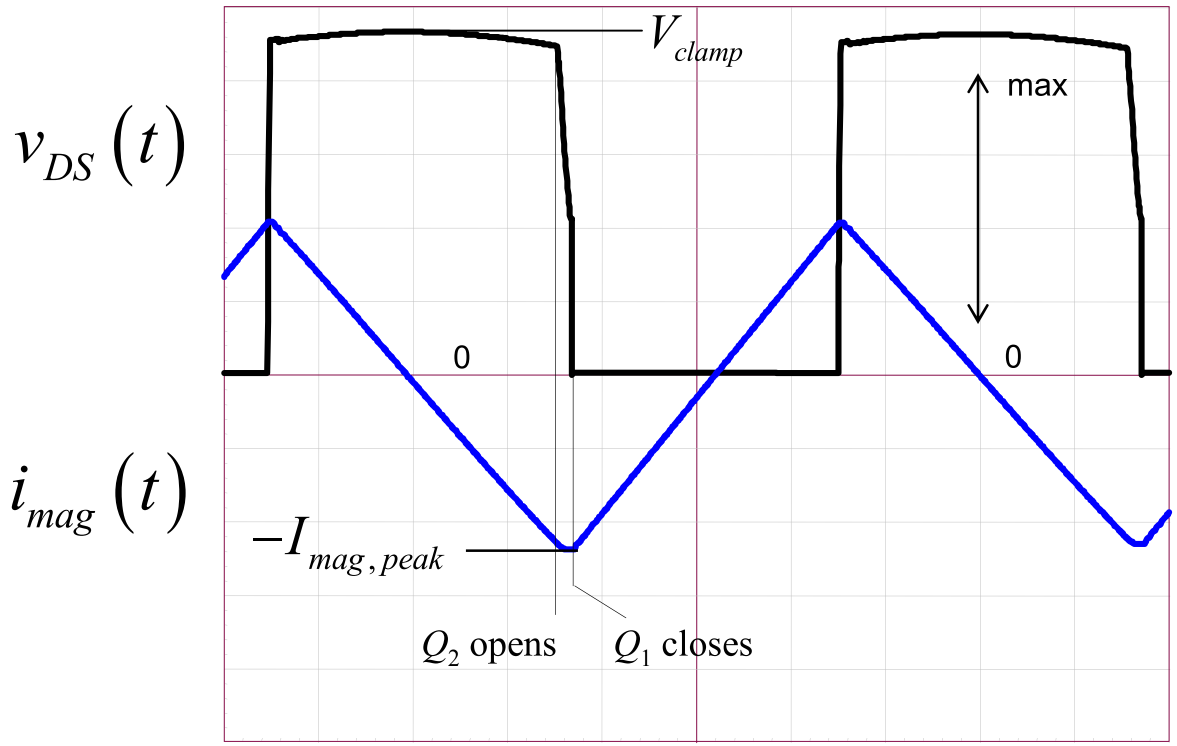
- The deadtime lets magnetizing current reach $-I_{mag,peak}$



- At this moment, Q_2 opens and current discharges C_{lump}

Full ZVS at Nominal Power is Difficult

- True ZVS is difficult to reach, quasi-ZVS is usually obtained



True ZVS

$$\frac{1}{2}L_{mag}\left(I_{mag,peak-}-NI_L\right)^2>\frac{1}{2}C_{lump}V_{in}^2$$

Building an Ac Model

- ❑ A model can be built following different methods

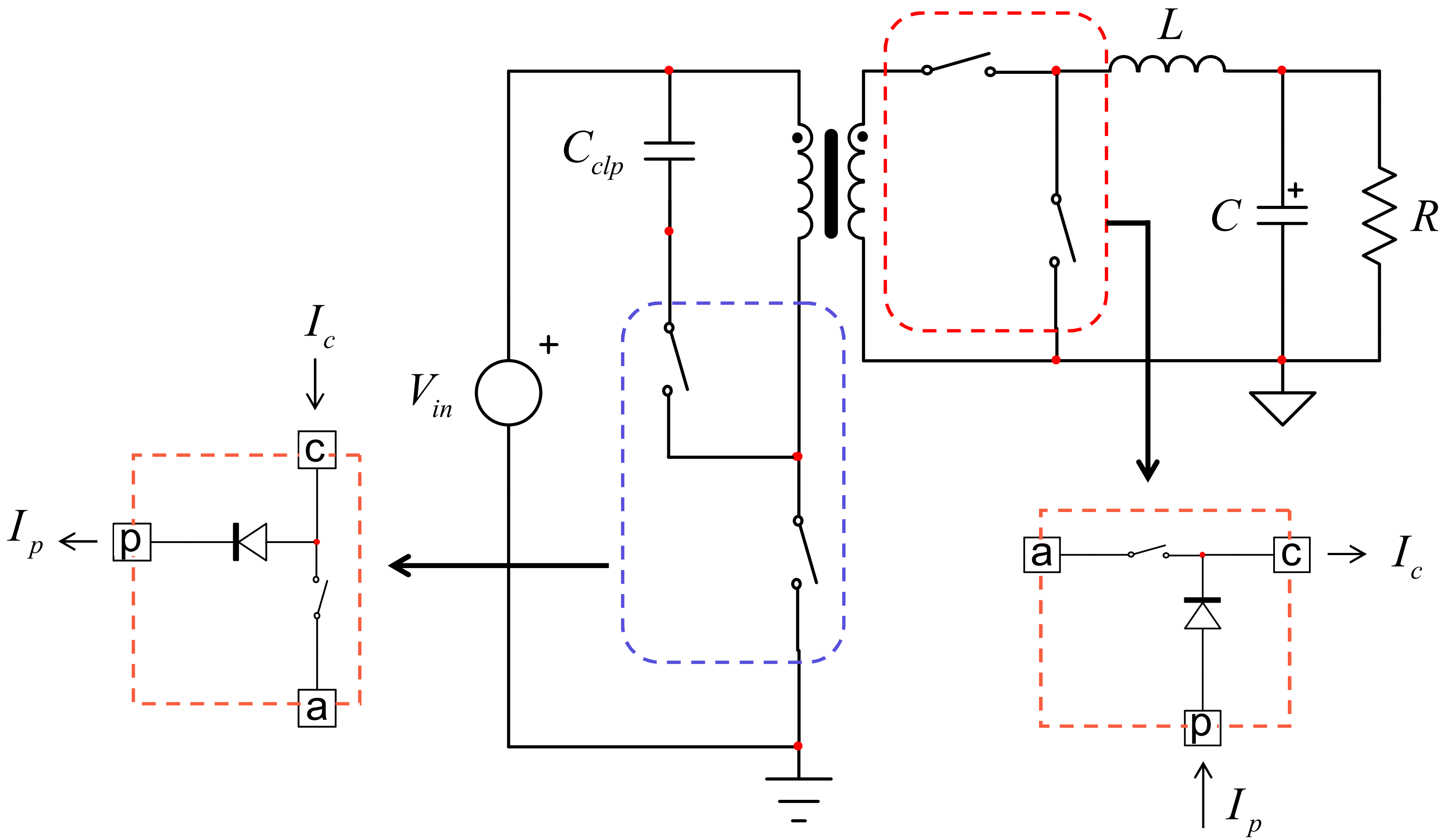
- ❖ Write large-signal equations of voltages and currents
- ❖ Assemble sources to build a large-signal model
- ❖ Linearize expressions and derive transfer functions

- o Reveal the presence of the PWM Switch model
- o Implant its already-available small-signal model
- o Solve for the transfer function expression

- ❑ Both approaches have pros and cons
- ❑ Going along both paths helps to cross-check results

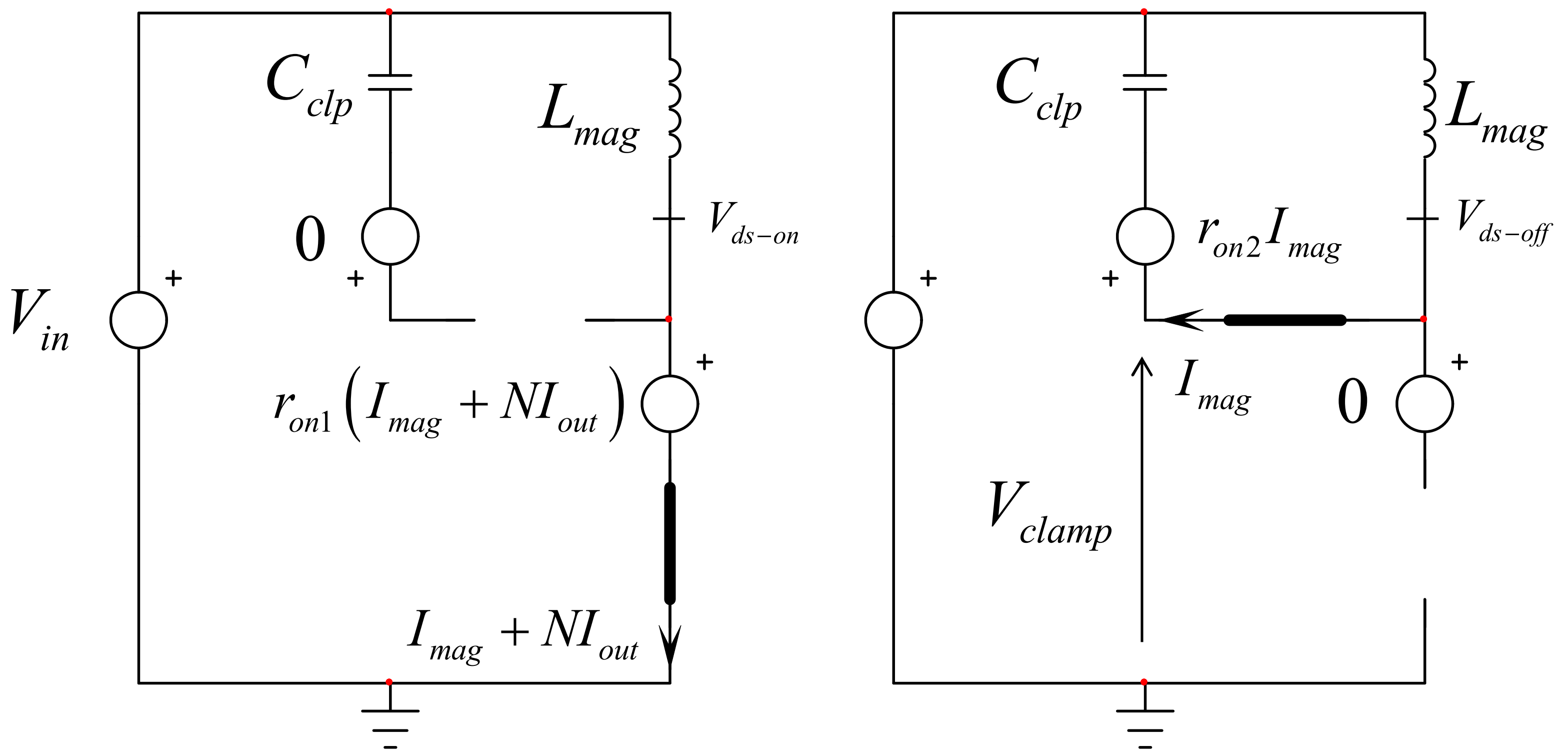
Identify PWM Switches Pair

- A pair of switches appear in primary and secondary sides



Primary-Side PWM Switch Model

❑ Identify currents and voltages during transitions



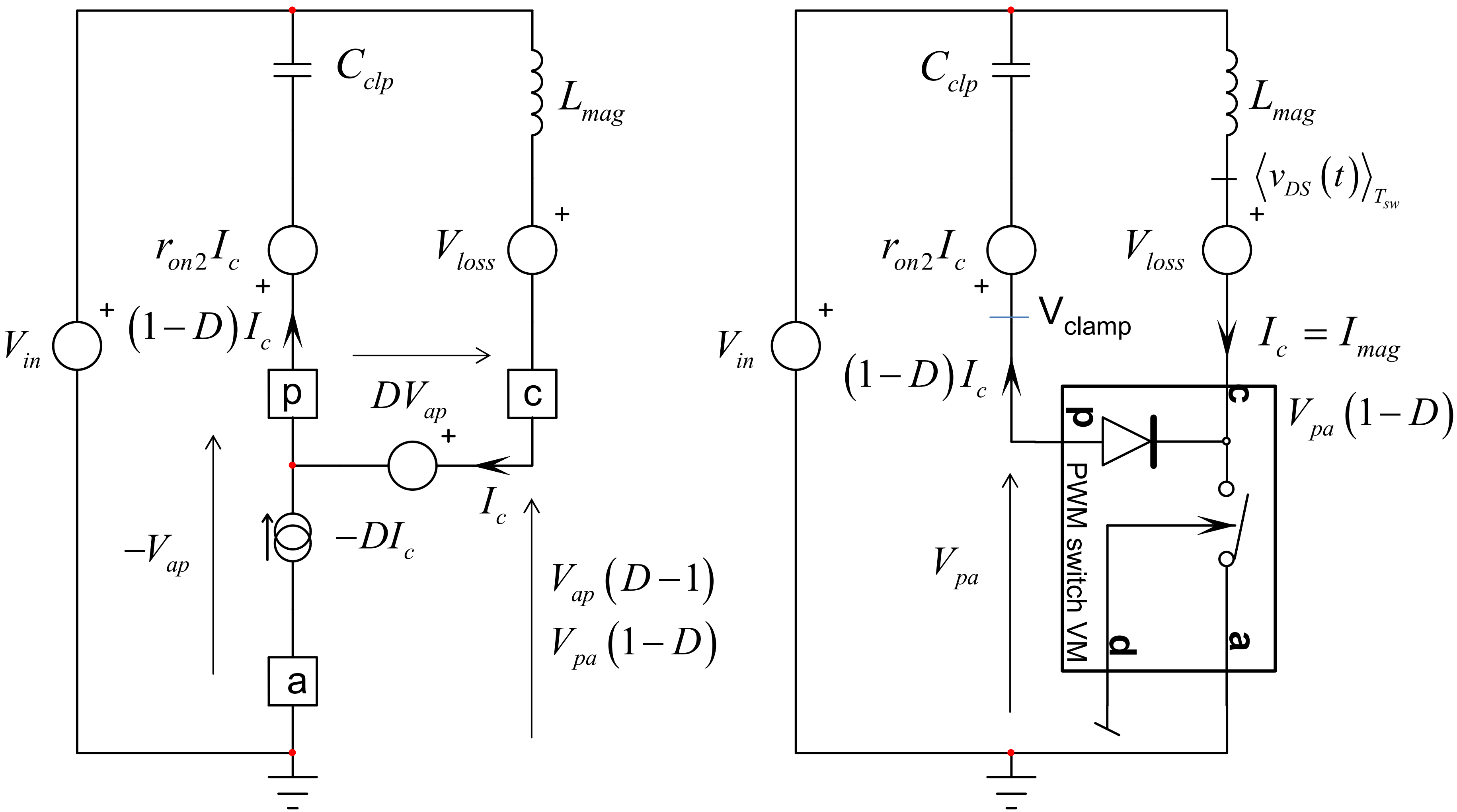
During the on-time DT_{sw} | During the off-time $(1-D)T_{sw}$

$$\left\langle v_{DS-on}(t)\right\rangle_{DT_{sw}}=\left(I_{mag}+NI_{out}\right)r_{on1}$$

$$\left\langle v_{DS-off}(t)\right\rangle_{(1-D)T_{sw}}=V_{in}+V_C+r_{on2}I_{mag}$$

