Simulation and Analysis Applied to the Design of Buck Topologies

Christophe Basso – Technical Fellow
IEEE Senior Member

ON Semiconductor®

Chris Basso APEC Seminars
2009 – 2019
Tenth anniversary!

Course Agenda

- ☐ The Buck Converter
- ☐ Control Schemes
- ☐ Introduction to Modeling
- ☐ The PWM Switch in Current-Mode Control
- ☐ Introduction to the Fast Analytical Techniques
- ☐ The Four Transfer Functions in CCM Current-Mode Control
- ☐ Compensation Strategy
- ☐ Prototype Measurements

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☐ Compensation Strategy

☐ Prototype Measurements

Input Voltage Reduction: Linear or Switching?

☐ How do you step-down the input source?

❖ A linear regulator

- Poor efficiency at high input-output differentials → $\eta \approx \frac{V_{out}}{V_{in}}$
- Constrained to low input voltages

✓ low-noise linear operation

❖ A switching regulator

- Noisy operation, requires filtering
- Built with energy-storing components

✓ Excellent efficiency

✓ Works across a wide input voltage range

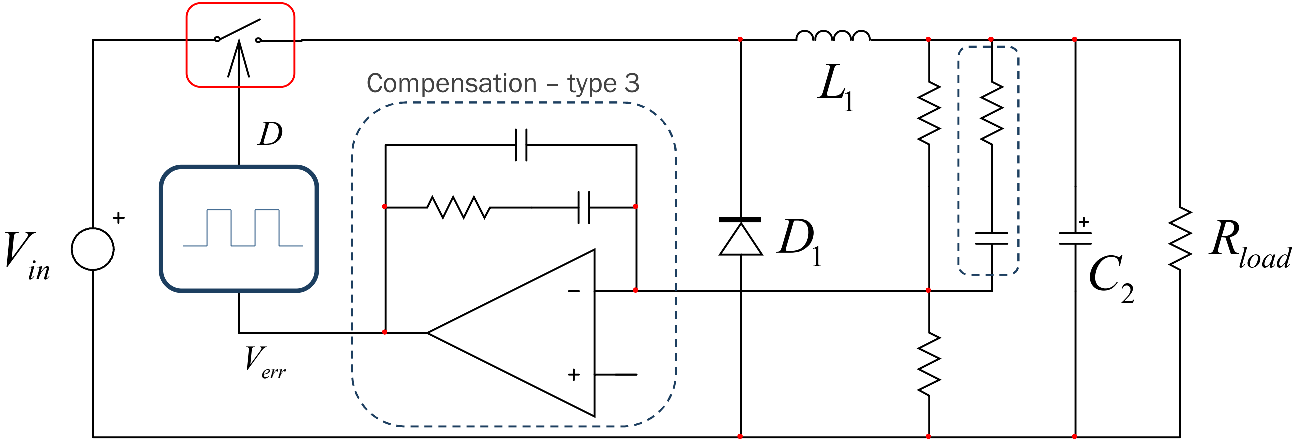
The buck converter

Principles of Operation in Voltage-Mode Control

☐ The buck converter can be operated in voltage-mode control

- Control-to-output transfer function changes between operating modes

✓ No need to sense the inductor current*



✓ Inherently-low open-loop output impedance

❖ Mediocre input line rejection

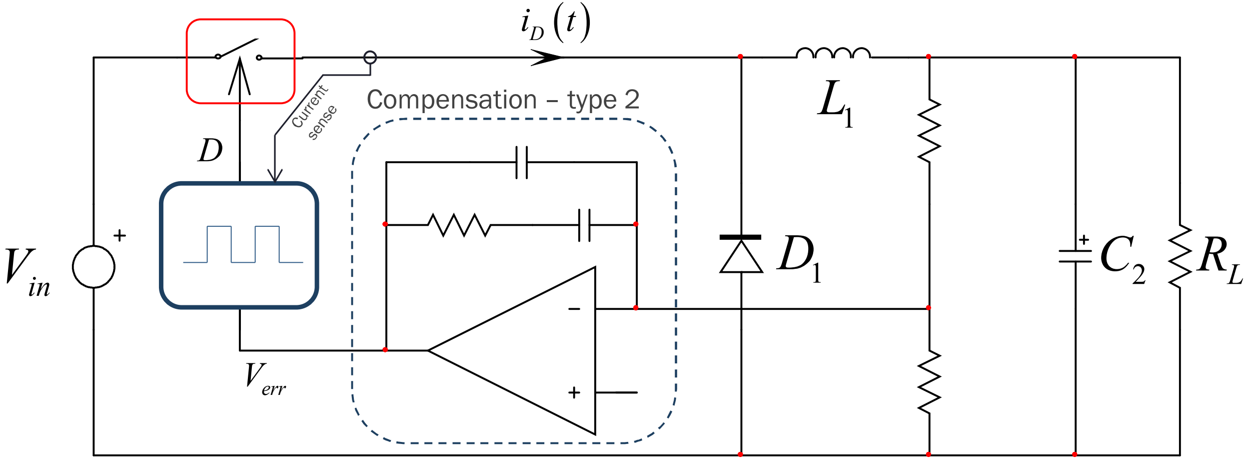
* Beside over-current protection purposes

Principles of Operation in Current-Mode Control

☐ The buck converter can be operated in current-mode control

✓ 1^{st}–order response in low frequencies whether CCM or DCM operation

- Unstable for duty ratios approaching 50% — CCM needs slope compensation

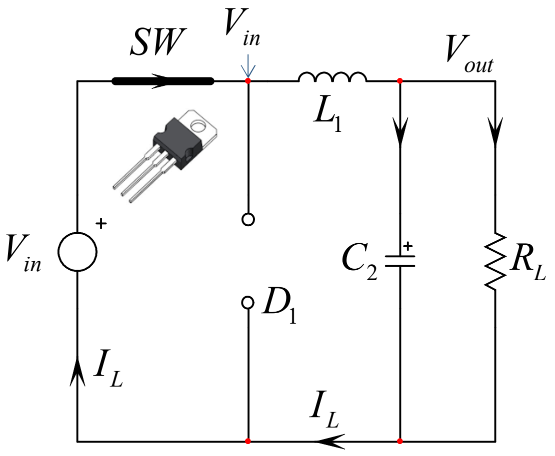


- Mediocre open-loop output impedance

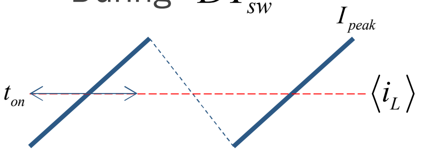
✓ Inherent good input line rejection

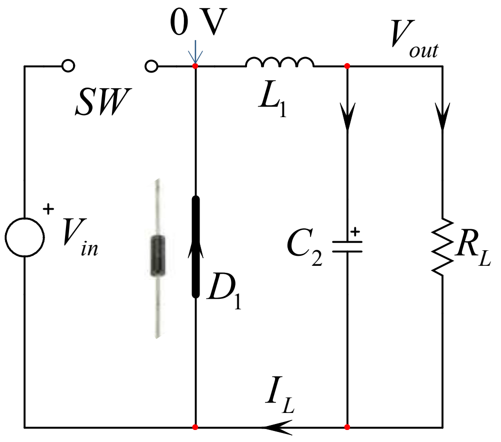
The Two Operating Phases of the Buck Converter

- ☐ Neglecting drops, the common *SW-D* node swings between V_{in} and 0 V

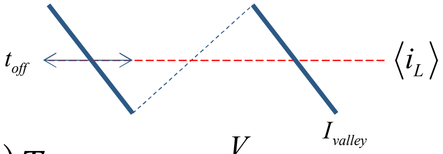


During DT_{sw}



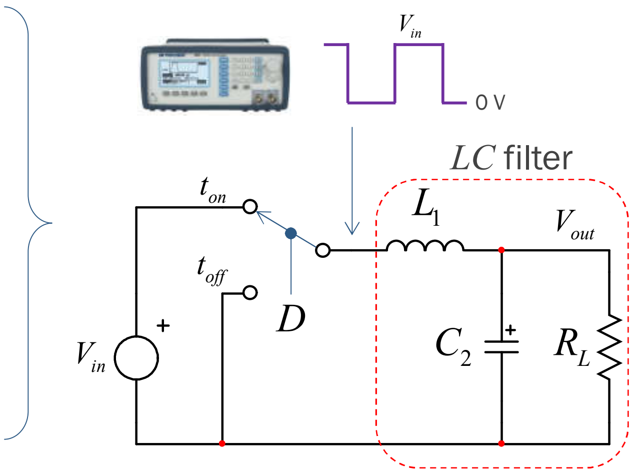


During $(1-D)T_{sw}$



$$S_{on} = \frac{V_{in} - V_{out}}{L_1} \qquad T_{sw} = DT_{sw} + (1-D)T_{sw} \qquad S_{off} = -\frac{V_{out}}{L_1}$$

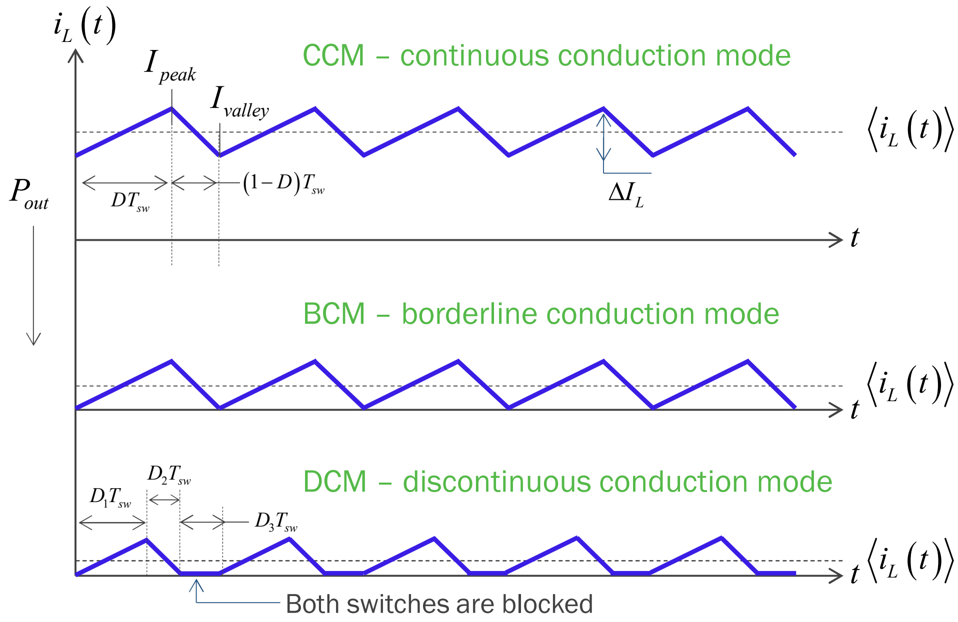
Low-impedance square generator

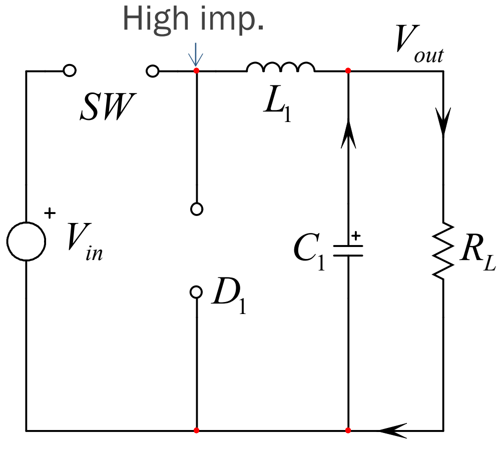


2nd-order system

A Third State Exists when the Inductor Depletes

☐ Observing the inductor current to infer conduction mode





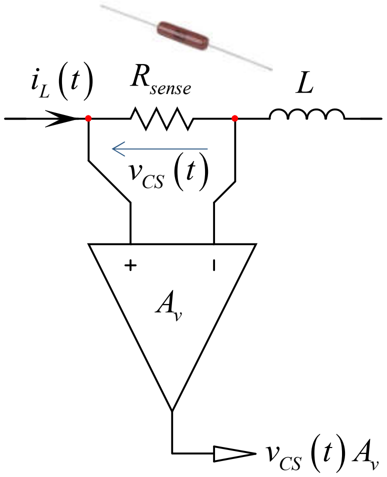
During D_3T_{sw}

$$T_{sw} = t_{on} + t_{off} + D_3T_{sw}$$

❖ Inductive current is in the high-side branch: how to sense?

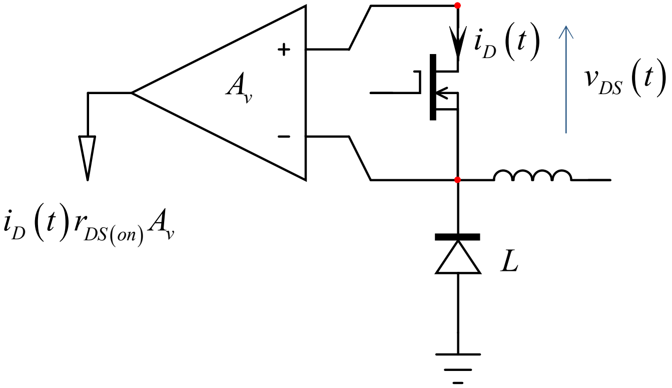
Challenging Current Sensing for Current Mode

❑ Insert a resistance with L



- ✓ Simplest solution
- ✓ Can read I_{out}
- ❖ Efficiency suffers

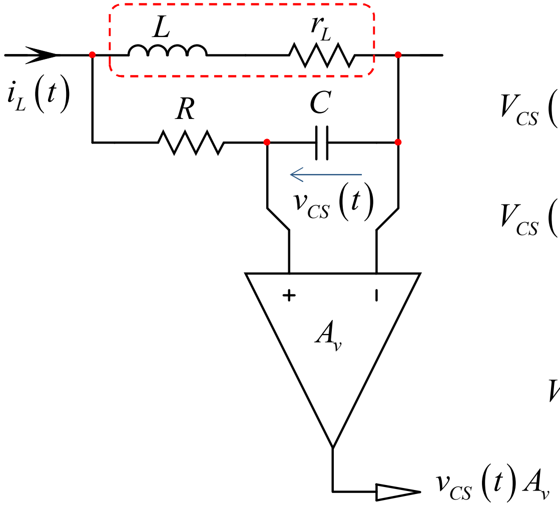
❑ Use MOSFET $r_{DS(on)}$



- ✓ No extra sensing element
- ✓ Can be integrated
- ❖ Temperature-dependent values
- ❖ $r_{DS(on)}$ not tested in production

Dc Resistance Inductor Current Sensing

- ❑ Integrate inductor voltage



$$V_{CS}(s) \approx I_L(s) r_L \frac{1+s\frac{L}{r_L}}{1+sRC}$$

$$V_{CS}(s) = I_L(s) r_L \frac{1+s\tau_1}{1+s\tau_2}$$

$$\Downarrow \quad \tau_1 = \tau_2$$

$$V_{CS}(s) \approx I_L(s) \underset{\uparrow}{r_L}$$

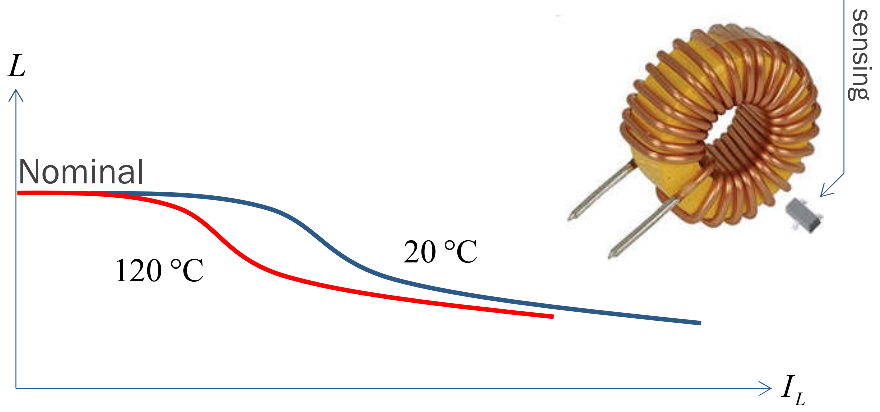
Sensing resistance

- ✓ Most efficient solution
- ✓ Low cost

- ❖ Time constant mismatch

⬇

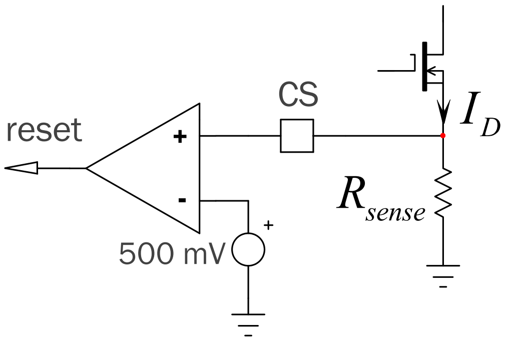
- ➢ inductance varies with A and °C
- ➢ beware of cheap material



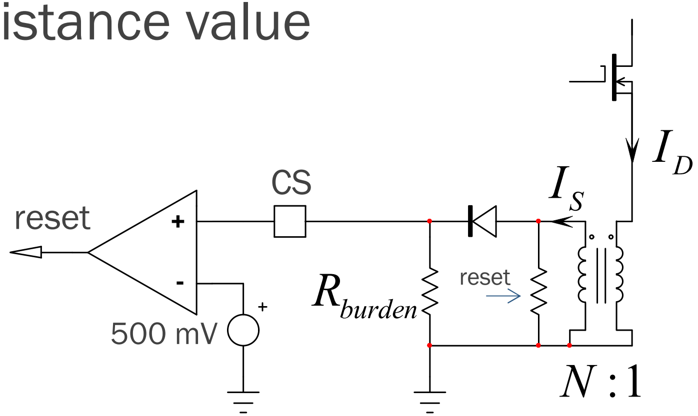
T. Hegarty, *Inductor DCR Current Sensing With Temperature Compensation*, How2Power, November 2013

Sense Transformer for High-Current Applications

- ❑ The transformer references the measured variable to ground
- ❑ Turns ratio N emulates any sense resistance value



$$R_{sense} = \frac{V_{sense,\max}}{I_{D,\max}} = \frac{0.5}{30} = 16.6\,\mathrm{m}\Omega$$

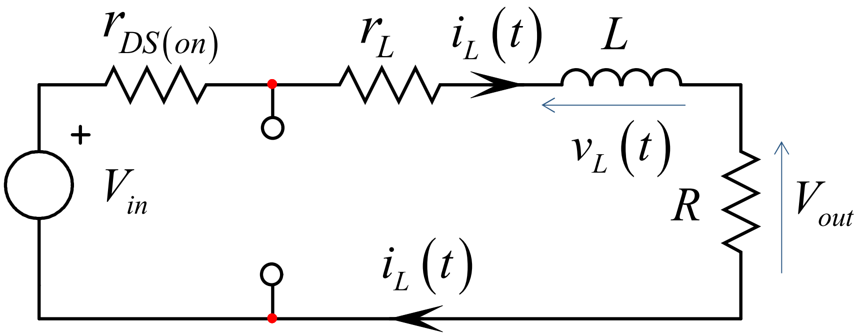


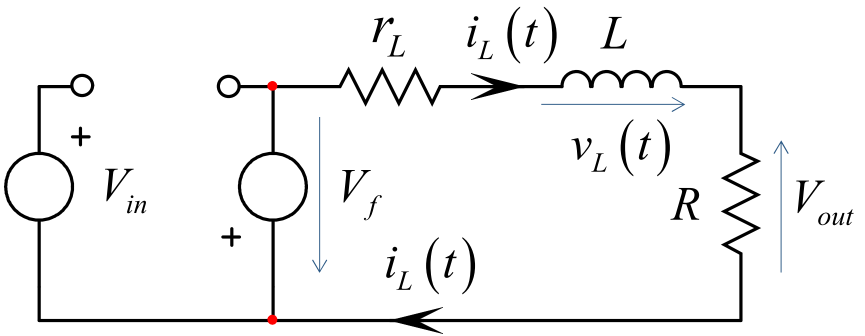
$$R_{burden} = \frac{V_{sense,\max}}{I_{D,\max}} N = \frac{0.5}{30} \times 100 = 1.66\,\Omega$$