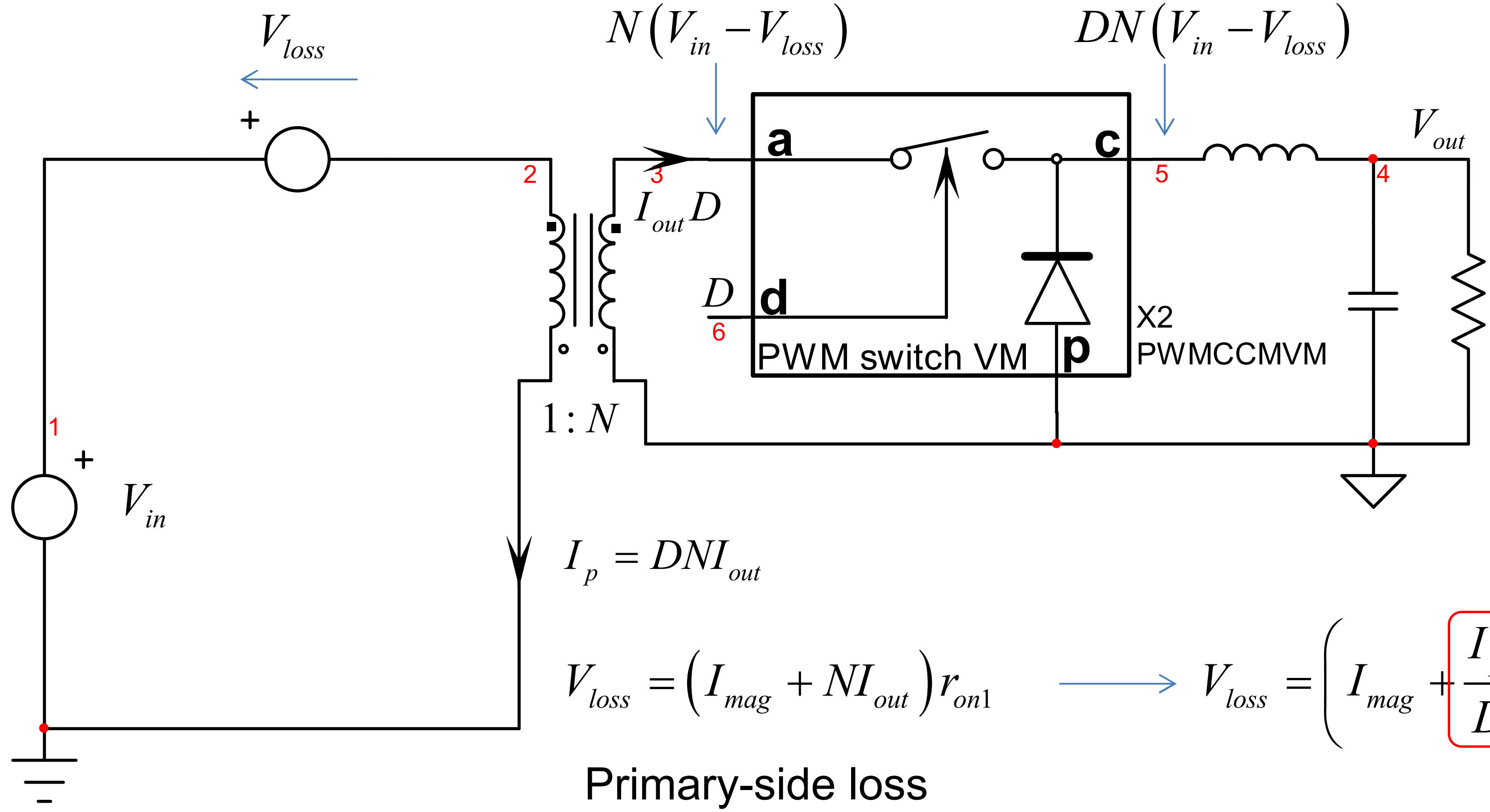
The First PWM Switch Model is Here

- Connect the PWM Switch the right way, check polarities



Secondary-Side PWM Switch Model

- ❑ The forward converter is a buck-derived topology



$$I_p = DNI_{out}$$

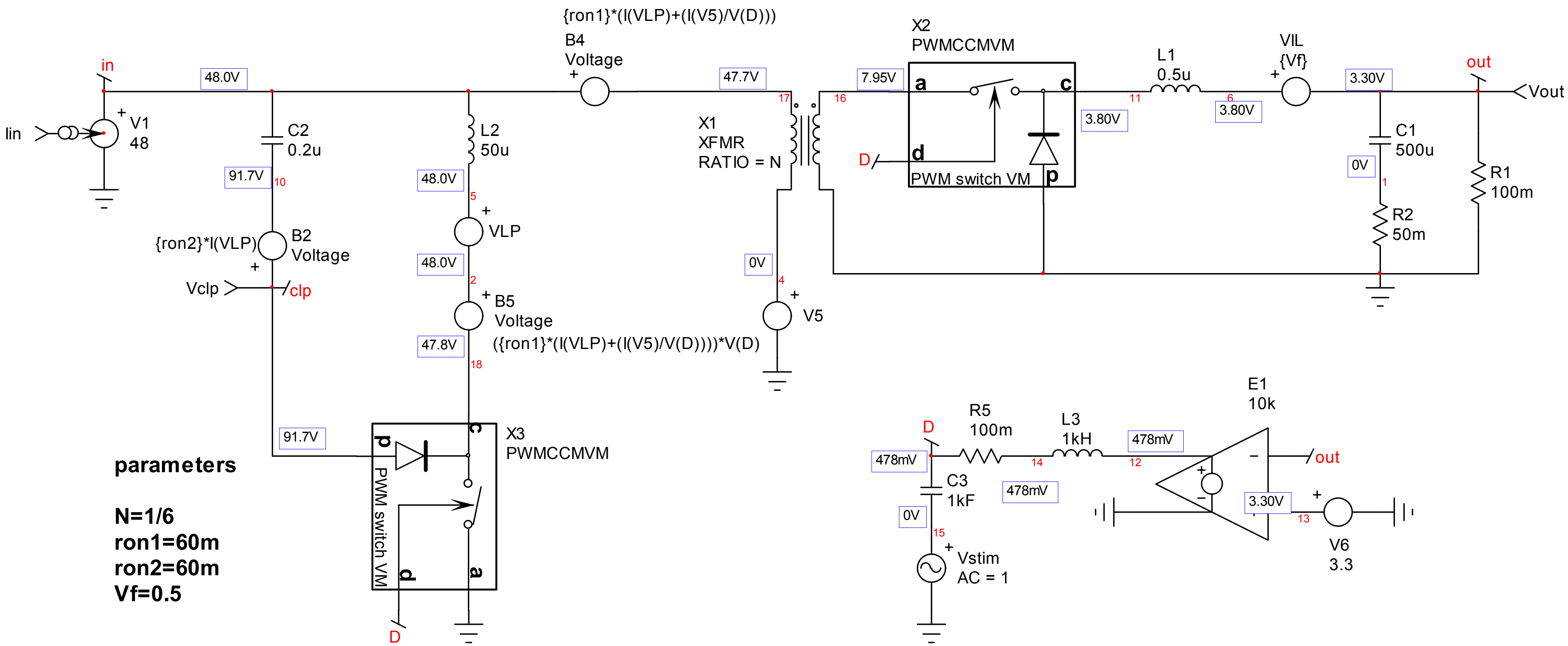
$$V_{loss} = \left(I_{mag} + NI_{out}\right)r_{on1} \longrightarrow V_{loss} = \left(I_{mag} + \frac{I_p}{D}\right)r_{on1}$$

Primary-side loss

$\frac{I_p}{D} \rightarrow NI_{out}$

Check the Dual-PWM Switch Response

- Compare ac response with that of the large-signal version



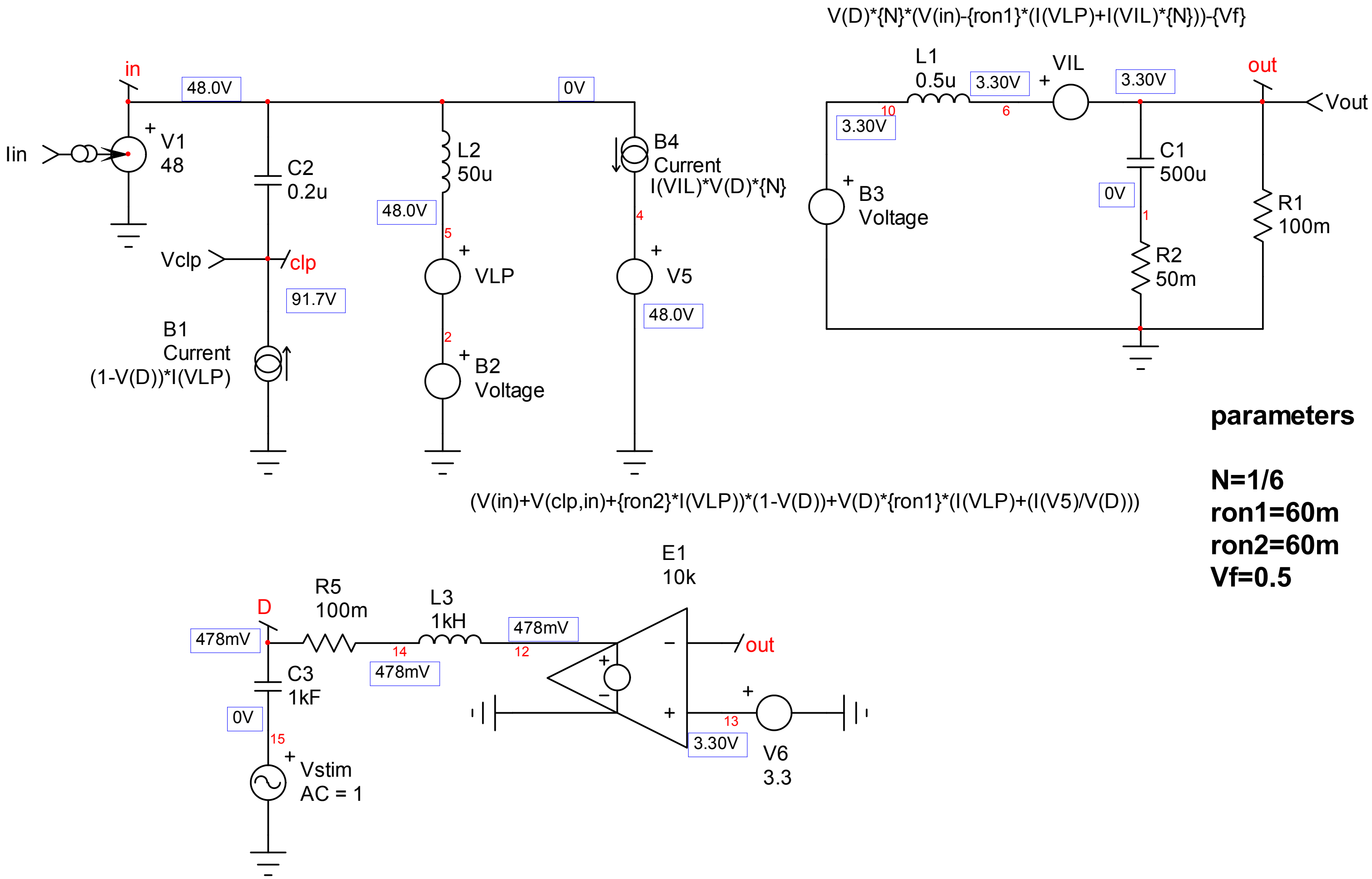
parameters

N=1/6
ron1=60m
ron2=60m
Vf=0.5

- Check first if dc operating points are similar
- Use this fixture to also run transient simulations