- ❖ Transformer reset is necessary (e.g. with a Zener diode)
- ❖ Can only sense ac currents

Parasitic Terms Affect Conversion Ratio

- ☐ Apply volt-second balance law*: $\langle v_L(t)\rangle_{T_{sw}} = 0$





$$\langle v_L(t)\rangle_{t_{on}} = DT_{sw}\left[V_{in} - \frac{V_{out}}{R}\left(r_{DS(on)}+r_L\right) - V_{out}\right]$$

$$\langle v_L(t)\rangle_{t_{off}} = (1-D)T_{sw}\left[V_{out}+V_f+\frac{V_{out}}{R}r_L\right]$$

Inductor flux balance implies:

$$\langle v_L(t)\rangle_{t_{on}} - \langle v_L(t)\rangle_{t_{off}} = 0$$

Solve for M

$$\frac{V_{out}}{V_{in}} = M = D\left[\frac{1}{1+\frac{r_{DS(on)}}{R}D+\frac{r_L}{R}+\frac{V_f}{V_{out}}D'}\right]$$

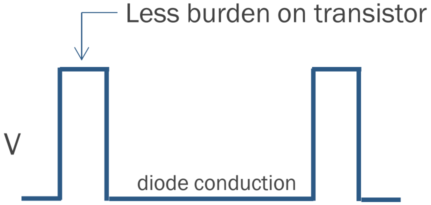
* At steady-state

Analyzing the Conversion Ratio Equation

- Depending on the duty ratio, optimize one of the paths

$$D\left[\frac{1}{1+\frac{r_{DS(on)}}{R}D+\frac{r_L}{R}+\boxed{\frac{V_f}{V_{out}}D'}}\right]$$

12 V to 1.8 V



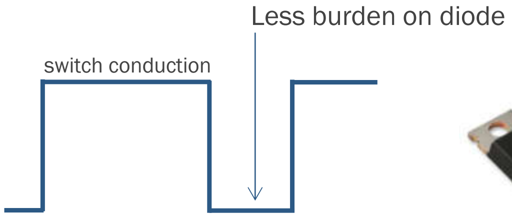
Small duty ratio

Synchronous rectification

duty ratio scales losses

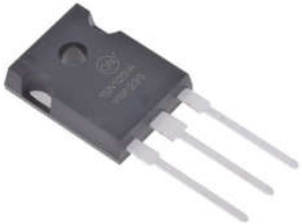
$$D\left[\frac{1}{1+\boxed{\frac{r_{DS(on)}}{R}D}+\frac{r_L}{R}+\frac{V_f}{V_{out}}D'}\right]$$

5 V to 3.3 V



Large duty ratio

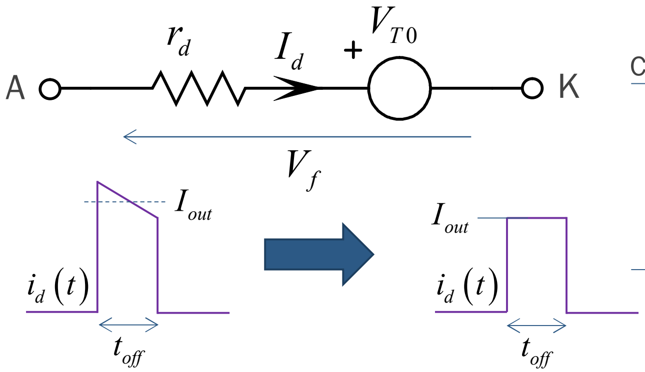
Reduce $r_{DS(on)}$



Continuous conduction mode (CCM) is assumed

Synchronous Rectification

- ❑ Diode conduction losses depend on average and rms currents
  - ❖ As a 1st-order approximation, losses are insensitive to ripple current



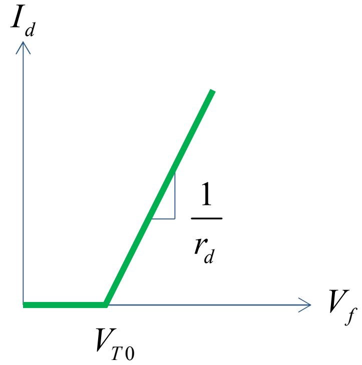
Conduction losses →

$$P_d = V_{T0} I_{d,avg} + r_d I_{d,rms}^{2} \approx V_f I_{d,avg}$$

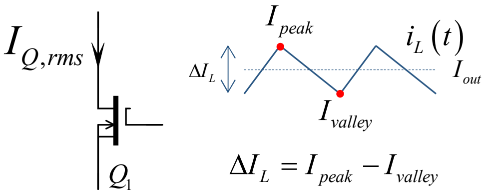
Average current →

$$P_d \approx I_{out}(1-D)V_f$$

Neg. temp co



➢ Replace the diode by a MOSFET and ripple current now matters



Ac ripple contribution

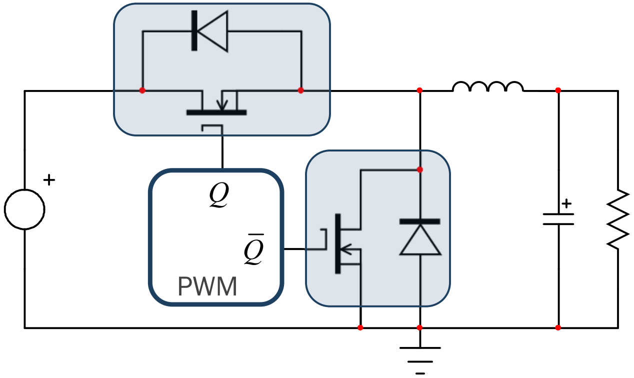
$$P_Q = I_{Q,rms}^{2} r_{DS(on)} = (1-D)\left(I_{out}^{2} + \frac{\Delta I_L^{2}}{12}\right) r_{DS(on)}$$

Pos. temp co

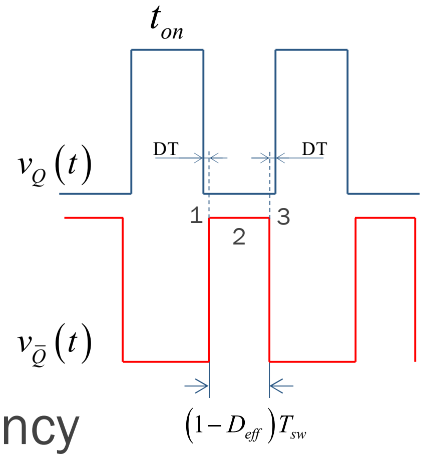
C. Basso, *Switch-Mode Power Supplies: SPICE Simulations and Practical Designs*, McGraw-Hill, 2nd edition, 2014

Synchronous Rectification Body Diode Conduction

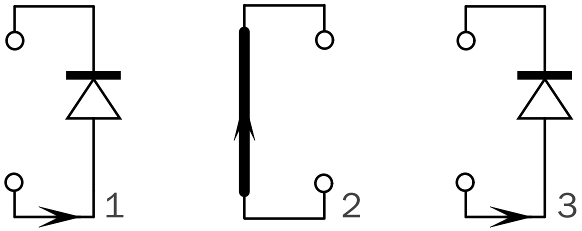
- ☐ Limit shoot-through currents by inserting a dead-time



Insert dead-time



- ➢ Body diode conduction can hamper efficiency



3rd quadrant

MOSFET + body diode

Conduction losses

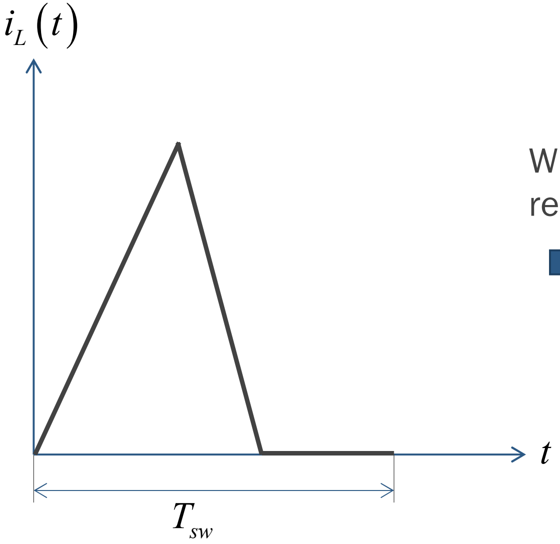
$$P_Q = I_{Q,rms}{}^2 r_{DS(on)} = (1 - D_{eff})\left(I_{out}{}^2 + \frac{\Delta I_L{}^2}{12}\right) r_{DS(on)}$$

$$P_{body} = 2 \cdot \mathrm{DT} \cdot F_{sw} I_{out} V_f$$

Minimize deadtime without shoot-through

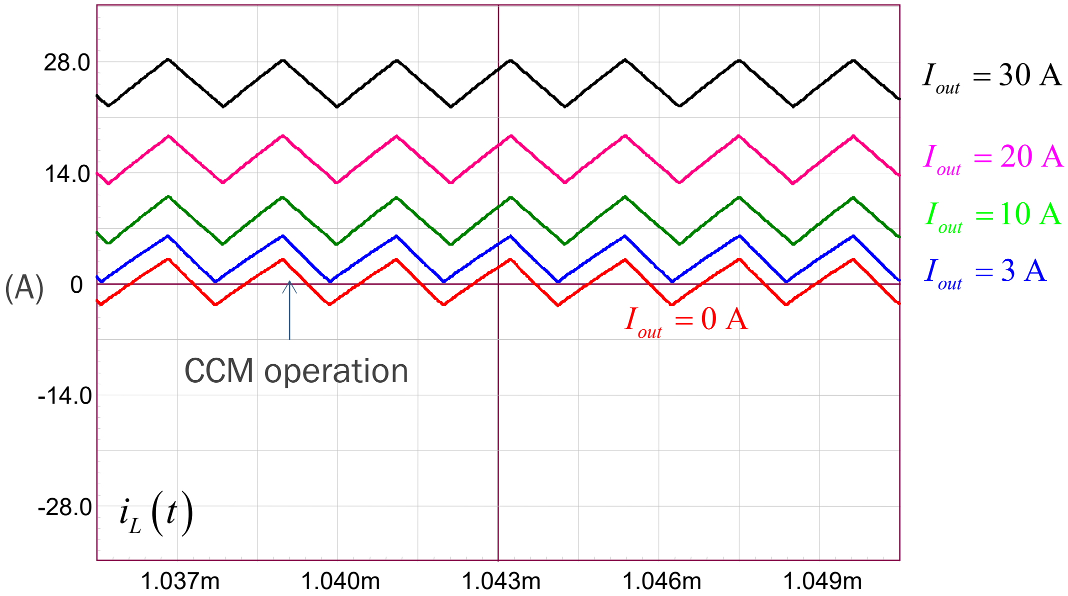
Continuous Conduction in No-Load Conditions