Large-Signal Equations with SPICE

- ❑ Non-linear equations are linearized by SPICE



parameters

N=1/6
ron1=60m
ron2=60m
Vf=0.5

Proposed by Dr. José Capilla, ON Semi, November 2012

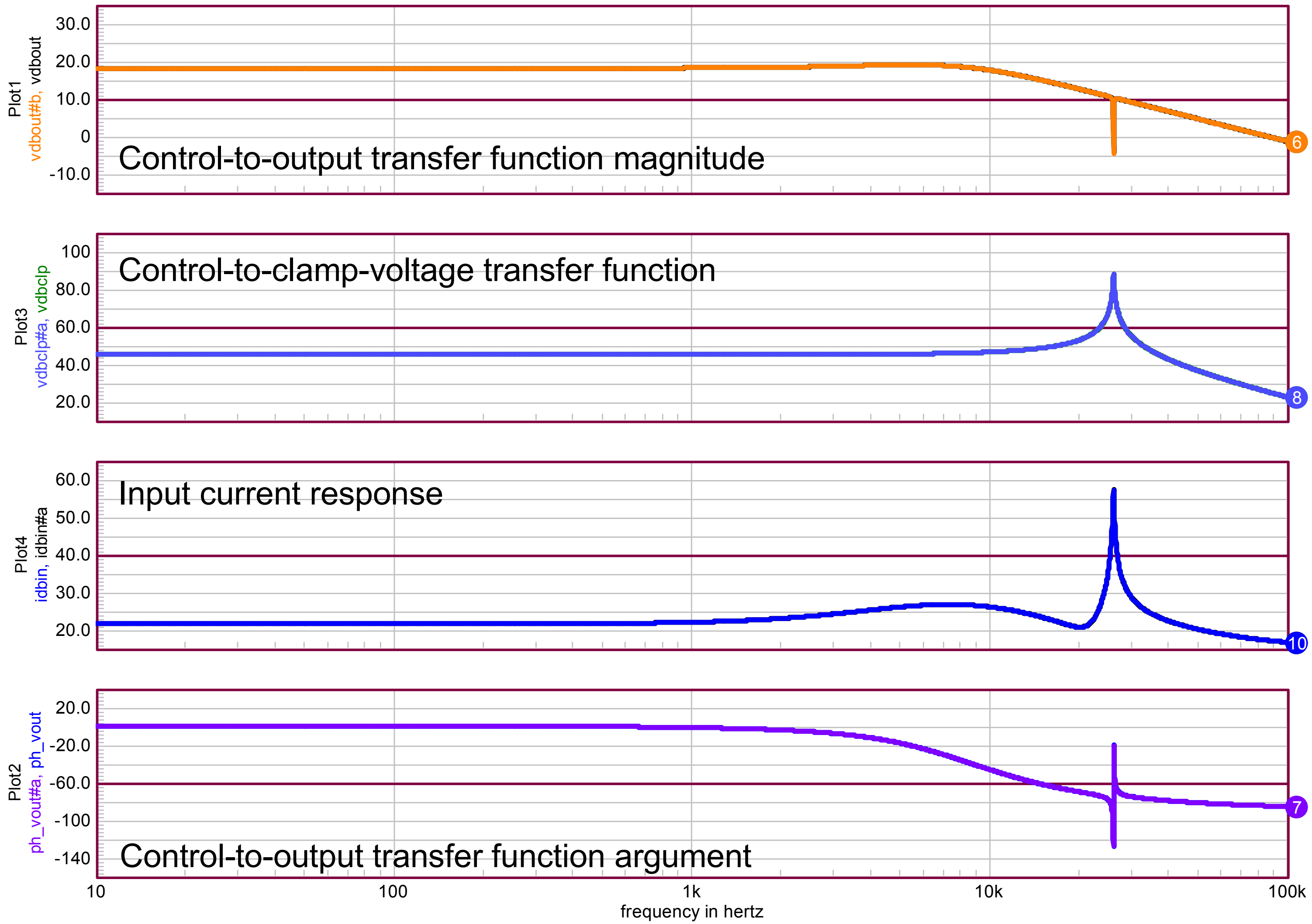
Curves Perfectly Superimpose

$$\left|\frac{V_{out}(f)}{D(f)}\right|$$

$$\left|\frac{V_{clamp}(f)}{D(f)}\right|$$

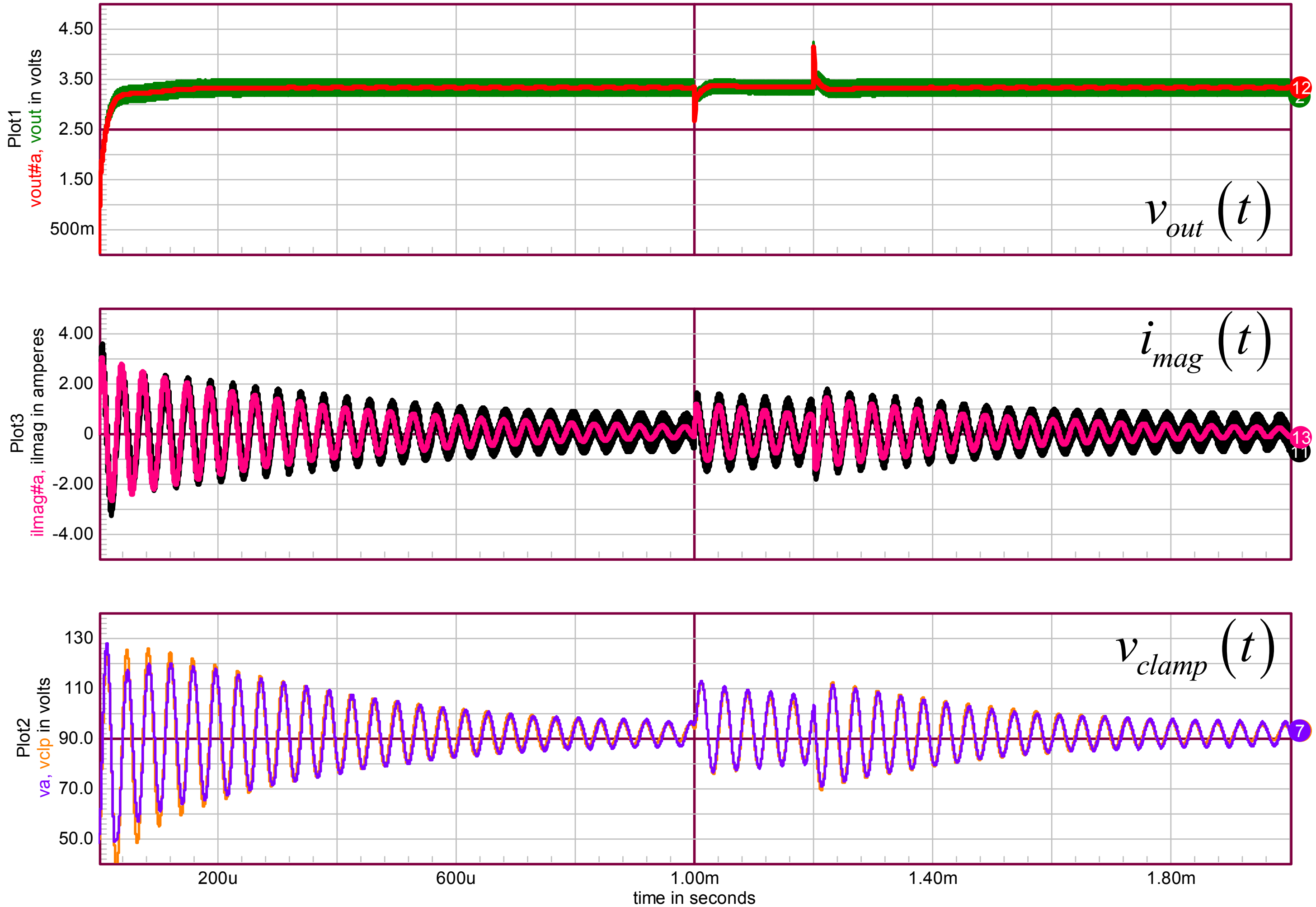
$$\left|\frac{I_{in}(f)}{D(f)}\right|$$

$$\angle\frac{V_{out}(f)}{D(f)}$$



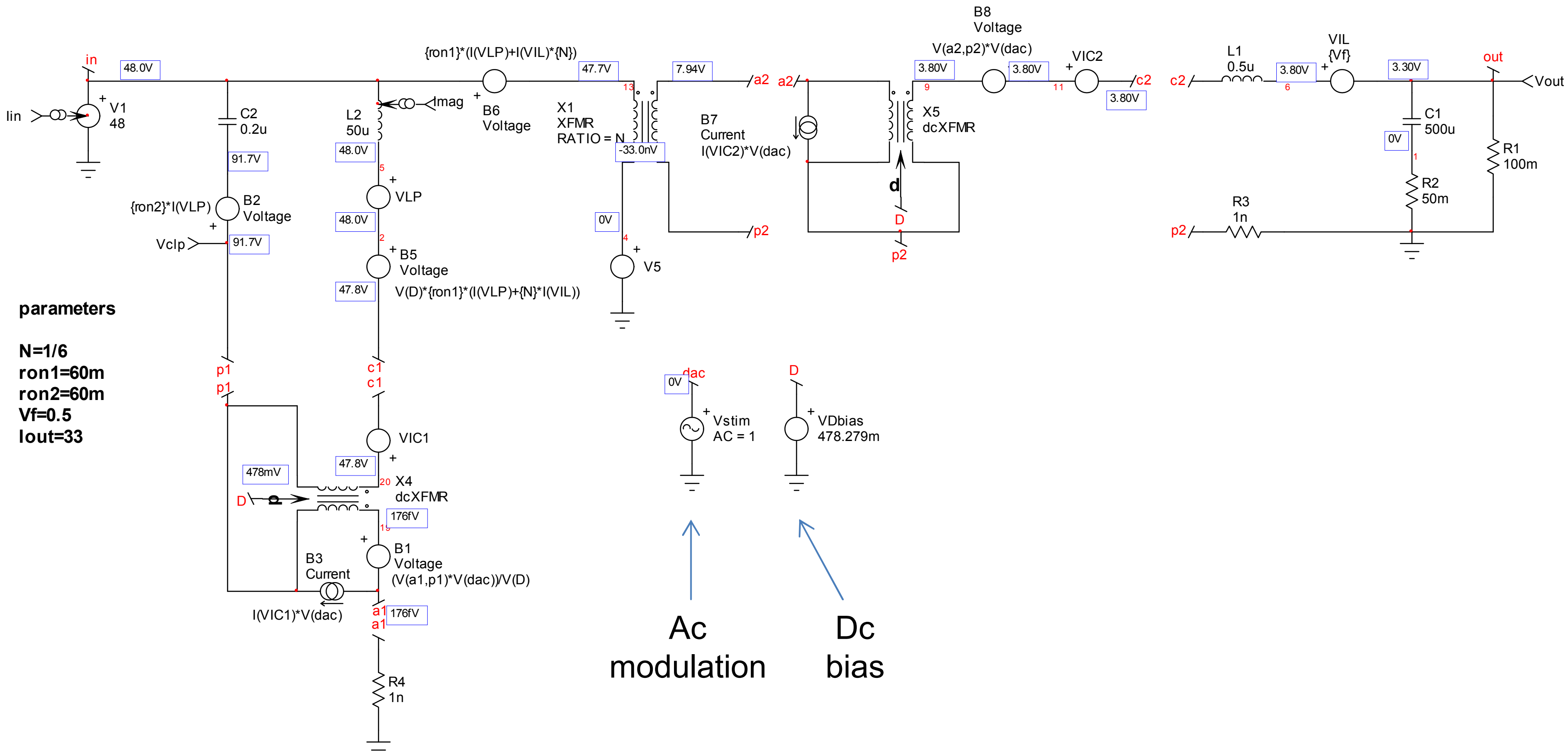
Compare Transient Responses

❑ Cycle-by-cycle results are similar to that of average model



Simulate with Small-Signal Sources

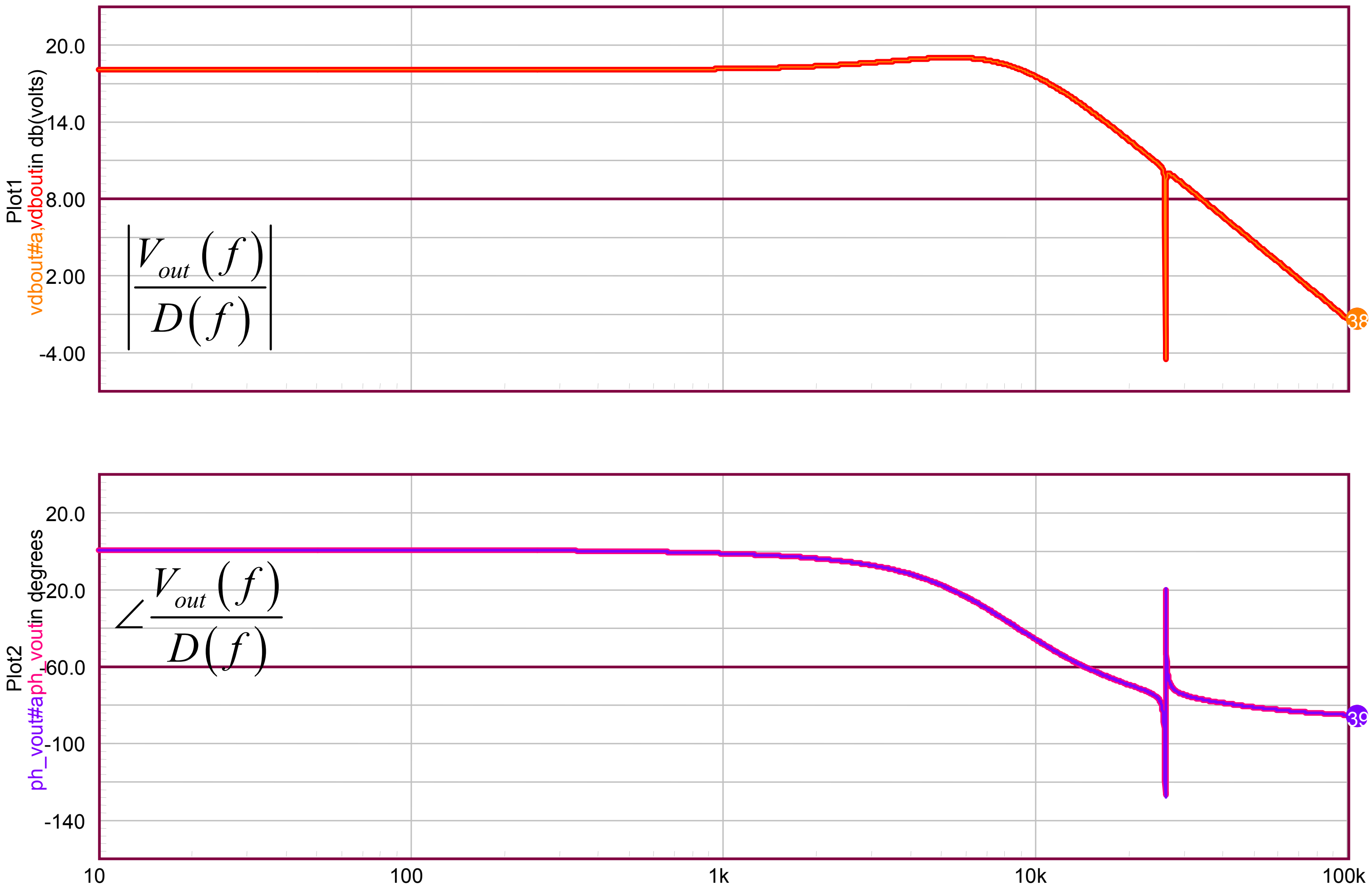
- ❑ Plug the small-signal models of the PWM Switch in



- ❑ Check if ac response is ok before proceeding!
- ❑ Simplify circuitry to concentrate on control-to-output only

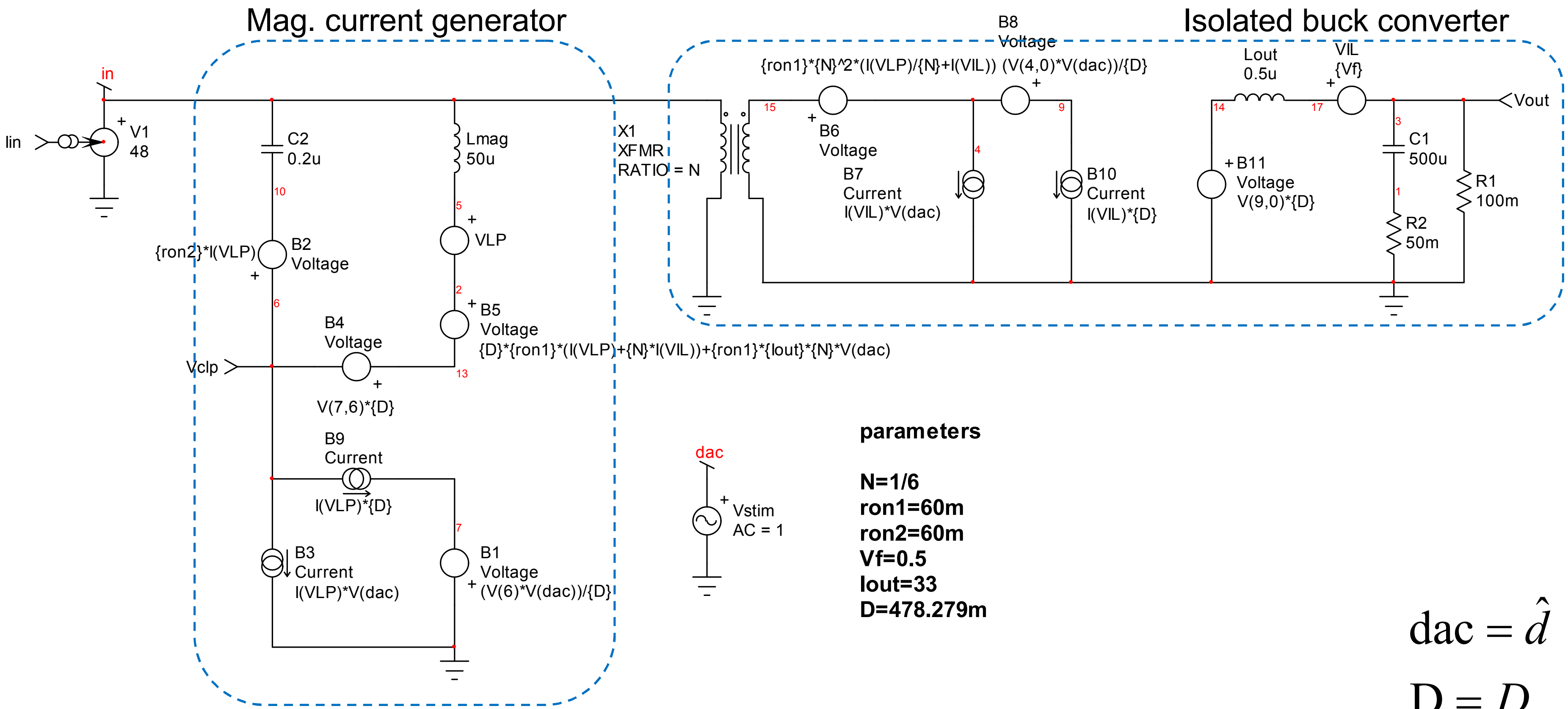
ON Semiconductor®

Same Response Between Fixtures



Make the Circuit Look Friendlier

- ❑ Re-arrange sources to simplify the electrical circuit



parameters

N=1/6
ron1=60m
ron2=60m
Vf=0.5
Iout=33
D=478.279m

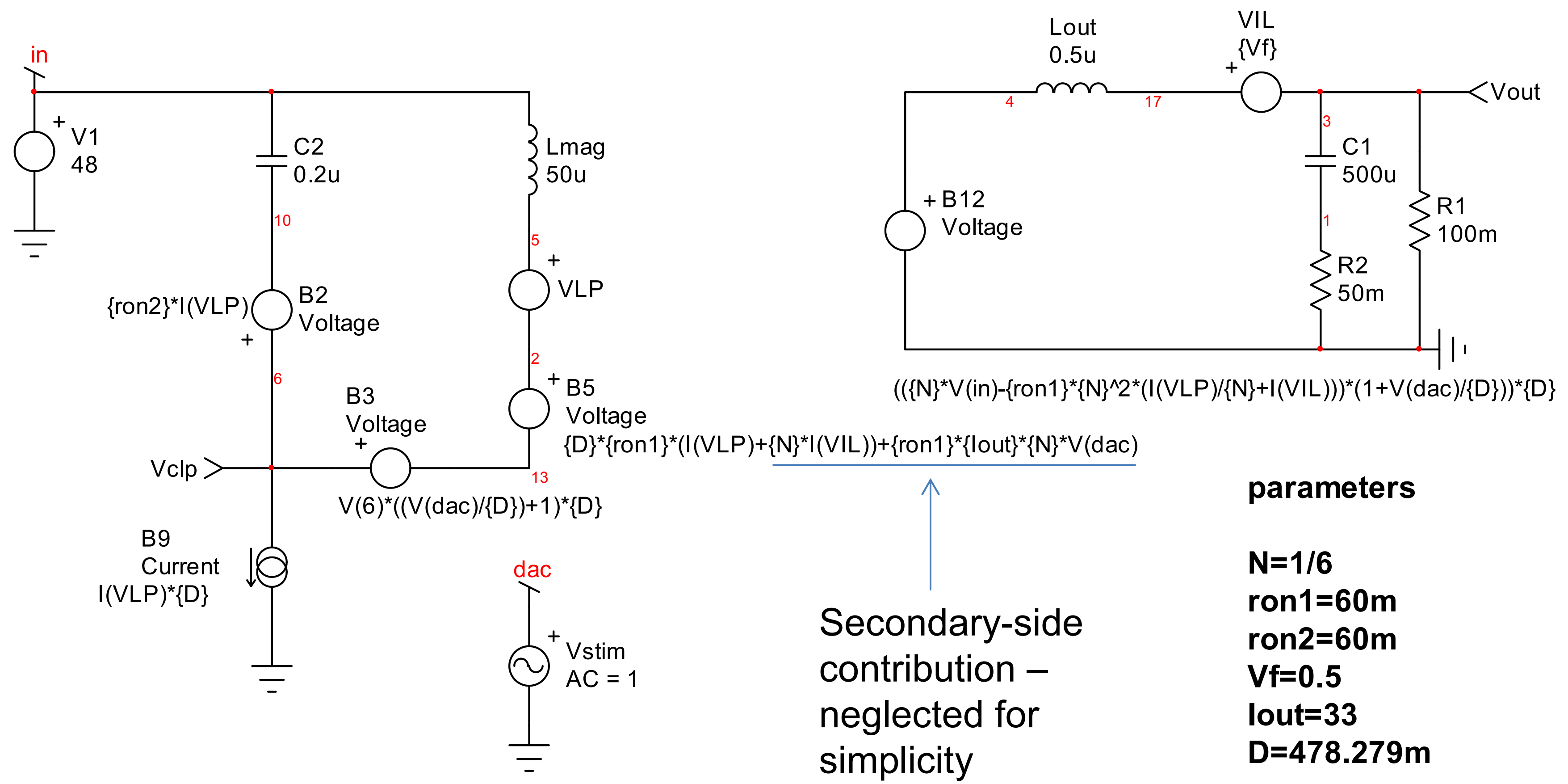
$$\mathrm{dac} = \hat{d}$$

$$\mathrm{D} = D_0$$

- ❑ Run a sanity check further to any simplification

Circuit is Looking Simpler Now

- Look for ways to get simpler equations, final arrangement



parameters

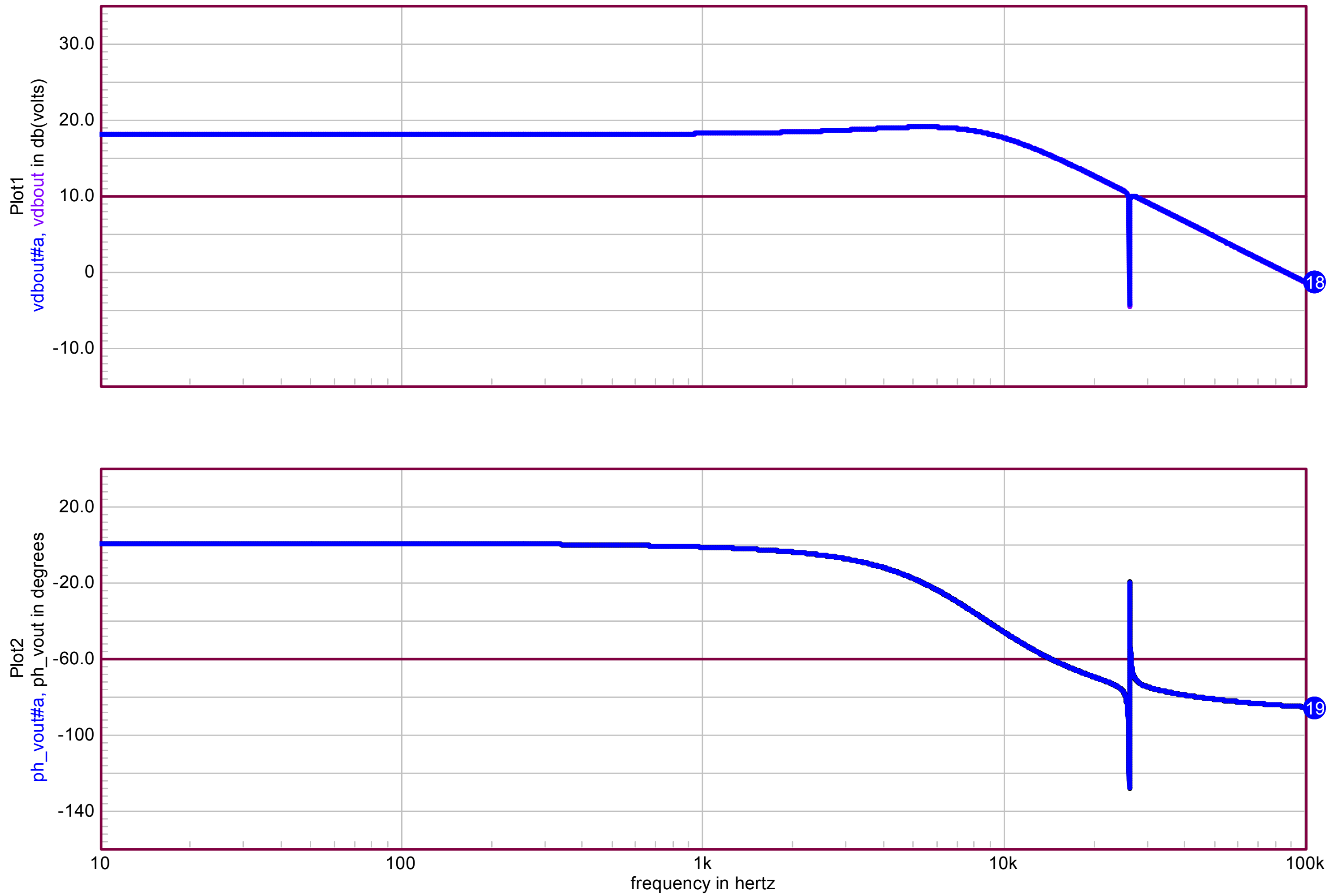
N=1/6
ron1=60m
ron2=60m
Vf=0.5
Iout=33
D=478.279m

- Final ac check is mandatory!