- DCM operation is often associated with light- and no-load operation
  - Synchronous rectification allows inductor current to swing below 0 A



Single-switch DCM operation
Light-load condition

With sync. rectifiers



CCM operation

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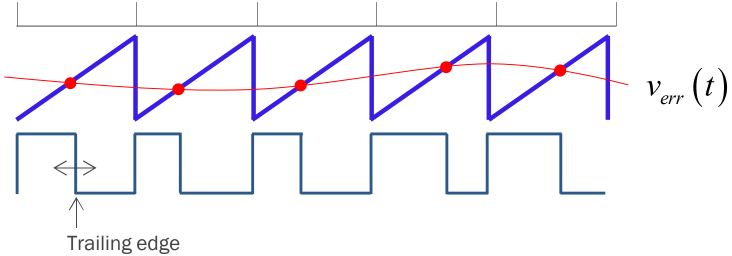
Different Modulation Strategies

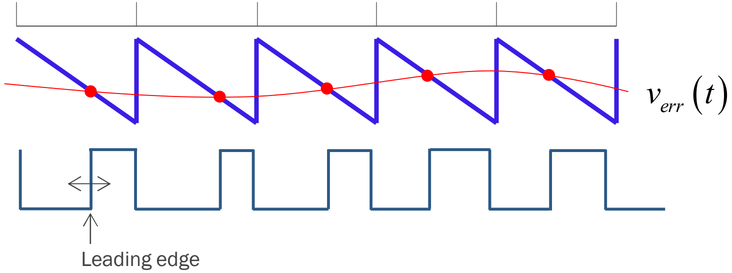
Trailing edge



Clocked turn-on
Fast turn-off
Delayed turn-on

Conventional ac-dc, dc-dc

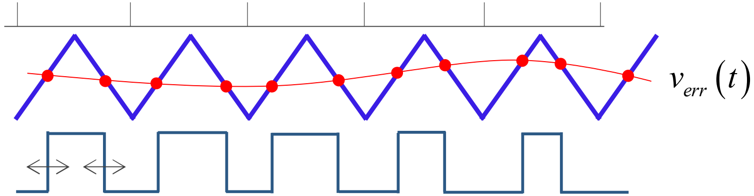
Leading edge



Clocked turn-off
Fast turn-on
Delayed turn-off

Post regulators

Dual edge

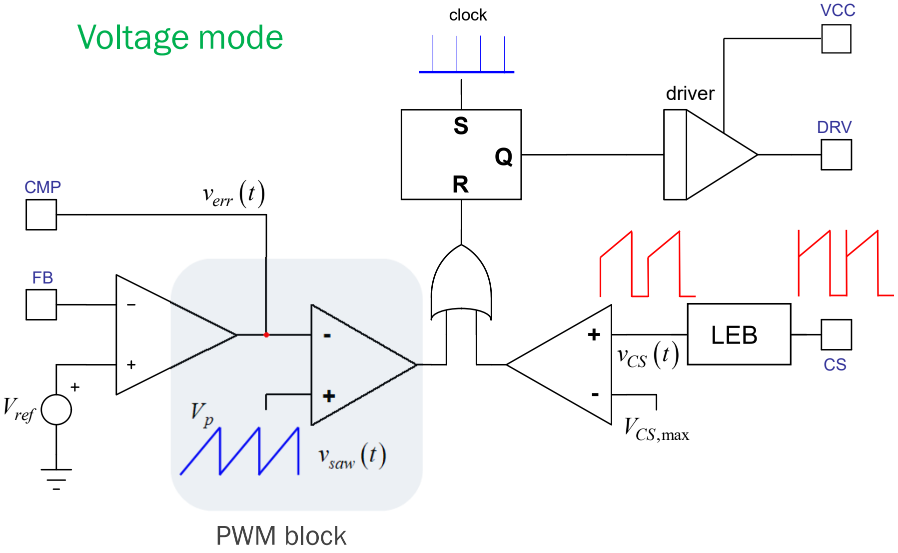


Fast turn-on
Fast turn-off

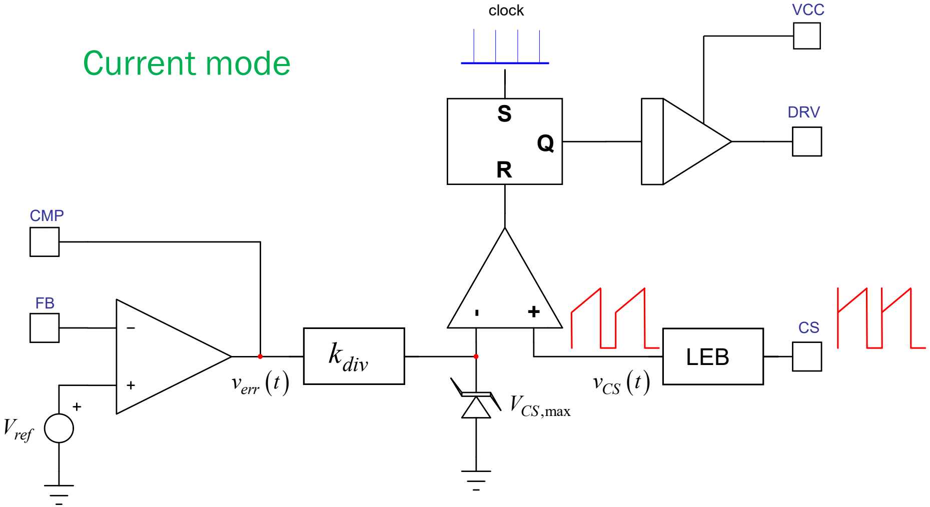
High-speed dc-dc

Fixed Switching Frequency Operation

- ☐ The main switch turns on at the clock occurrence



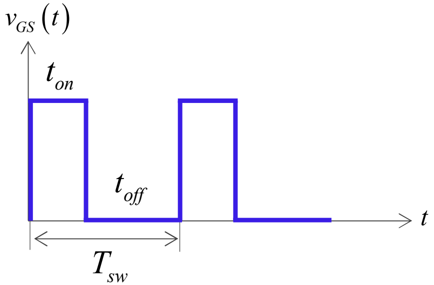
- ➢ The duty ratio is directly controlled by V_{err}



- ➢ The peak current is controlled by V_{err}
- ➢ D is indirectly controlled by V_{err}

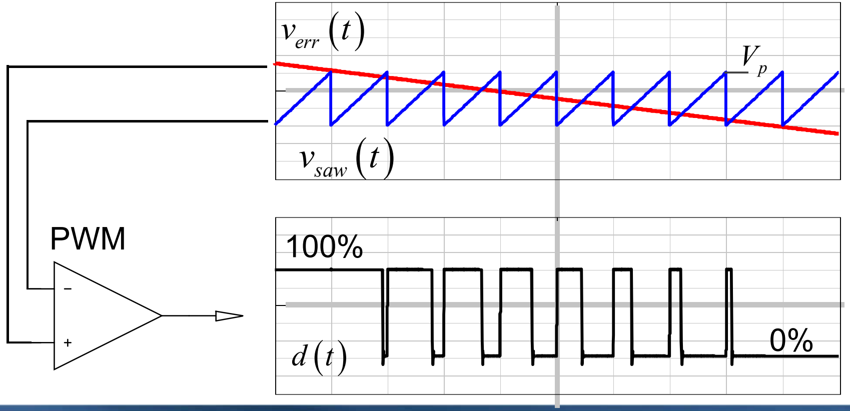
Duty Ratio Generation – Pulse Width Modulation

- The power stage is controlled via the duty ratio D



$$D = \frac{t_{on}}{T_{sw}} = \frac{3u}{10u} = 30\%$$

- An artificial ramp is compared to the control voltage



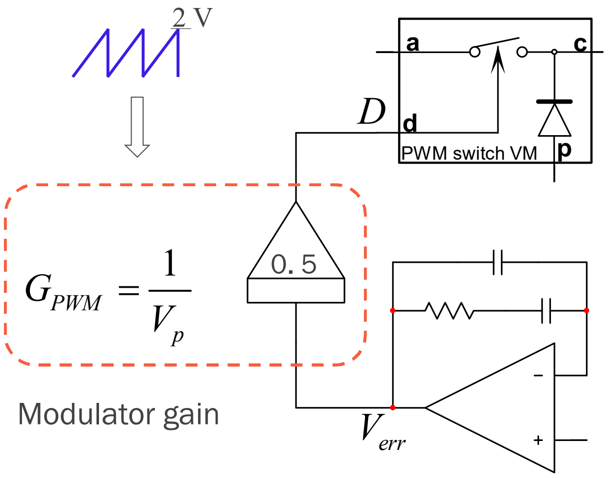
$$\left.\begin{aligned} v_{saw}(t) &= V_p \frac{t}{T_{sw}} \\ v_{err}(t) &= V_p d(t) \end{aligned}\right\} \quad d(t) = \frac{v_{err}(t)}{V_p}$$

Modeling the PWM Block

- ❑ Its role is to convert a voltage V_{err} into a duty ratio D

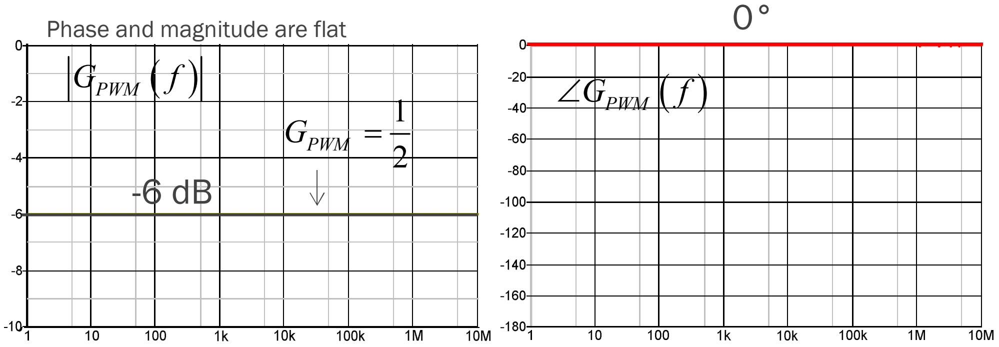
$$d(t) = \frac{v_{err}(t)}{V_p} \xrightarrow[\text{over } T_{sw}]{\text{average}} D(V_{err}) = \frac{V_{err}}{V_p} \longrightarrow G_{PWM} = \frac{dD(V_{err})}{dV_{err}} = \frac{1}{V_p}$$

- ❑ It is a simple block inserted before the D input of the model



$$G_{PWM} = \frac{1}{V_p}$$

Modulator gain

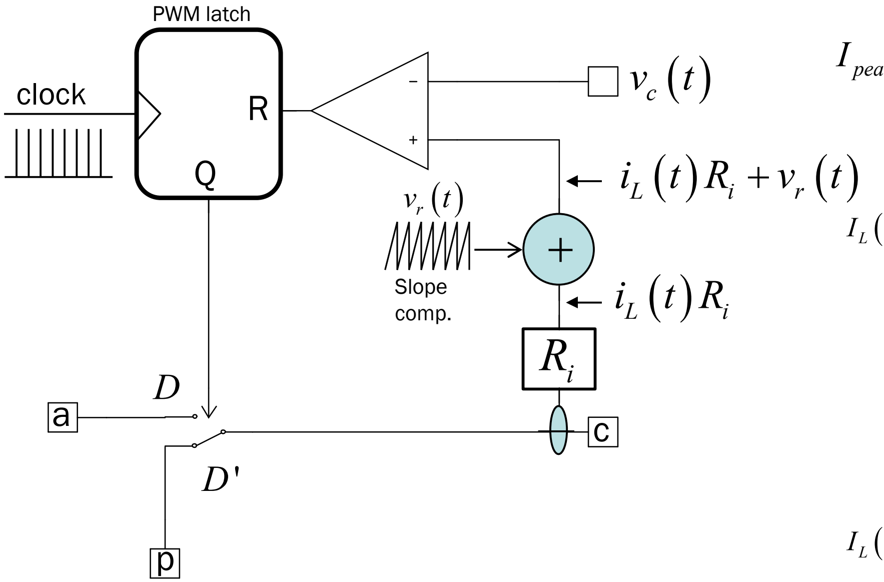


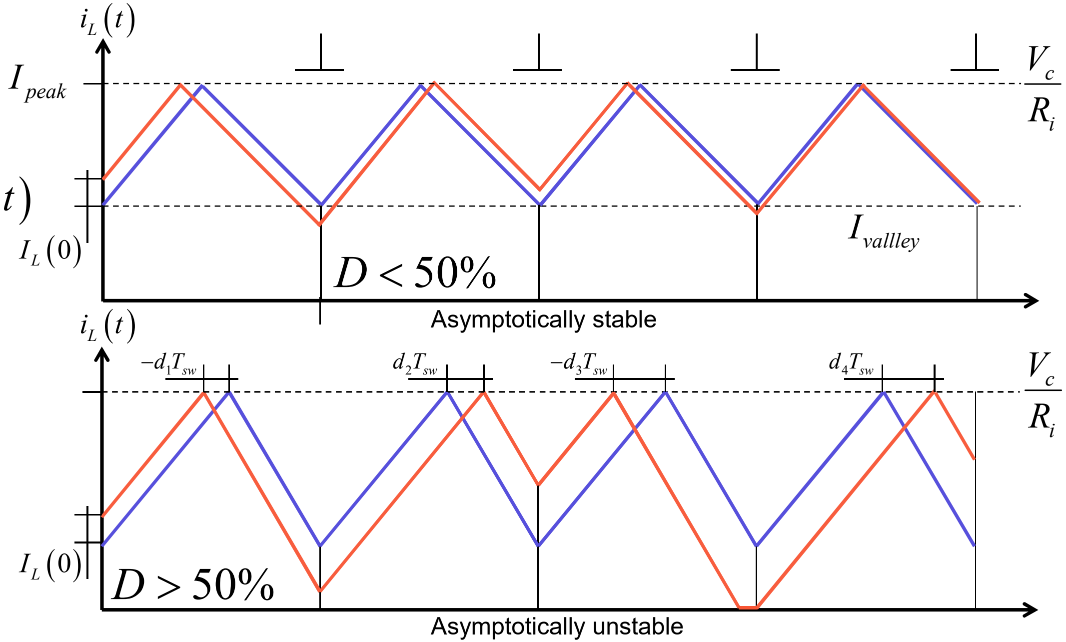
Frequency response of the naturally-sampled modulator is flat