ON Semiconductor®

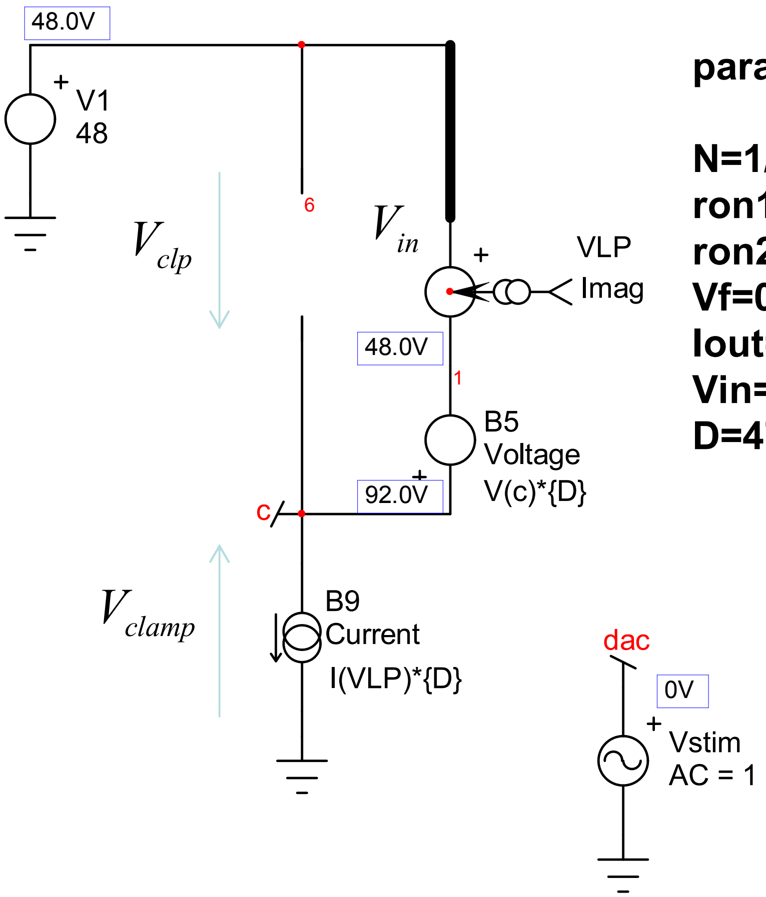
Simpler Circuit Does Not Distort Response

- We are good to start analyzing the equivalent circuit



Start with the Clamp Circuitry

- For the dc transfer function, open caps and short inductors



parameters

N=1/6
ron1=60m
ron2=60m
Vf=0.5
Iout=33
Vin=48
D=478.279m

$$V_{clp} = V_{(c)} D = V_{clamp} D$$

$$V_{clamp} = V_{in} + V_{clp}$$

$$V_{clp} = \left(V_{in} + V_{clp}\right) D$$

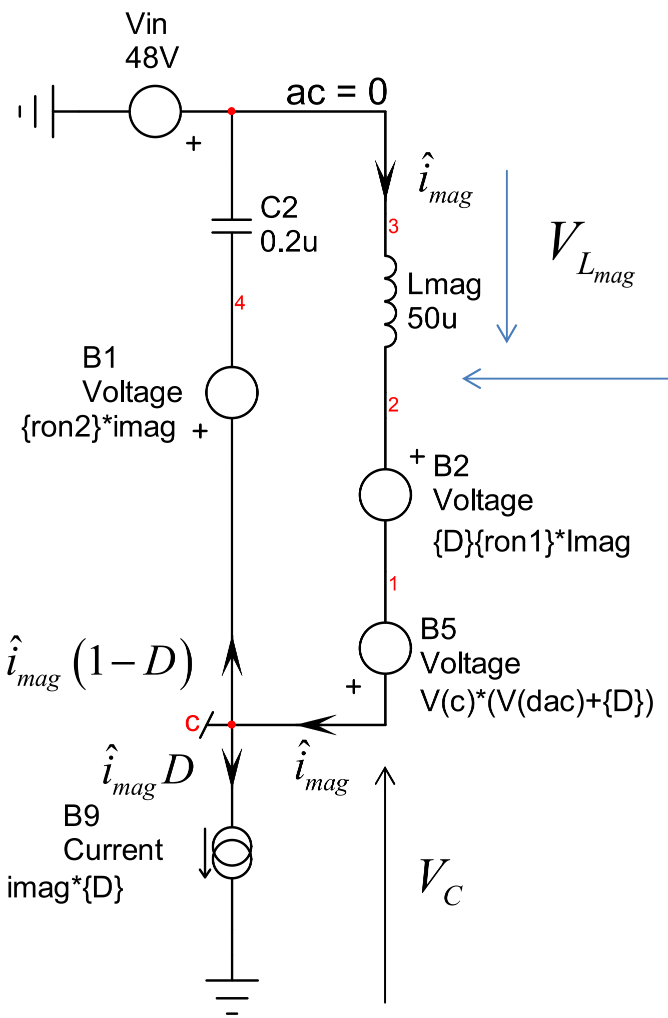
$$V_{clp} = V_{in} \frac{D}{1-D} = 48 \frac{0.478}{1-0.478} \approx 44\text{ V}$$

$$V_{clamp} = V_{in} + V_{clp} = 92\text{ V}$$

This is a buck-boost dc transfer function

We Need the Magnetizing Current

- Use KVL and KCL to get the mag. current expression



$$V_{L_{mag}} = V_{(C)} - V_{(C)}\left(\hat{d} + D_0\right) + D_0 r_{on1}\hat{i}_{mag}$$

The secondary side contribution has been purposely neglected, I_{out} and $\hat{i}_{out}$

Collect Terms and (Carefully) Simplify

- Use KVL and KCL to get the mag. current expression

$$V_{L_{mag}} = V(2) - V_{in} \qquad \text{with} \qquad V(2) = V_{(C)} - V_{(C)}\left(D_0 + \hat{d}\right) + D_0 r_{on1} \hat{i}_{mag}$$

The voltage at node (c) has a dc and an ac component

$$V_{L_{mag}} + \hat{v}_{L_{mag}} = V_{(C)} + \hat{v}_{(C)} - \left(V_{(C)} + \hat{v}_{(C)}\right)\left(D_0 + \hat{d}\right) + D_0 r_{on1} \hat{i}_{mag} - V_{in} - \hat{v}_{in}$$

$$V_{L_{mag}} + \hat{v}_{L_{mag}} = V_{(C)} + \hat{v}_{(C)} - V_{(C)}\hat{d} - V_{(C)}D_0 - \underbrace{\hat{v}_{(C)}\hat{d}}_{\approx 0} - \hat{v}_{(C)}D_0 + D_0 r_{on1} \hat{i}_{mag} - V_{in} - \underbrace{\hat{v}_{in}}_{0}$$

- Sort out an ac and a dc equation

dc $\quad V_{L_{mag}} = V_{(C)} - V_{(C)}D_0 - V_{in} \xrightarrow{\left\langle V_{L_{mag}} \right\rangle_{T_{sw}} = 0} V_{in} = V_{(C)}\left(1 - D_0\right) \longrightarrow V_{clamp} = \dfrac{V_{in}}{\left(1 - D_0\right)}$

ac $\quad \hat{v}_{L_{mag}} = \hat{v}_{(C)}\left(1 - D_0\right) - V_{clamp}\hat{d} + D_0 r_{on1} \hat{i}_{mag}$

Express the Magnetizing Current i_{mag}

- The clamp capacitor ac voltage depends on i_{mag}

$$\hat{v}_{(C)} = \hat{i}_{mag}\left(1-D_0\right)\left(\frac{1}{sC_{clp}}\right) + \hat{i}_{mag} r_{on2} = \hat{i}_{mag}\left[\left(1-D_0\right)\left(\frac{1}{sC_{clp}}\right) + r_{on2}\right]$$

substitute

$$\hat{i}_{mag} = -\frac{\hat{v}_{L_{mag}}}{sL_{mag}} = -\frac{\hat{v}_{(C)}\left(1-D_0\right) - V_{clamp}\hat{d} + D_0 r_{on1}\hat{i}_{mag}}{sL_{mag}}$$

Solve for i_{mag}

$$I_{mag}(s) = D(s) V_{clamp} \frac{sC_{clp}}{D_0^2 - 2D_0 + 1 + sC_{clp}\left(r_{on2} + D_0 r_{on1} - D_0 r_{on2}\right) + s^2 L_{mag} C_{clp}}$$

Identify Second-Order Coefficients

- Identify terms with a second-order polynomial form

$$\frac{I_{mag}(s)}{D(s)} = \frac{V_{clamp}}{(1-D_0)^2} \frac{sC_{clp}}{1 + sC_{clp}\left[\frac{r_{on2}(1-D_0) + D_0 r_{on1}}{(1-D_0)^2}\right] + s^2 \frac{L_{mag}C_{clp}}{(1-D_0)^2}}$$

Develop and rearrange:

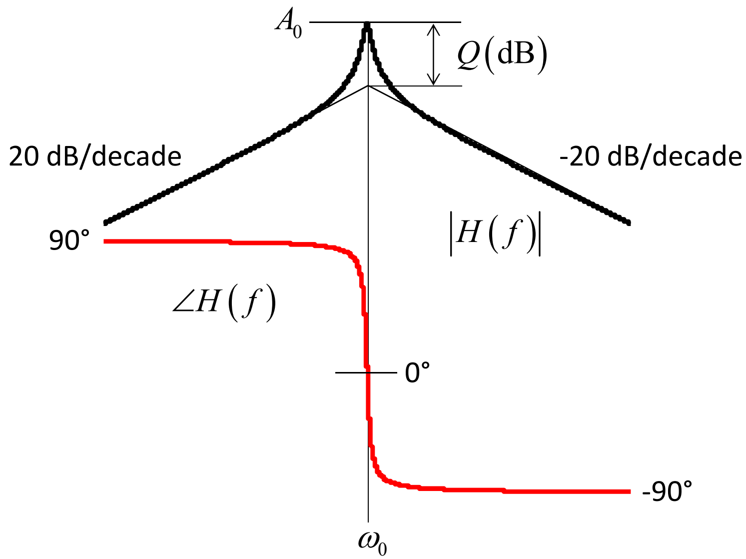
$$\frac{I_{mag}(s)}{D(s)} = M_0 \frac{sC_{clp}}{1 + \frac{s}{\omega_{0M}Q_M} + \left(\frac{s}{\omega_{0M}}\right)^2}$$

$$M_0 = \frac{V_{clamp}}{(1-D_0)^2} = \frac{V_{in}}{(1-D_0)^3} \qquad \omega_{0M} = \frac{1-D_0}{\sqrt{L_{mag}C_{clp}}}$$

$$Q_M = \sqrt{\frac{L_{mag}}{C_{clp}}} \frac{1-D_0}{r_{on2}(1-D_0) + D_0 r_{on1}}$$

Re-Write the Expression Nicely

- A tuned network offers the following transfer function



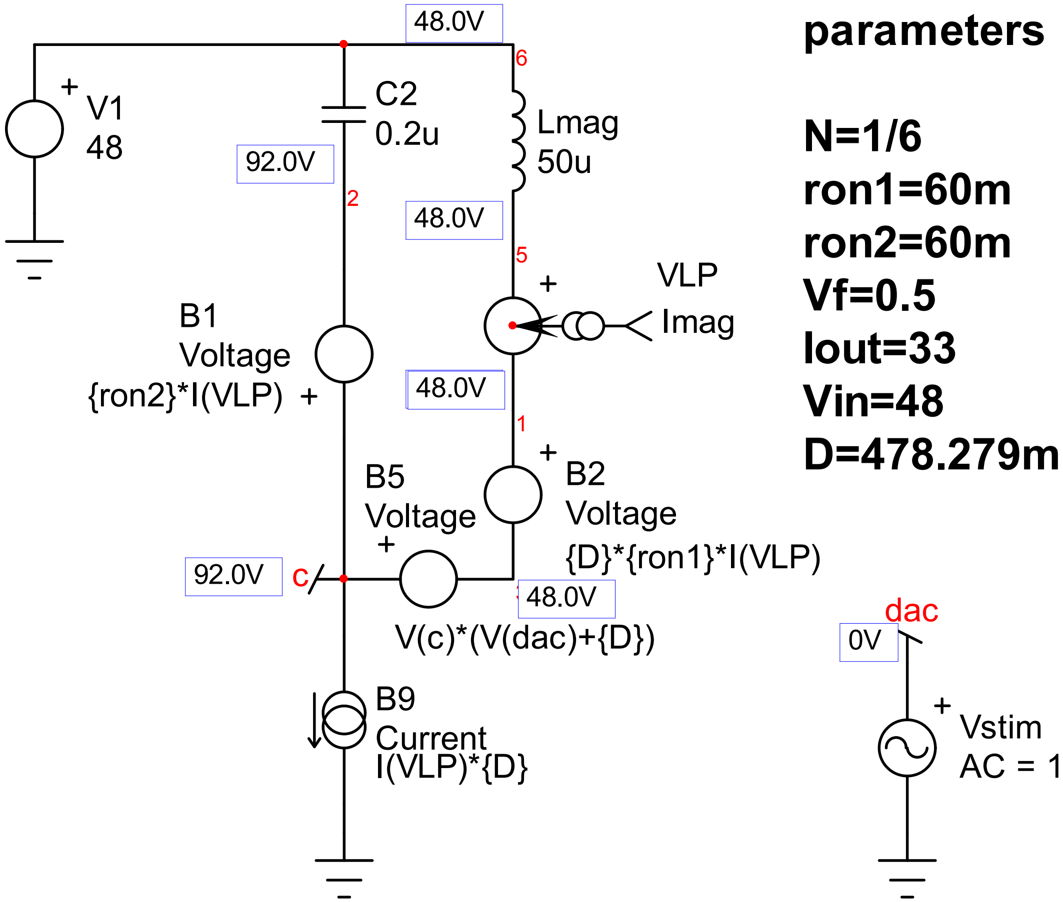
$$H(s) = A_0 \frac{1}{1+\left(\frac{\omega_0}{s}+\frac{s}{\omega_0}\right)Q}$$

$$\frac{I_{mag}(s)}{D(s)} = M_0 \frac{sC_{clp}}{1+\frac{s}{\omega_{0\mathrm{M}}Q_{\mathrm{M}}}+\left(\frac{s}{\omega_{0\mathrm{M}}}\right)^2} \longrightarrow \frac{I_{mag}(s)}{D(s)} = A_0 \frac{1}{1+\left(\frac{\omega_{0\mathrm{M}}}{s}+\frac{s}{\omega_{0\mathrm{M}}}\right)Q_{\mathrm{M}}}$$

$$A_0 = \frac{V_{clamp}}{r_{on2}(1-D_0)+D_0 r_{on1}} \longrightarrow \left|A_0(\omega_{0\mathrm{M}})\right| = 67.713\ \mathrm{dB}$$

Time for an ac Check

- SPICE can obtain the ac response in a snap-shot

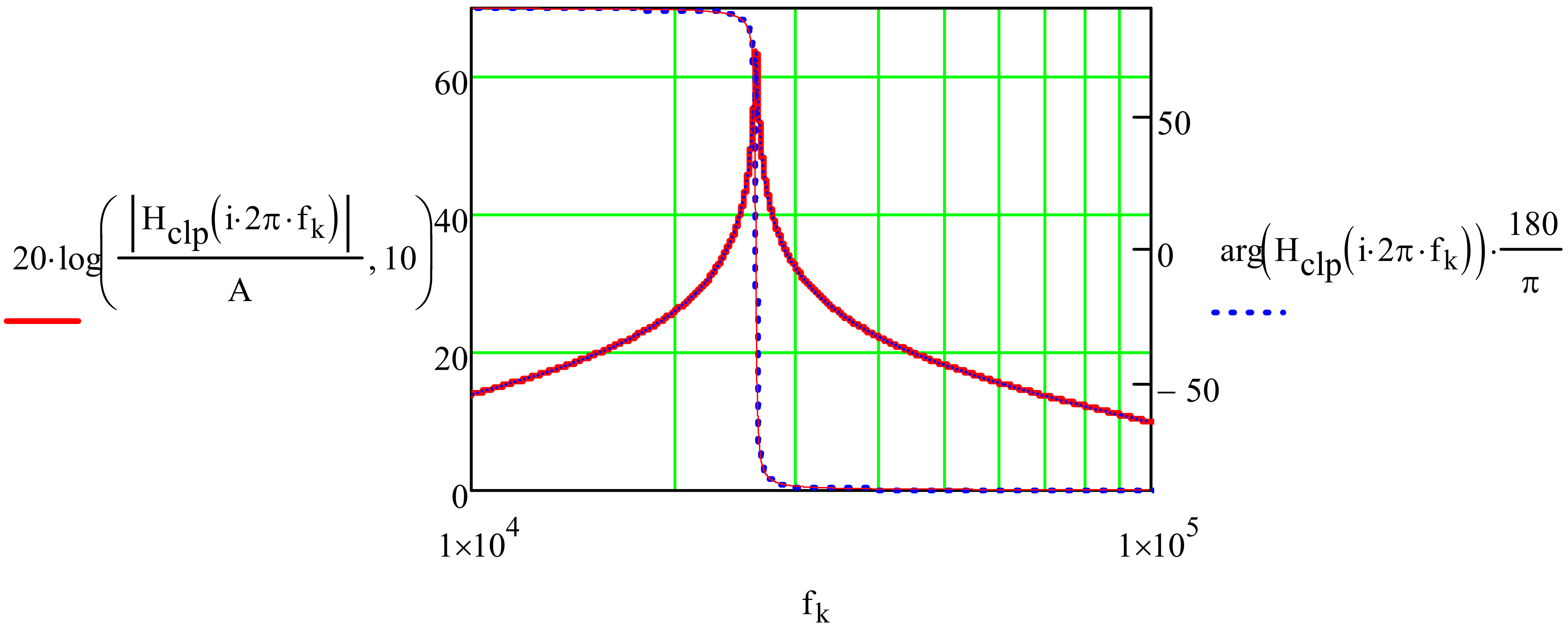


parameters

N=1/6
ron1=60m
ron2=60m
Vf=0.5
Iout=33
Vin=48
D=478.279m

Curves Perfectly Superimpose

- ❑ It is important to obtain similar plots, otherwise: error!

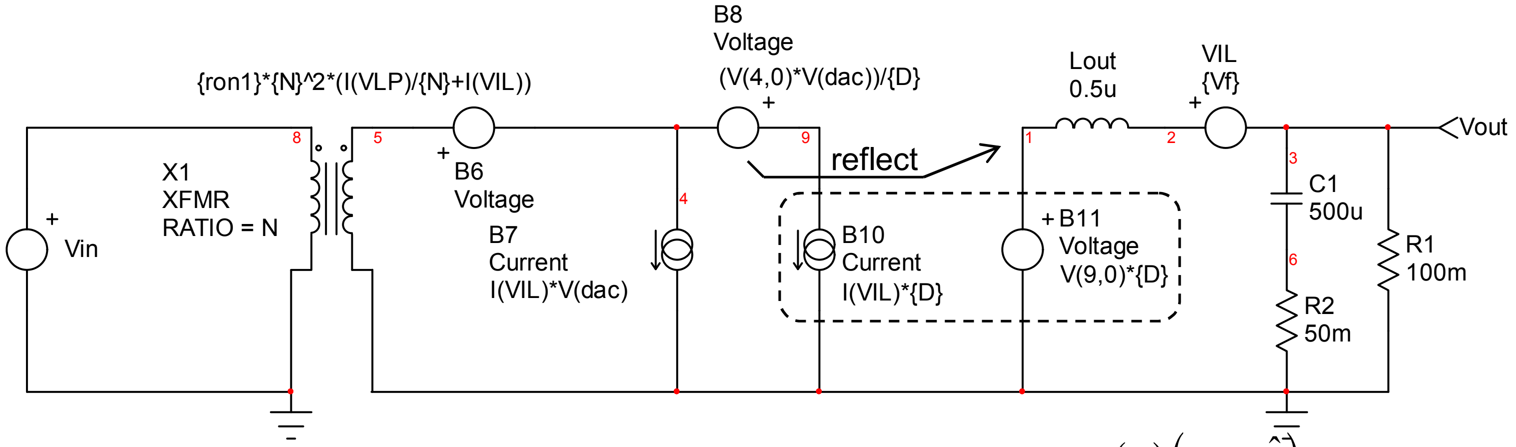


Max SPICE is 63.710 dB
Max Mathcad® is 63.713 dB

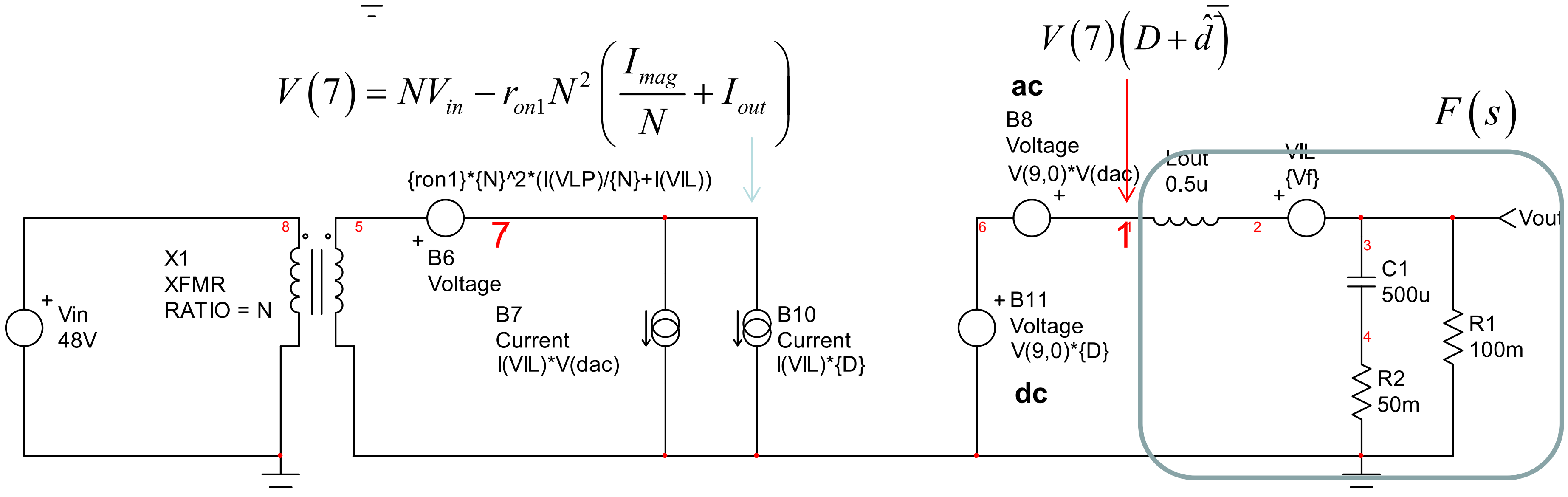
"Can do!"

Run Another Round of Rearrangement

- Now concentrate on the buck output stage only



$$V(7) = NV_{in} - r_{on1}N^2\left(\frac{I_{mag}}{N} + I_{out}\right)$$



Further Simplification is Necessary

- Extract the ac current from the equation, $\left\langle i_{mag}(t)\right\rangle_{T_{sw}} = 0$

$$V_{(7)} = NV_{in} - r_{on1}N^2\left(\frac{\hat{i}_{mag}}{N} + \left(I_{out} + \hat{i}_{out}\right)\right) \longrightarrow V_{(1)} = \left[NV_{in} - r_{on1}N^2\left(\frac{\hat{i}_{mag}}{N} + \left(I_{out} + \hat{i}_{out}\right)\right)\right]\left(D + \hat{d}\right)$$

Develop $V_{(1)}$ keep ac terms only

$$DNV_{in} + NV_{in}\hat{d} - DN\hat{i}_{mag}r_{on1} - N\hat{d}\hat{i}_{mag}r_{on1} - DI_{out}N^2r_{on1} - DN^2\hat{i}_{out}r_{on1} - I_{out}N^2\hat{d}r_{on1} - N^2\hat{d}\hat{i}_{out}r_{on1}$$

dc ≈ 0 dc ≈ 0

Rearranging the result, we obtain:

$$NV_{in}\hat{d} - DN\hat{i}_{mag}r_{on1} - DI_{out}N^2r_{on1} - DN^2\hat{i}_{out}r_{on1} - I_{out}N^2\hat{d}r_{on1}$$

$$\hat{d}\left(NV_{in} - I_{out}N^2\hat{d}r_{on1}\right) - DN\hat{i}_{mag}r_{on1} - DN^2\hat{i}_{out}r_{on1}$$

$$r_{on1} \ll 1 \qquad \hat{d} \ll 1 \qquad N^2 \ll 1$$

Neglecting small terms, we finally obtain:

$$\hat{v}_{(1)} = \hat{d}NV_{in} - DN\hat{i}_{mag}r_{on1}$$

Add the Second-Order Response of LC Filter

The magnetizing current definition is:

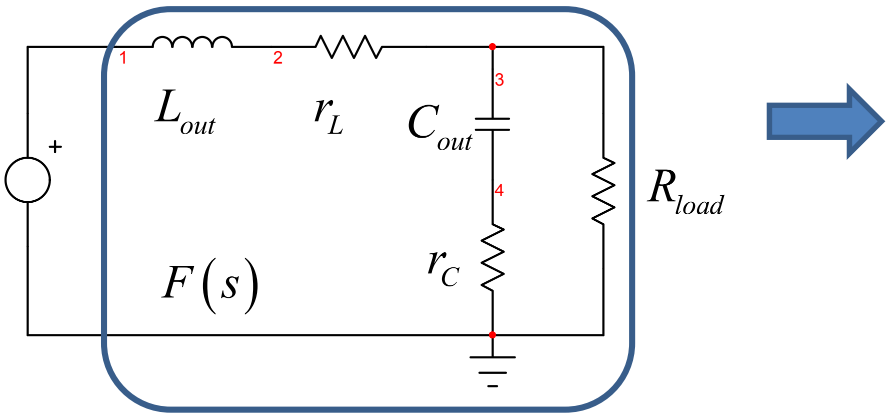
$$\frac{I_{mag}(s)}{D(s)} = M_0 \frac{sC_{clp}}{1+\frac{s}{\omega_{0M}Q_M}+\left(\frac{s}{\omega_{0M}}\right)^2} = M(s)$$

Once re-injected in to the previous definition, we have:

$$V_1(s) = D(s)NV_{in} - D_0 Nr_{on1}D(s)M(s)$$

$$V_1(s) = D(s)\left[NV_{in} - D_0 Nr_{on1}M(s)\right]$$

The source is filtered by the 2nd-order LC filter:



$$F(s) = F_0 \frac{1+\frac{s}{s_{zF}}}{1+\frac{s}{\omega_{0F}Q_F}+\left(\frac{s}{\omega_{0F}}\right)^2}$$

$$F_0 = \frac{R_{Load}}{R_{load}+r_L} \qquad \omega_{zF} = \frac{1}{r_C C_{out}}$$

$$\omega_{0F} = \frac{1}{\sqrt{L_{out}C_{out}}}\sqrt{\frac{r_L+R_{load}}{r_C+R_{load}}} \qquad Q_F = \frac{L_{out}C_{out}\omega_{0F}\left(r_C+R_{Load}\right)}{L_{out}+C_{out}\left(r_L r_C+R_{load}\left(r_L+r_C\right)\right)}$$

We Have the Final Transfer Function

- The transfer function reveals the mag. current contribution

$$\frac{V_{out}(s)}{D(s)} = F(s)\left(NV_{in} - D_0 N r_{on1} M(s)\right)$$ → YES!

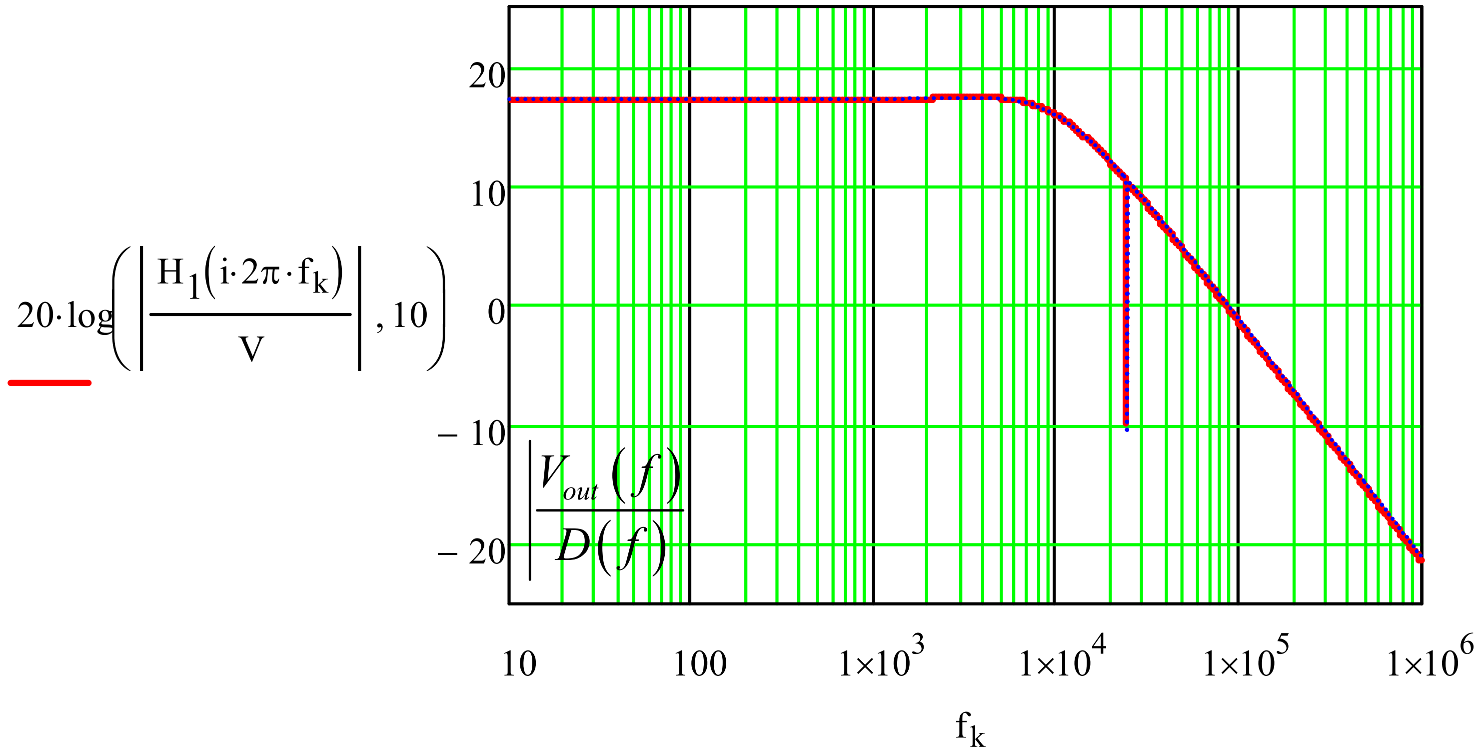
- The mag. current substracts and explains the notch

$$\frac{V_{out}(s)}{D(s)} = F_0 \frac{1+\frac{s}{s_{zF}}}{1+\frac{s}{\omega_{0F}Q_F}+\left(\frac{s}{\omega_{0F}}\right)^2} N \left(V_{in} - D_0 r_{on1} M_0 \frac{sC_{clp}}{1+\frac{s}{\omega_{0M}Q_M}+\left(\frac{s}{\omega_{0M}}\right)^2} \right)$$