→ With a perfect comparator, t_p = 0

Current-Mode Operations

- ☐ The artificial ramp is replaced by the inductor current

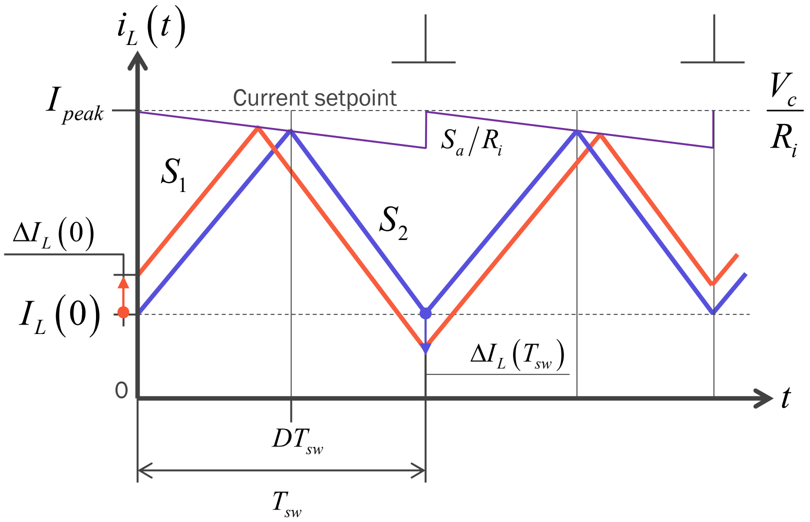


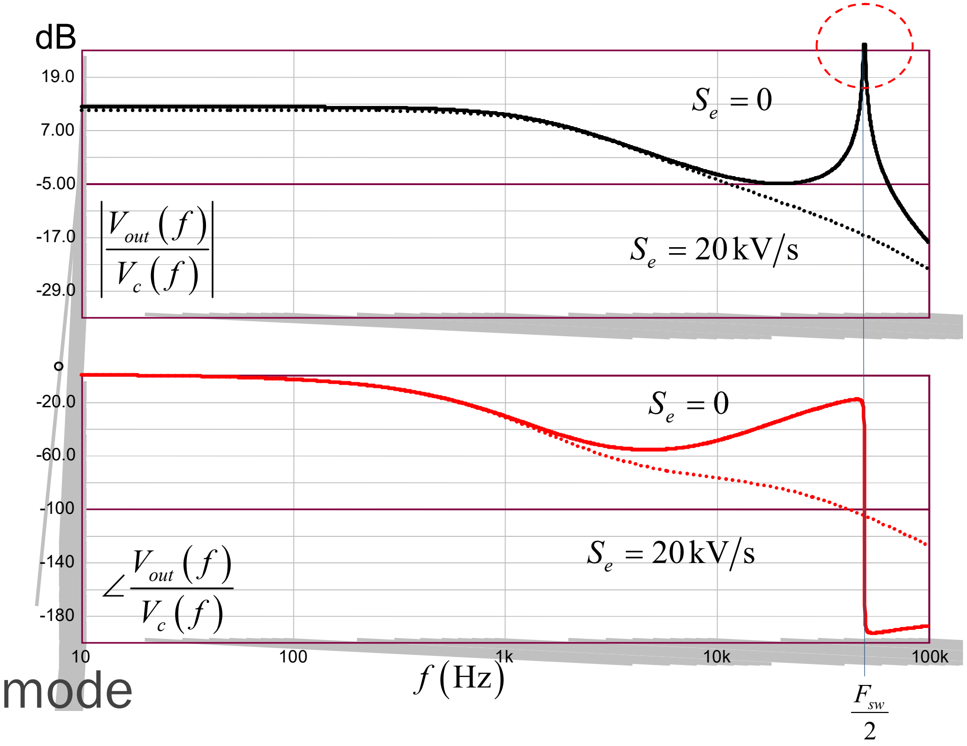


- ☐ In CCM with D > 50% subharmonic instabilities occur

Slope Compensation Cures Oscillations

- ☐ Injecting an external ramp damps the poles at $F_{sw}/2$

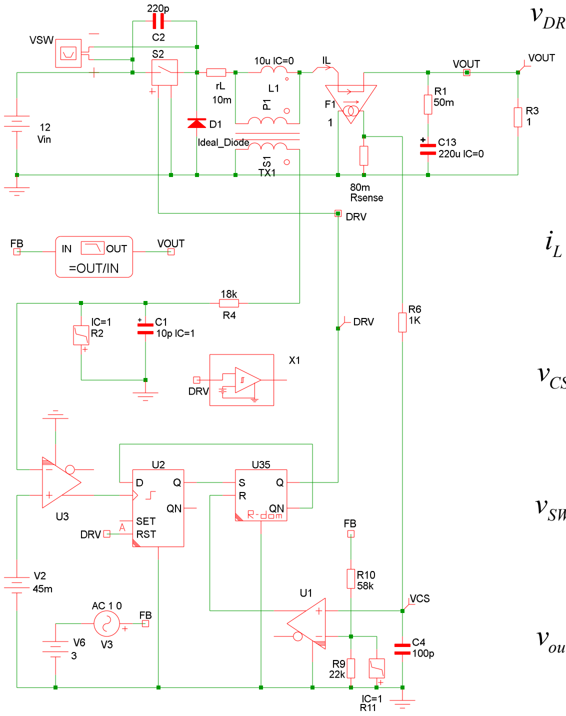


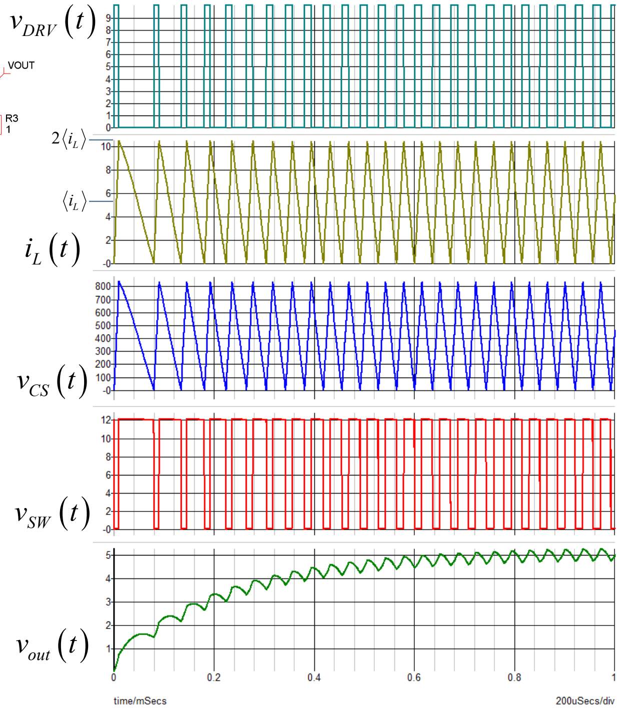


- ☐ Do not overcompensate
  - ❖ Current mode turns into voltage mode

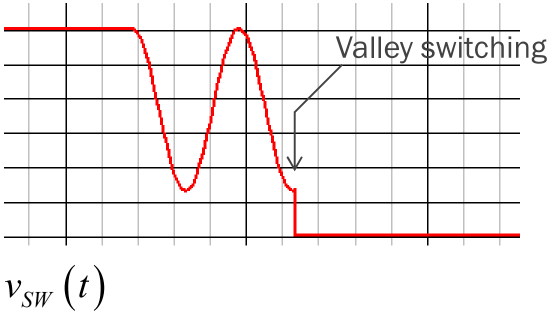
Self-Relaxing Quasi-Resonant Operations

- ☐ The converters operates in borderline conduction mode





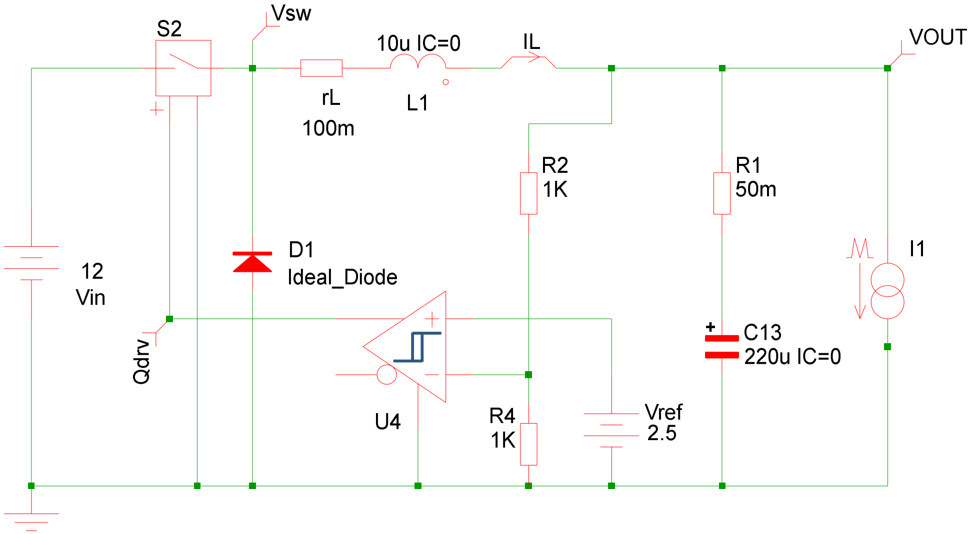
- ❖ 200% inductor ripple current
- ❖ load/line-dependent frequency
- ❖ needs an extra winding over L

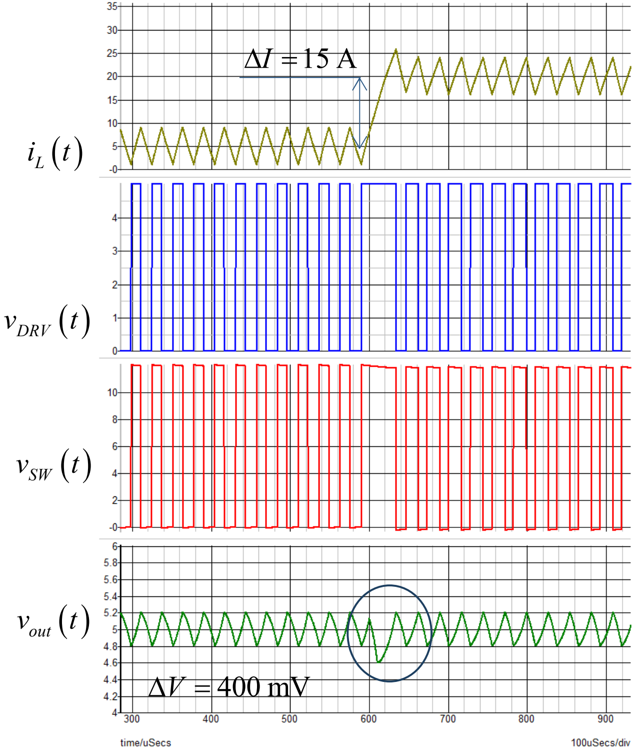


✓ Reduced switching losses

Hysteretic Control – the Basic System

- The simplest and fastest switching converter



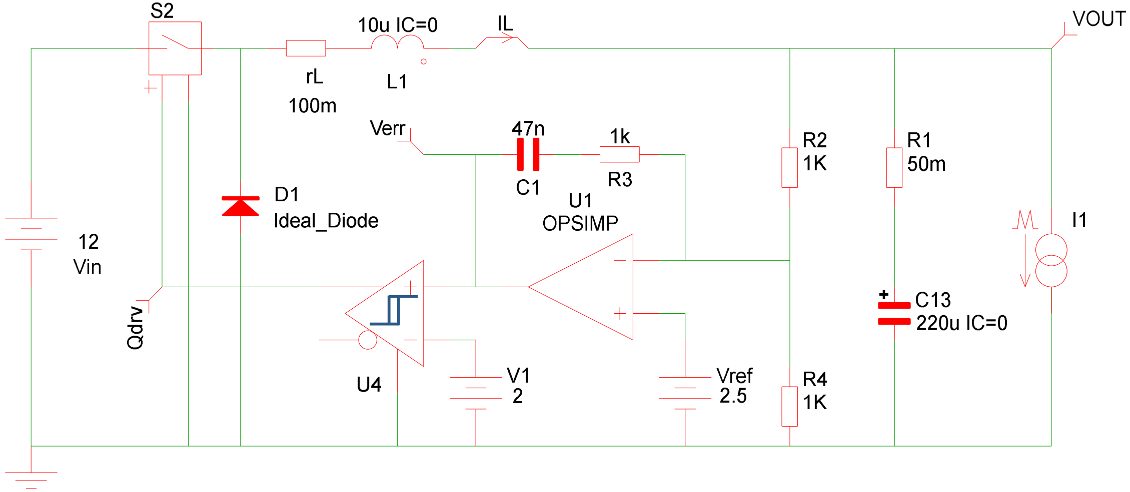


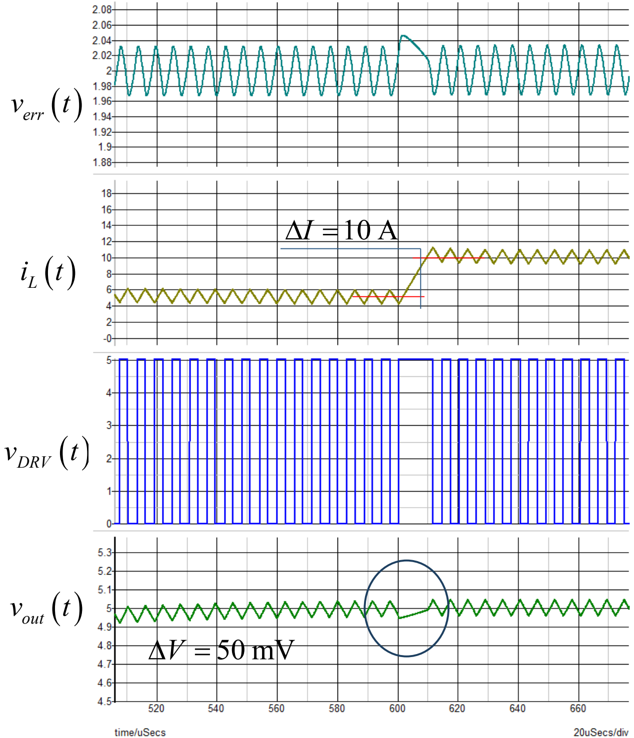
- Variable frequency operation
- Capacitor ESL affects stability in DCM

R. Redl and J. Sun, *Ripple-based control of switching regulators—an overview*, *IEEE Tr. Power Electronics*, Dec. 2009, 4 vol. 24, no. 12, pp. 2669 – 2680.