- This is the control-to-output transfer function of the ACF

Final Sanity Check - Magnitude

❑ Compare the analytical ac response with that of SPICE

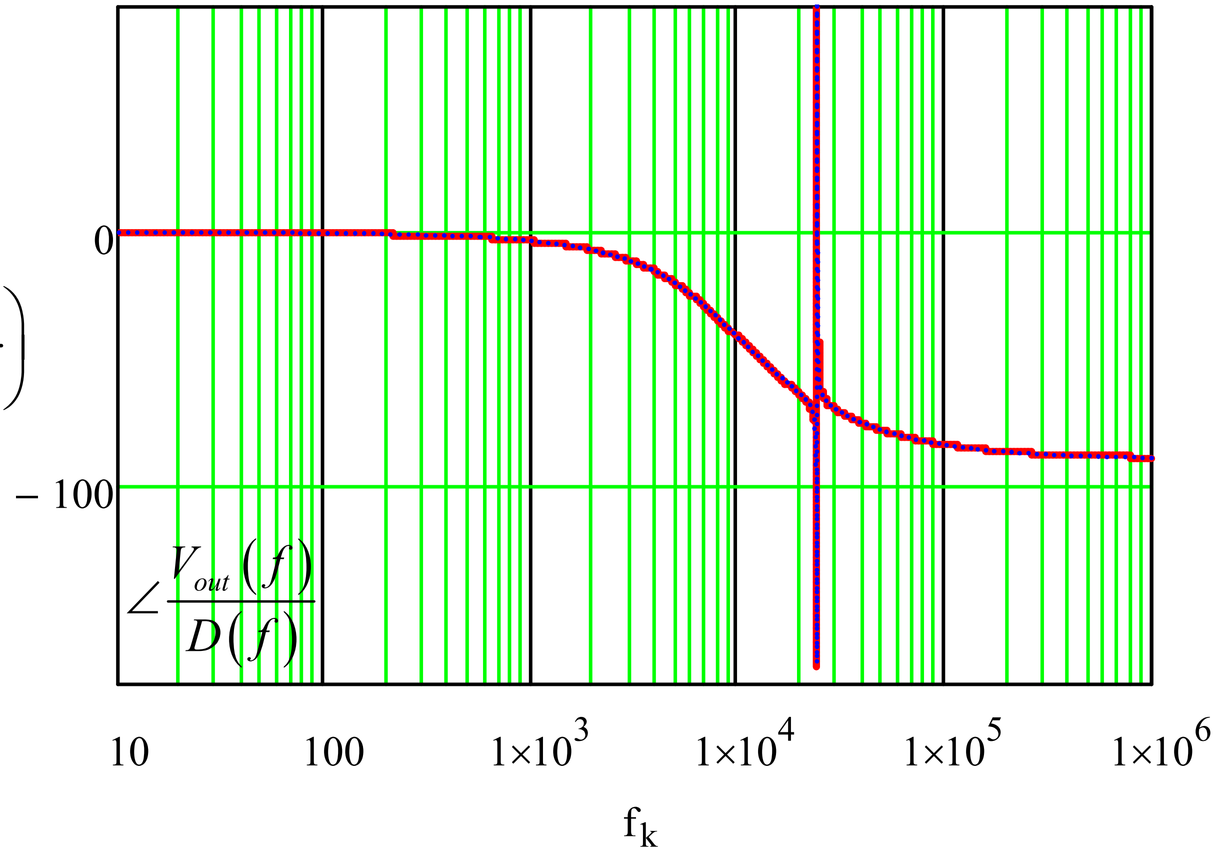


❑ Magnitude curves superimpose perfectly

Final Sanity Check - Phase

- Compare the analytical ac response with that of SPICE

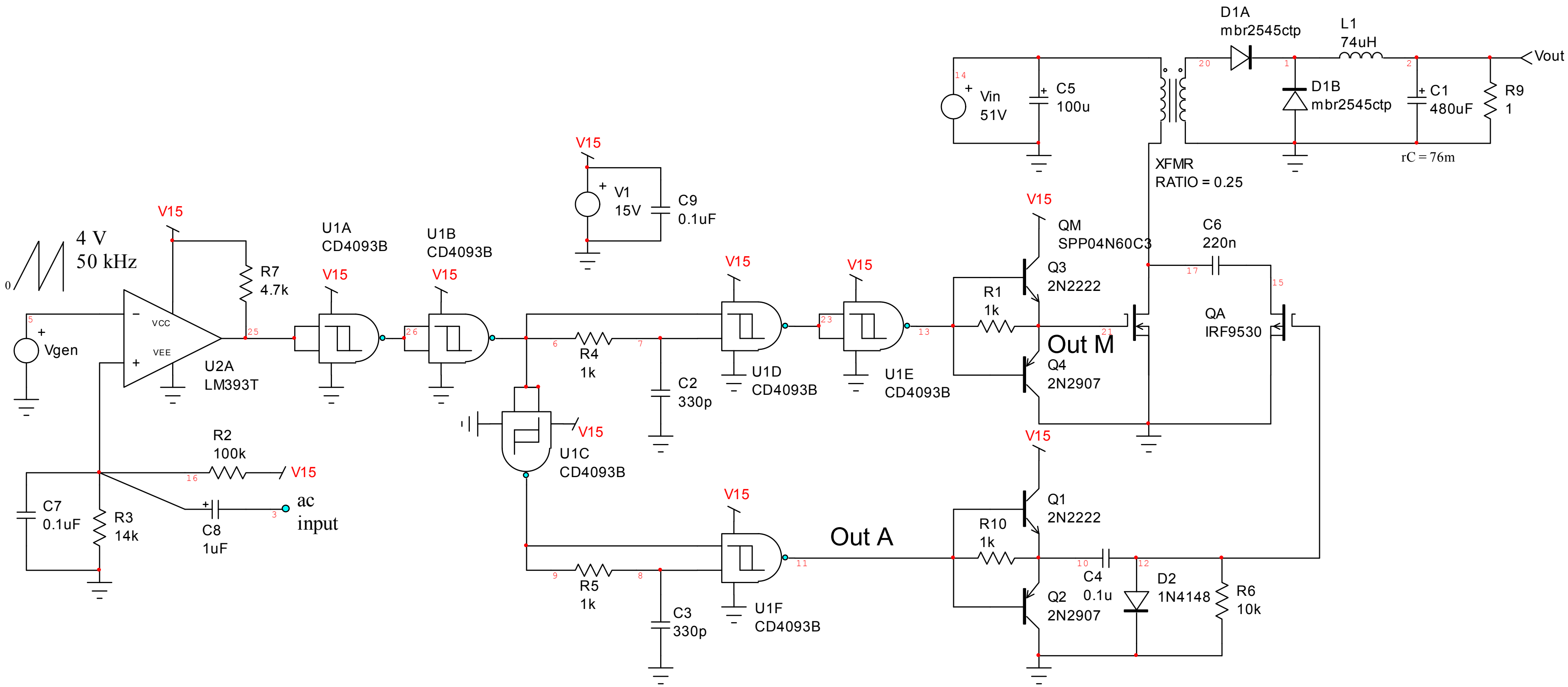
$$\left(\arg\left(H_1\left(i\cdot 2\pi\cdot f_k\right)\right)\cdot\frac{180}{\pi}\right)$$



- Argument curves are in excellent agreement too

The Bench is the Final Referee

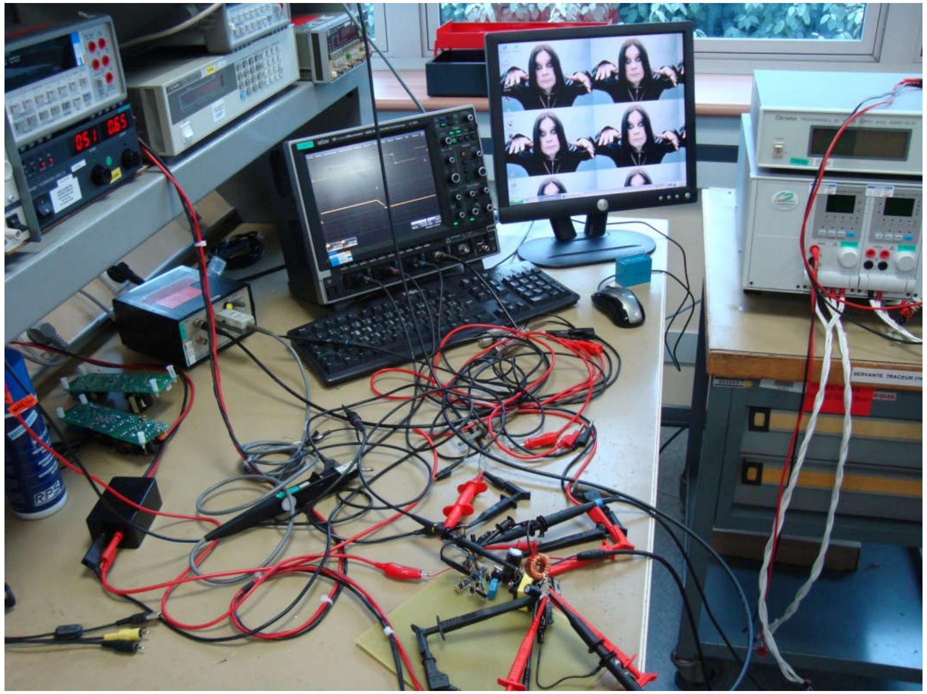
- ❑ Build a hardware using simple gates arrangements

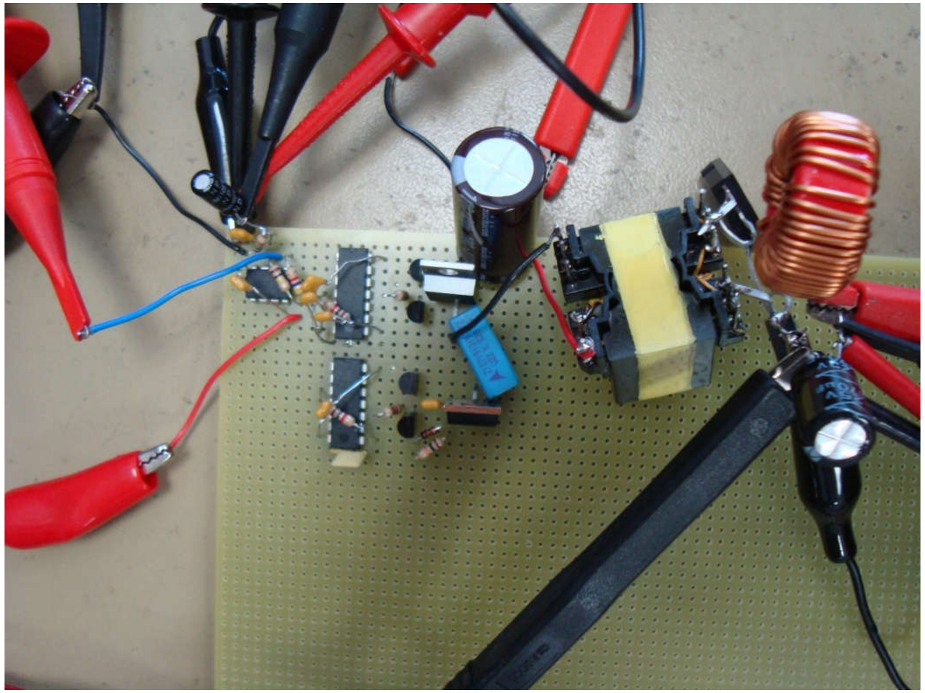


- ❑ This active clamp forward converter delivers 5 V/5 A

The Prototype Hardware

❑ The circuit is assembled using available components

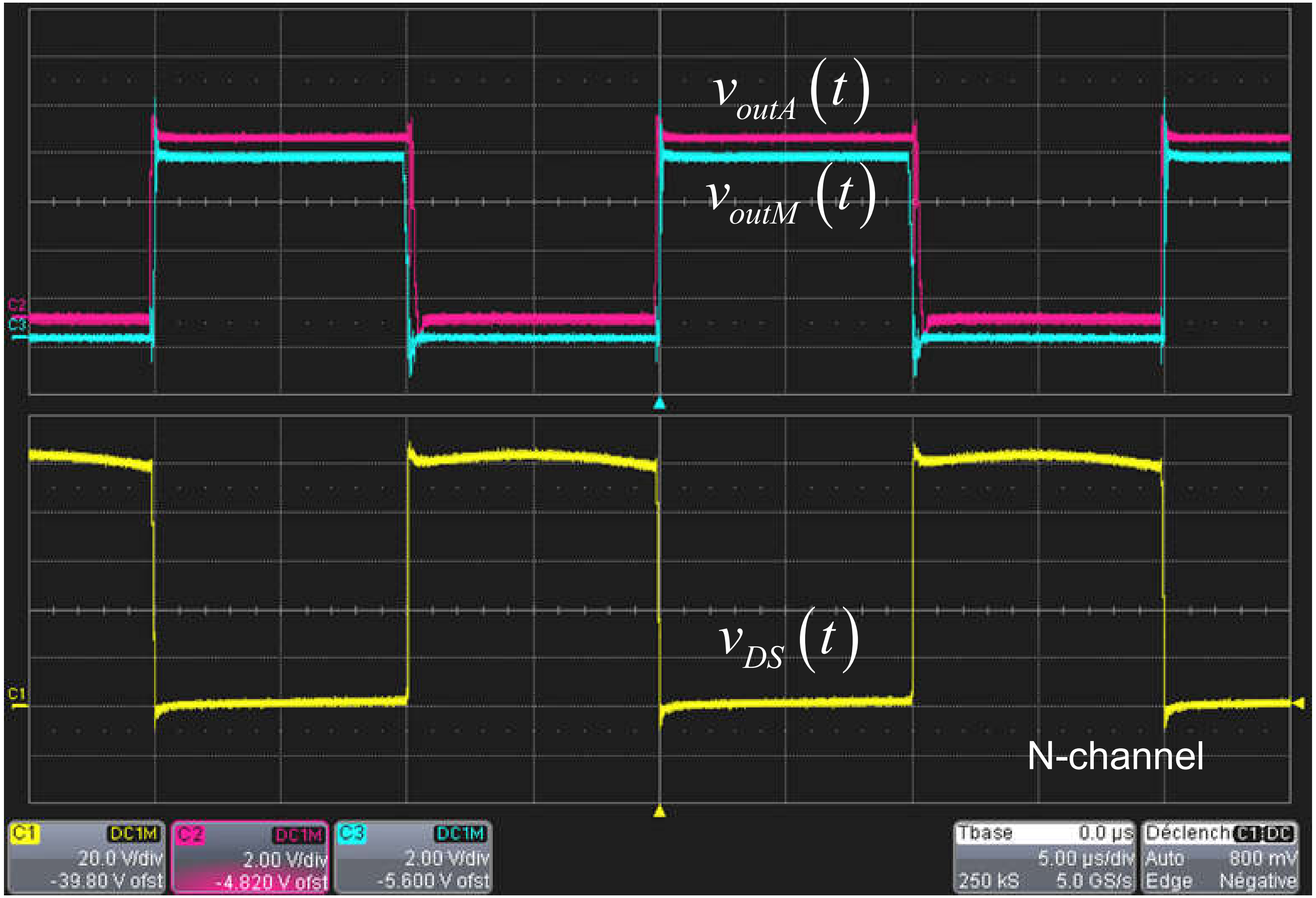




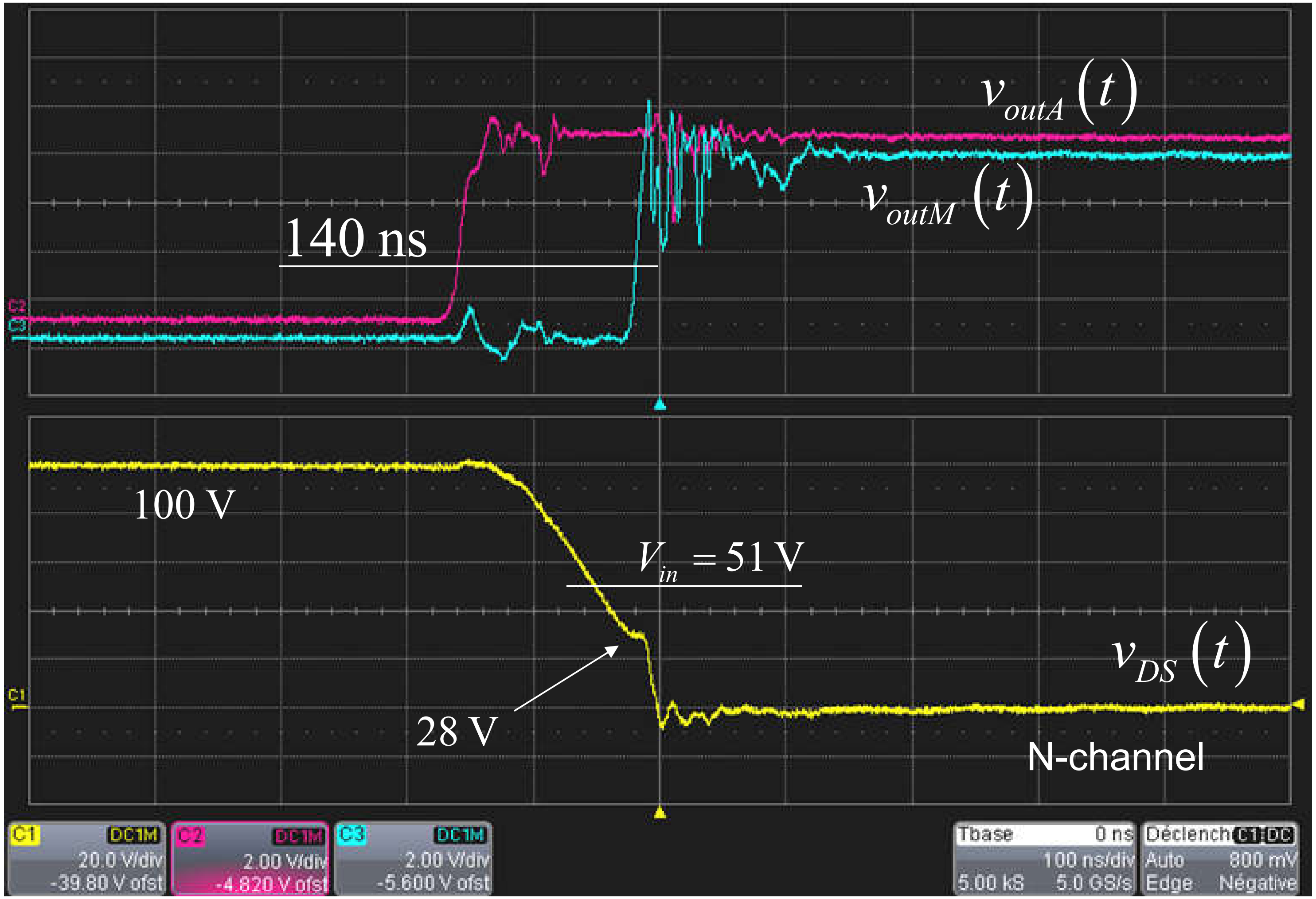
❑ Make sure caps. are well characterized before soldering

With the kind help of Yann Vaquette, Application engineer

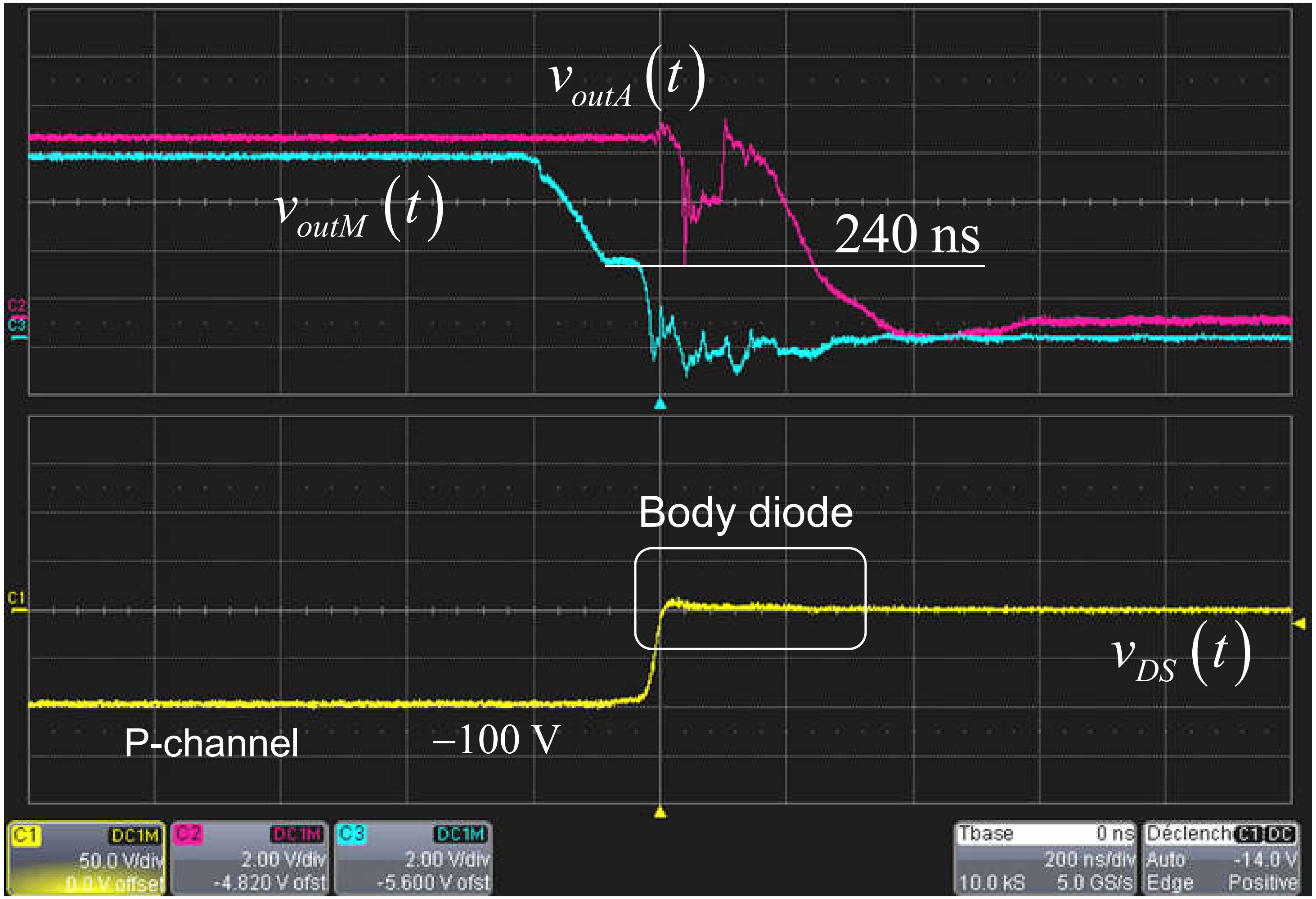
Typical Prototype Waveforms



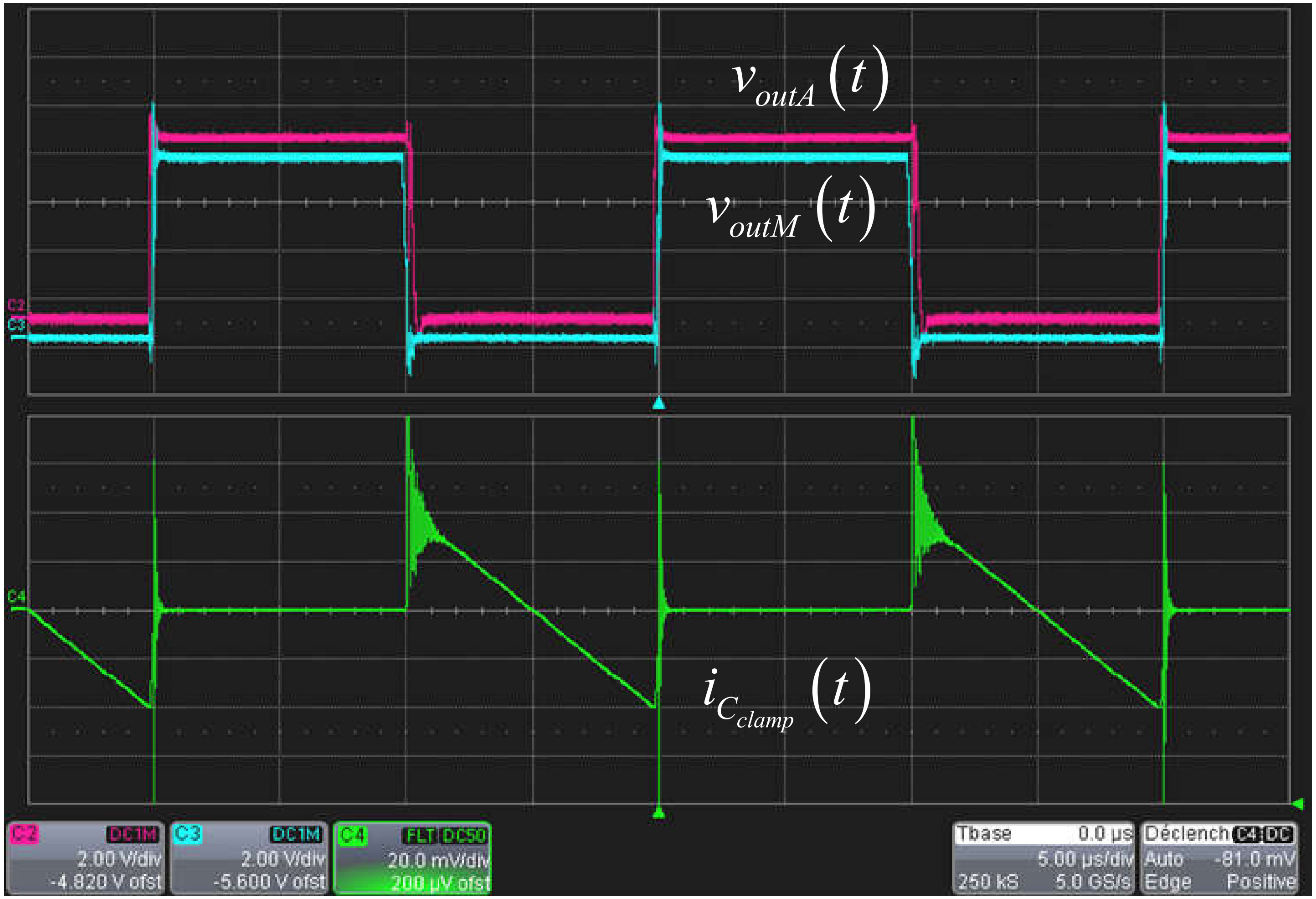
Quasi-ZVS is Ensured on the Drain



ZVS is Also Ensured on Clamp Switch

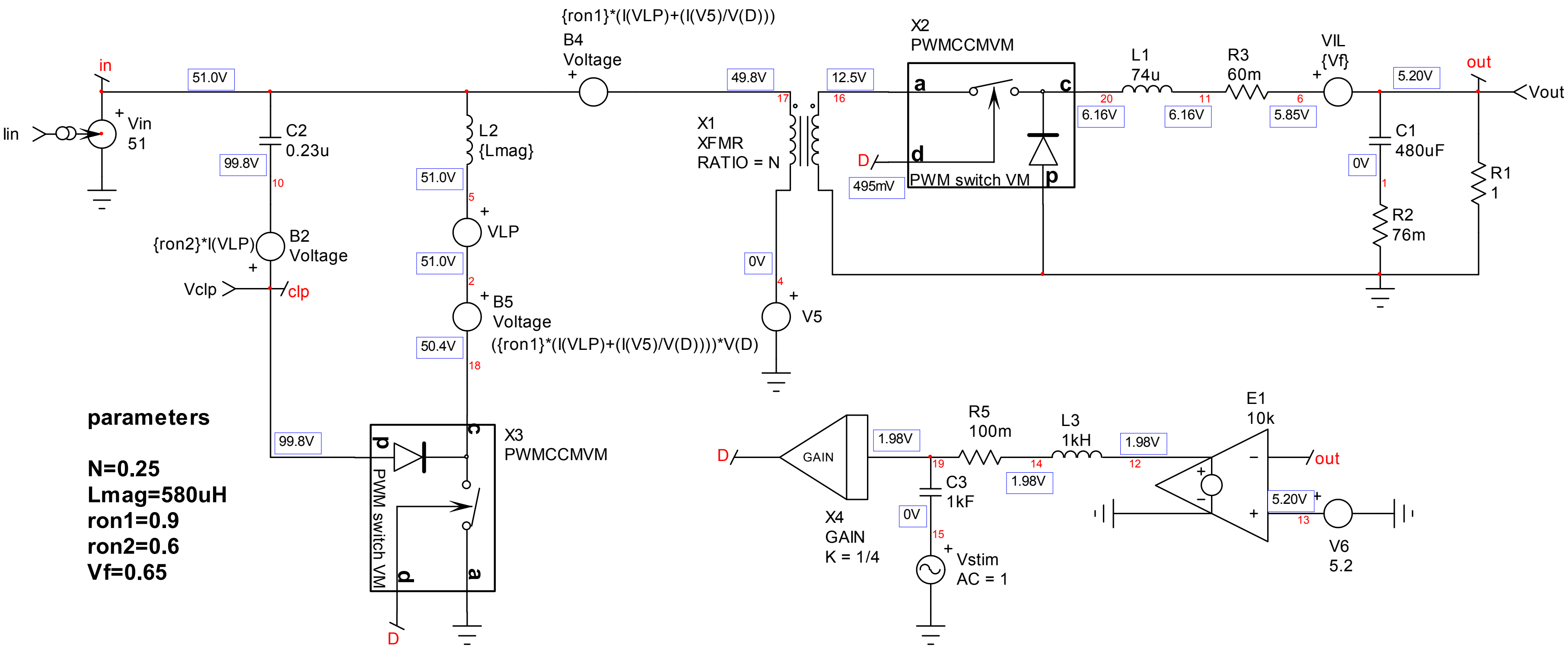


Current in the Clamping Network



SPICE Test Fixture

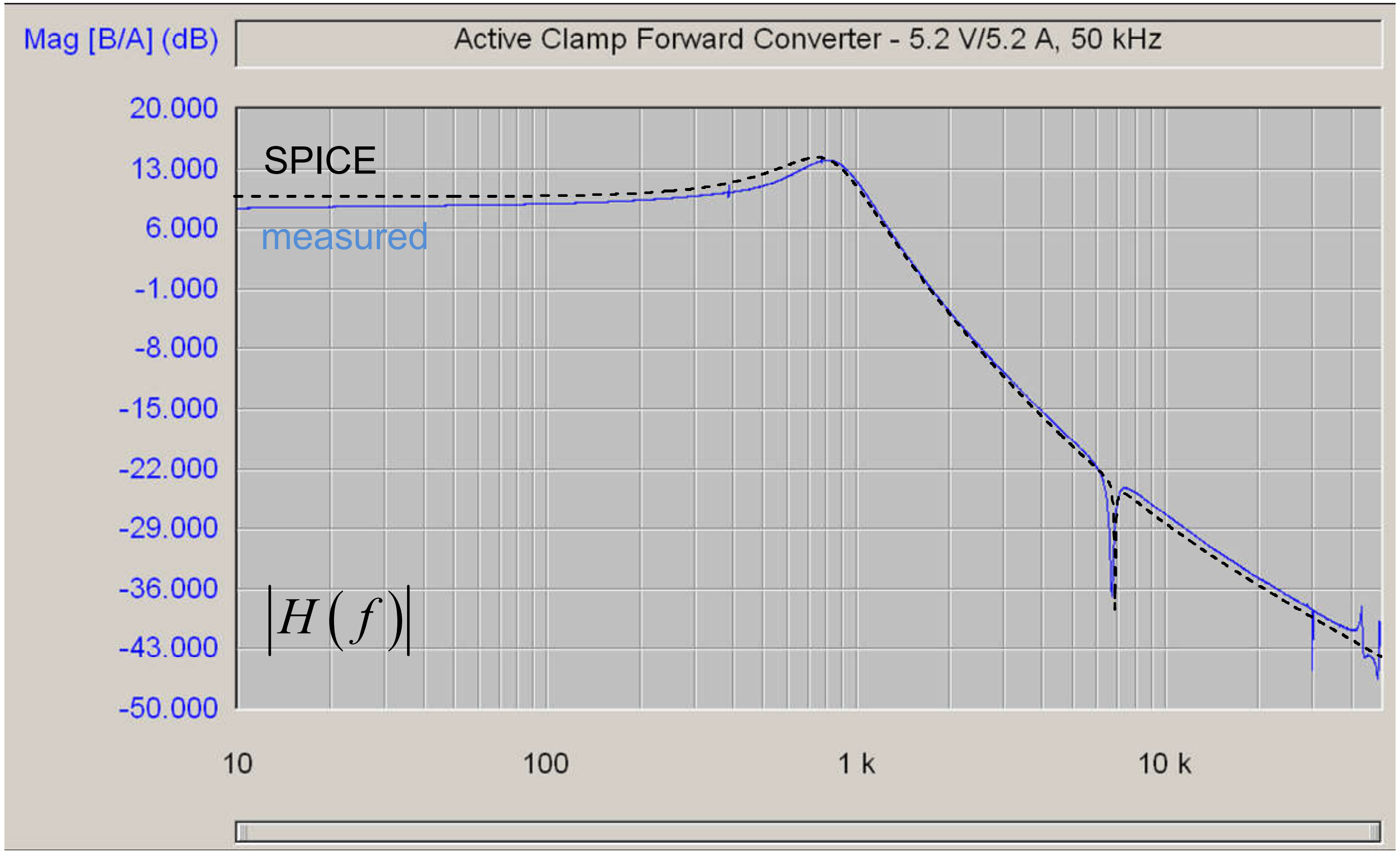
- We have used the following SPICE simulation fixture



- Check dc points versus hardware values: ok

Magnitude Response

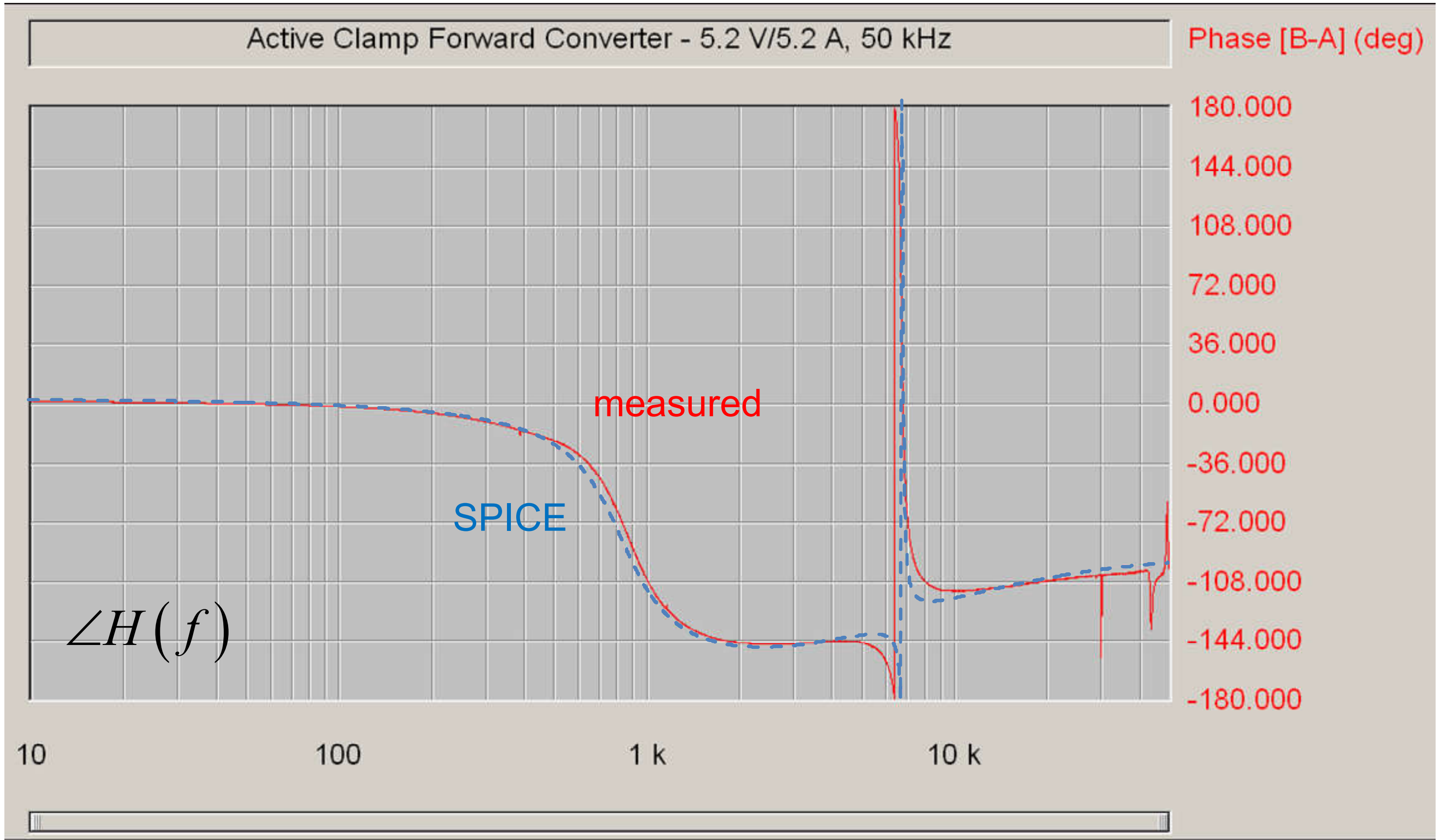
- There is a slight shift but overall agreement is good