Hysteretic Control – Adding an Op-Amp

- You can add an error amplifier to improve dc regulation





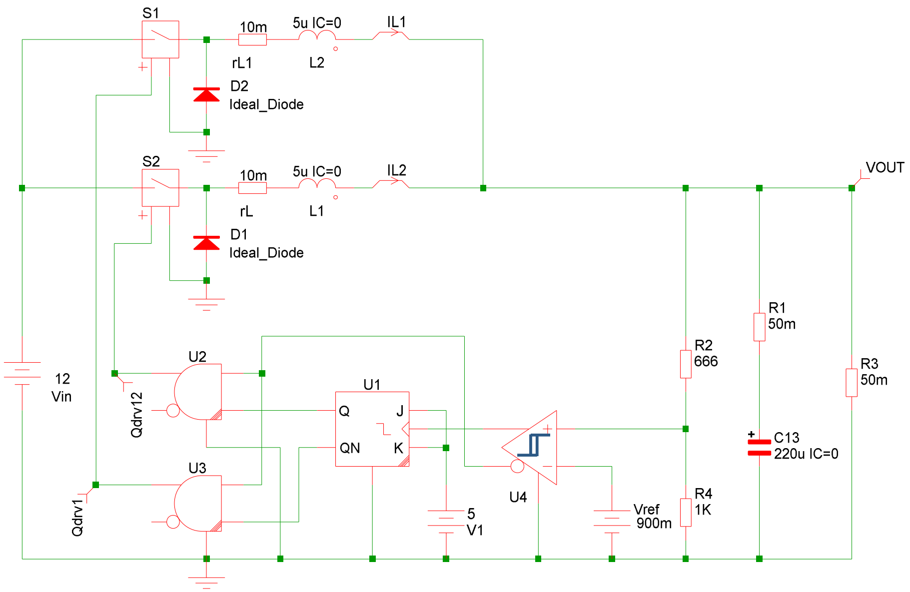
- A compensation network is needed

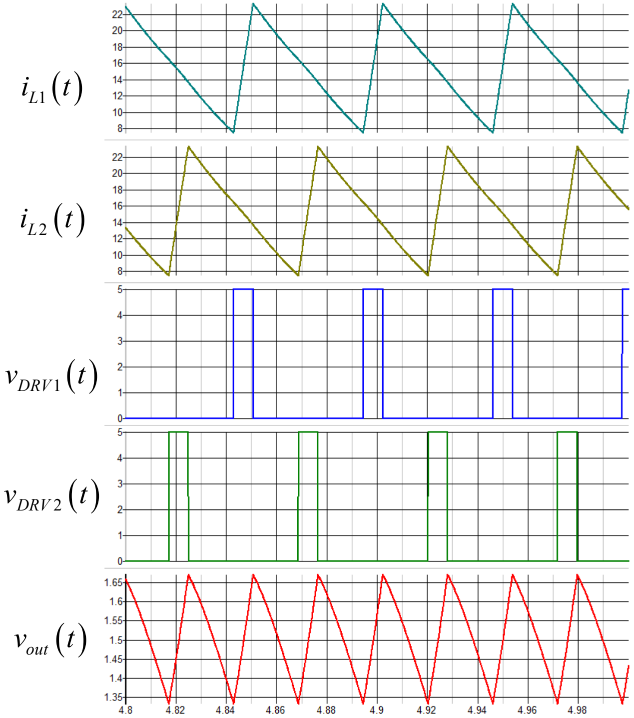
- Many techniques to:
  stabilize F_{sw}, improve stability, reduce drop…

R. Redl, *Ripple-Based Dc-Dc Converters, In-House Seminar, Toulouse* 2010.

Extension to Multiphase Conversion

- ❑ The two-phase extension requires a simple logic control



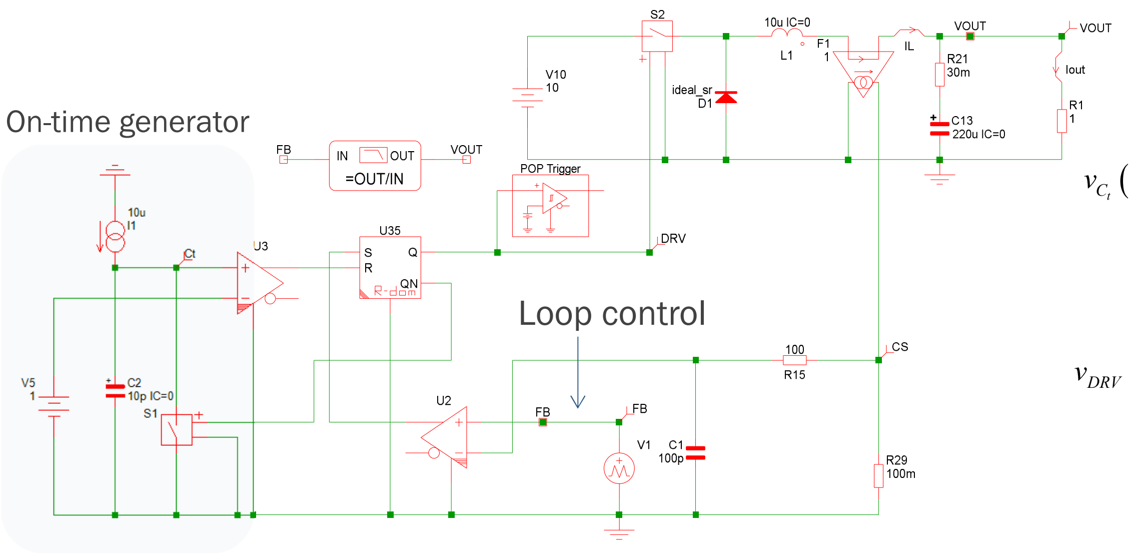


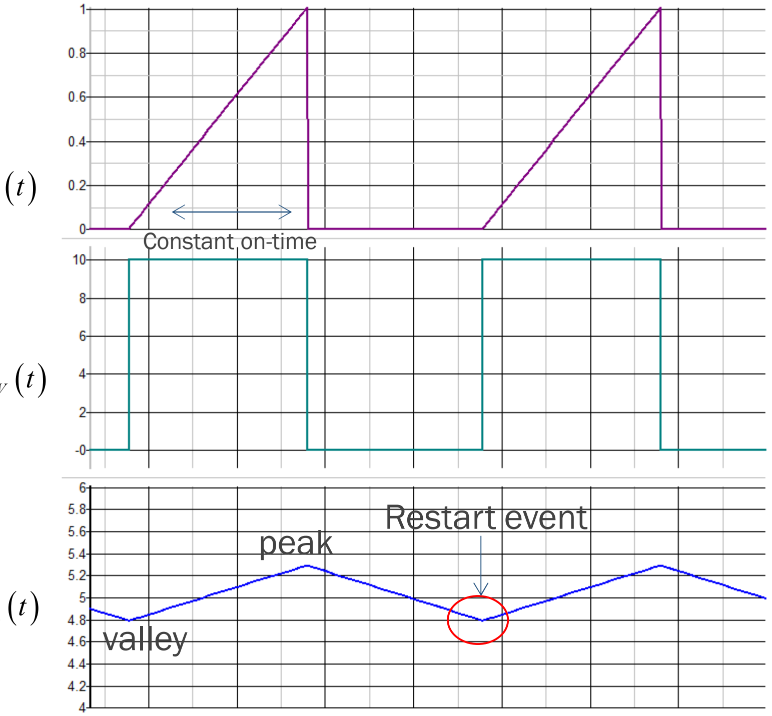
OVP: optimal voltage positioning

- ❖ Bad dynamic current sharing (no sensing)
- ❑ Popular in dc-dc with OVP (motherboards)

Constant-On-Time Valley-Current Control - COT

- ☐ On-time duration is fixed – restart is given by the valley current





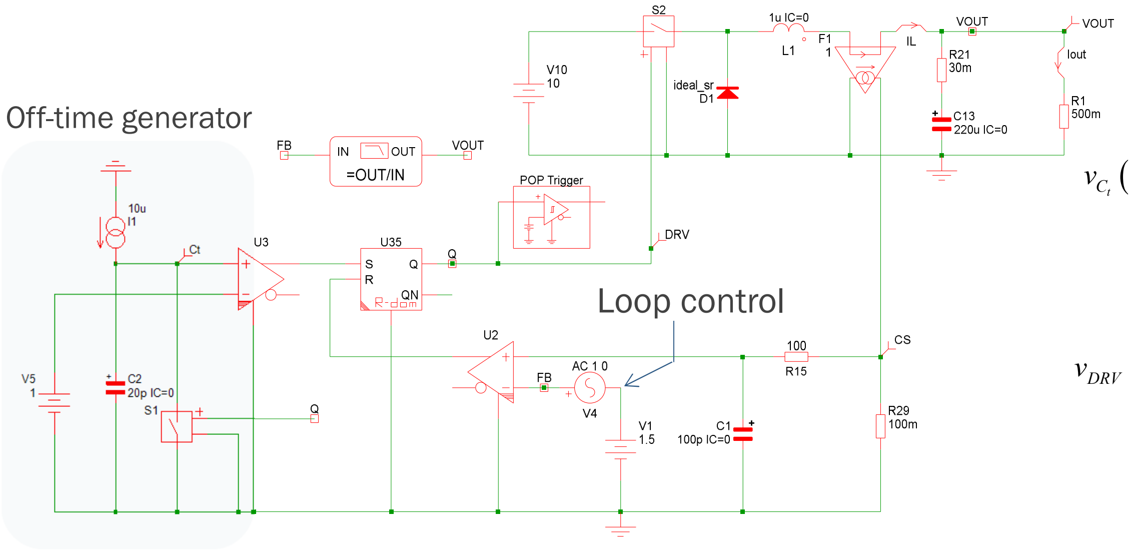
R. Redl, *Ripple Regulator Review*, Professional Education APEC seminar 2008.