- cap. ESR is often the offender in loop measurement

Phase Response

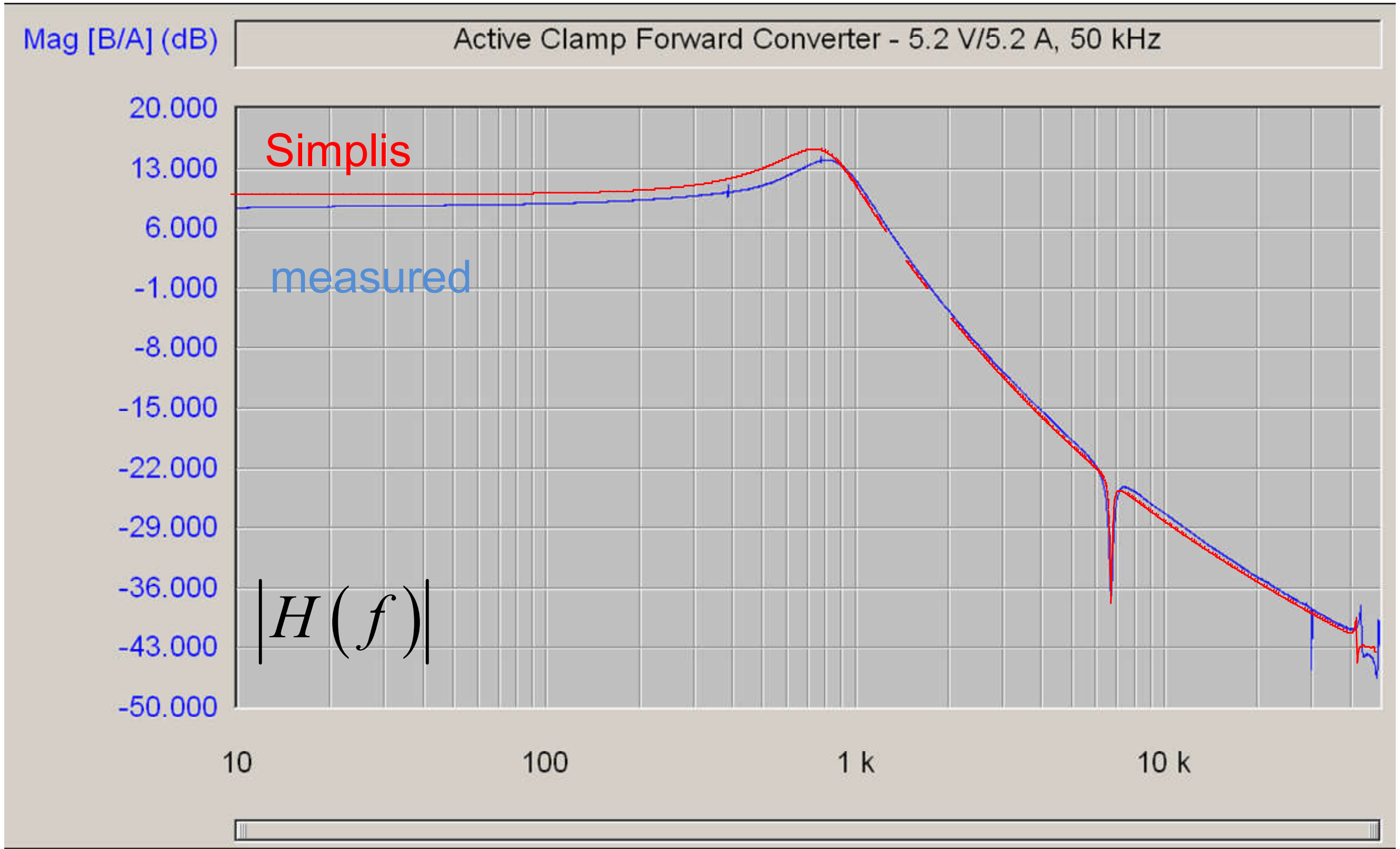
- ❑ Good agreement between curves, expecially peaking



- ❑ The notch Q depends on resistive elements $r_{DS(on)}$ etc.

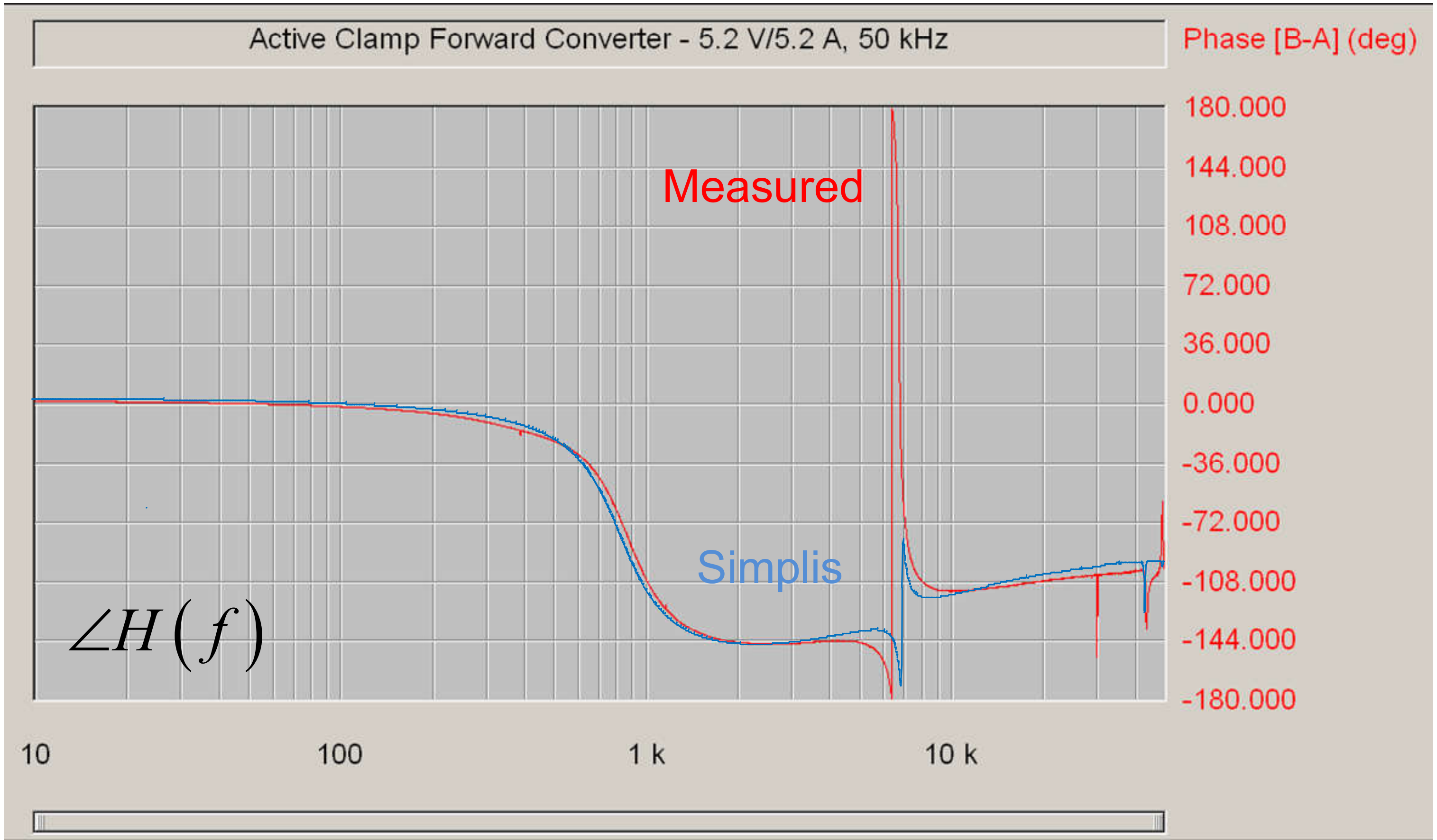
Comparison with Simplis Response

❑ Excellent agreement between curves, almost no shift



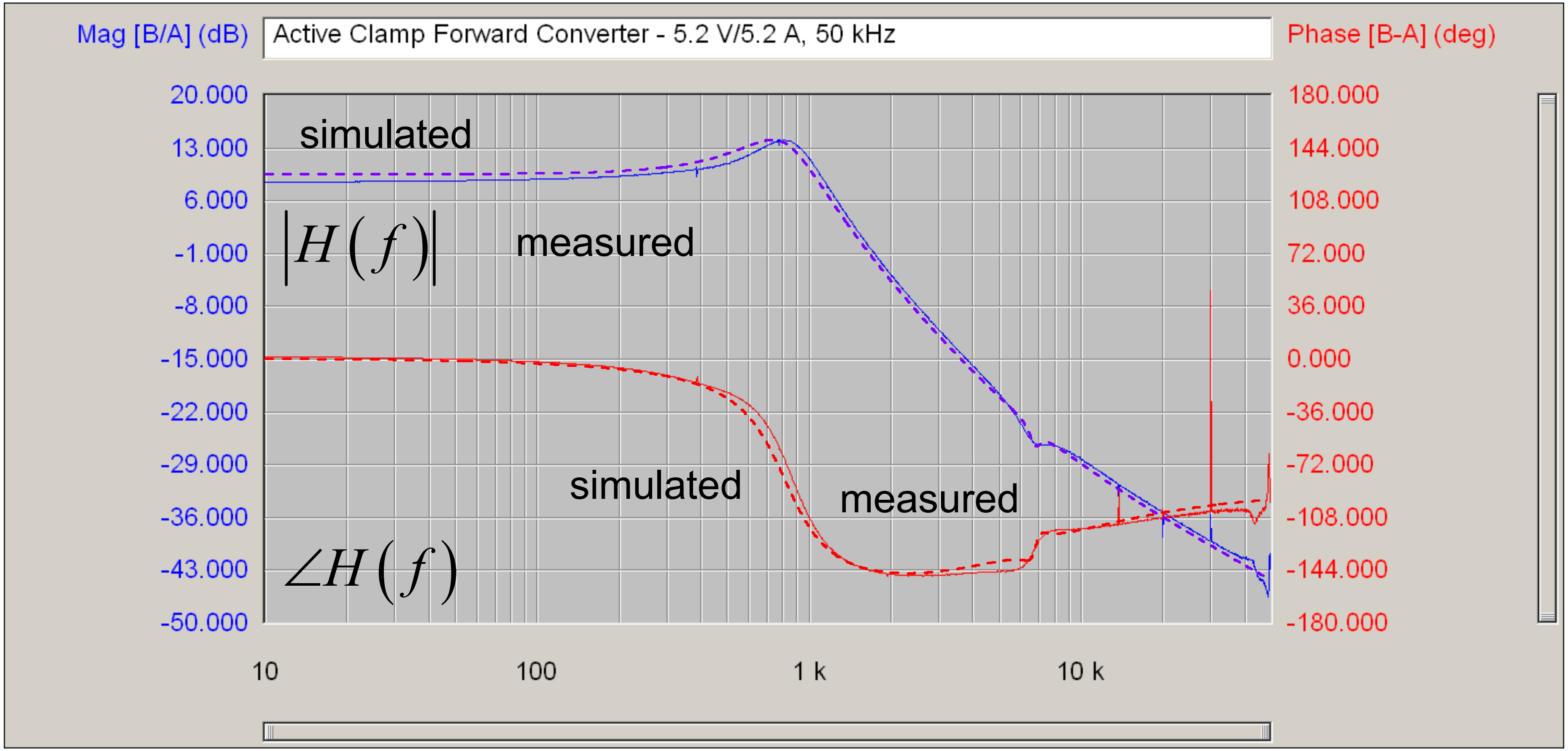
Comparison with Simplis Response

- Despite a slightly lower peaking, phase agreement is ok



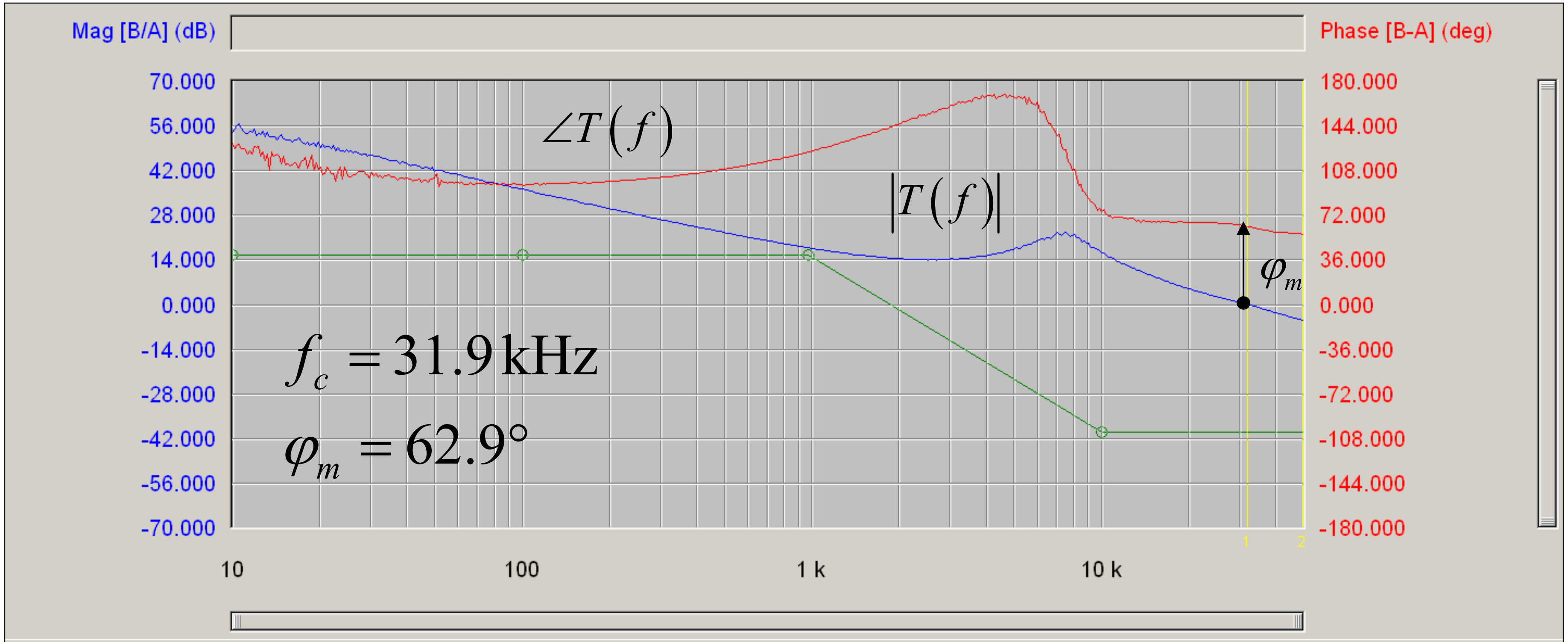
Simulation also Confirms Damping!

- Inserting a 10-Ω resistance damps the notch nicely



A Practical Case

- ❑ The model helped to stabilize this 3.3-V/30-A converter



Data	M1	M2	M2 - M1
Frequency	31.91 kHz	49.57 kHz	17.66 kHz
Magnitude	-0.066 dB	Range	Range
Phase	62.900 deg	Range	Range
Var. Source	5.00 mV	5.00 mV	0.00 V

Transient Response Test

- Step-load transient response confirms stability margins

V_{IN} = 36 V, 15 A to 22.5 A - Slew rate 1 A / µs

ON Semiconductor®

Conclusion

- ❑ The PWM switch model is an essential tool for modeling
- ❑ We have seen how to derive it in different operating modes
- ❑ Small-signal modeling using the PWM switch is simple and fast
- ❑ When modeling converters, always proceed step by step
- ❑ Always perform intermediate sanity checks (SPICE, Mathcad®…)
- ❑ Analytical analysis does not shield you against lab. experiments
- ❑ Analysis, simulation and bench: the path to success!