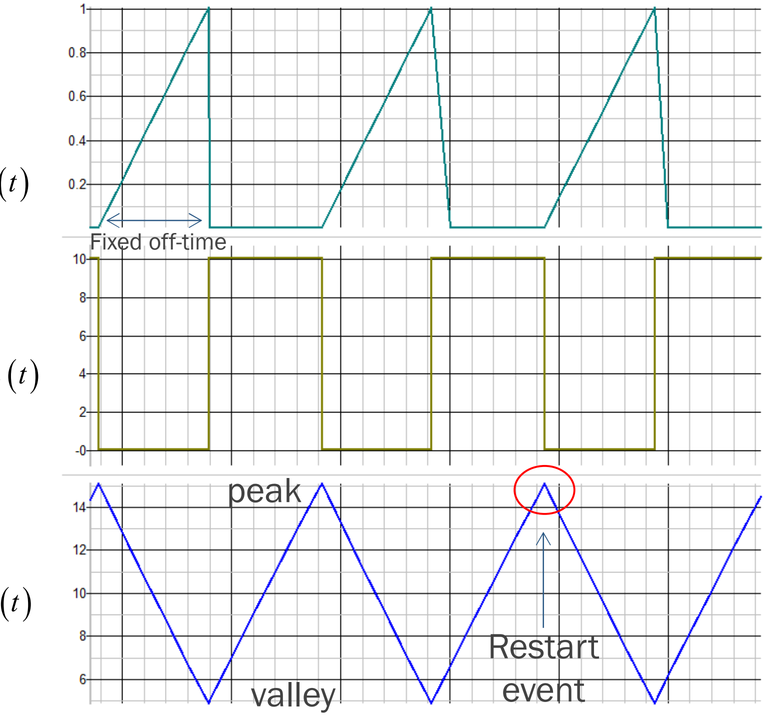
- ❖ Can be subject to instabilities

$$r_C C > \frac{t_{off}}{2}$$

Fixed-Off-Time Peak-Current Control - FOT

- ☐ Off-time duration is fixed – restart is given by the peak current





R. Redl, *Ripple Regulator Review,* Professional Education APEC seminar 2008.

- ❖ Can be subject to instabilities

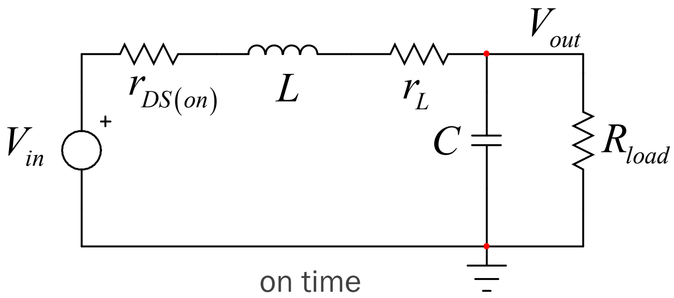
$$r_C C > \frac{t_{on}}{2}$$

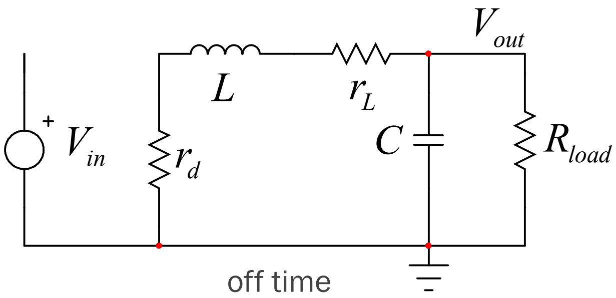
Course Agenda

- ☐ The Buck Converter
- ☐ Control Schemes
- ☐ **Introduction to Modeling**
- ☐ The PWM Switch in Current-Mode Control
- ☐ Introduction to the Fast Analytical Techniques
- ☐ The Four Transfer Functions in CCM Current-Mode Control
- ☐ Compensation Strategy
- ☐ Prototype Measurements

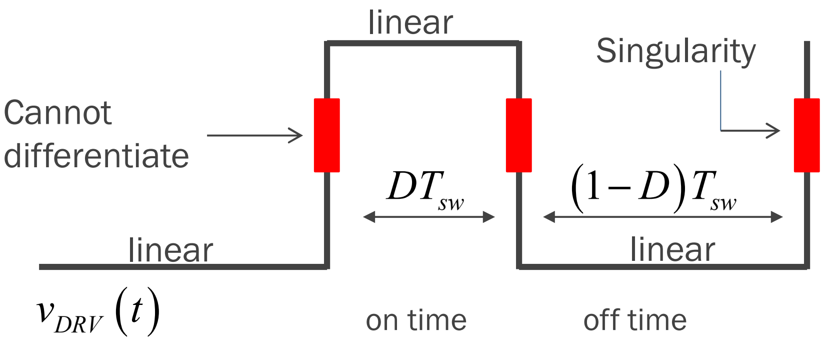
Time-Discontinuous Switching Waveforms

- ☐ A switching converter is made of linear elements



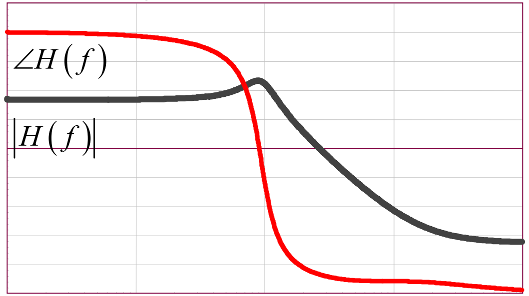


- ☐ The non-linearity or discontinuity is coming from transitions



How do we get this?

Control-to-output transfer function



State Space Averaging Technique

- ❑ Despite linear networks, equation is discontinuous in time
- ❑ Introduced in 76, SSA weights on and off expressions

valid during $(1\text{-}D)T_{sw}$

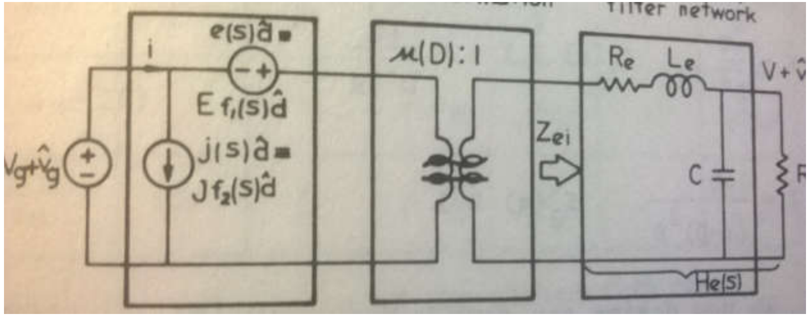
$$\dot{x} = \left[\mathbf{A}_1 D + \mathbf{A}_2\left(1-D\right)\right]x\left(t\right) + \left[\mathbf{B}_1 D + \mathbf{B}_2\left(1-D\right)\right]u\left(t\right)$$

valid during DT_{sw}

A is the state coefficient matrix
B is the source coefficient matrix

- ❑ Singularity is gone (time continuous)
- ❖ Equation is nonlinear now
- ❑ Linearize and feed the canonical model
- ❖ Add a new element: restart from scratch!

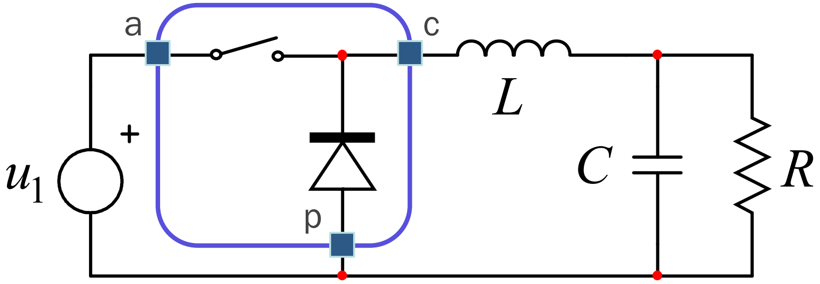
Canonical model



S.Ćuk, Modeling, Analysis and Design of Switching Converters, Ph. D. Thesis, Caltech November 1976

The PWM Switch Model in Voltage Mode

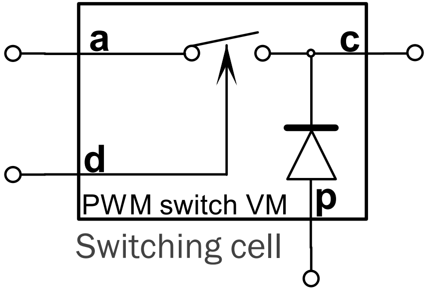
- ❑ We know that non-linearity is brought by the switching cell



a: active
c: common
p: passive

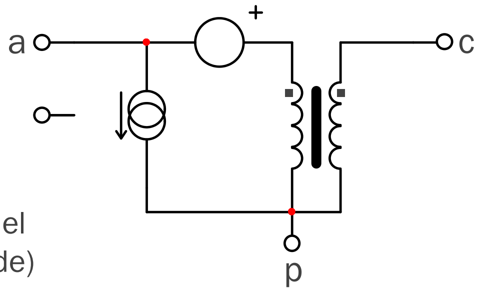
Add a filter or a resistance

- ❑ Why not linearize the cell alone?



Switching cell

Small-signal model
(CCM voltage-mode)

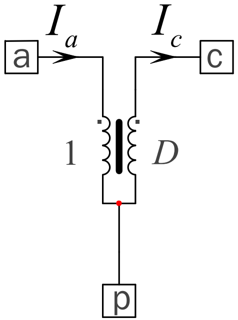


Model is unchanged

V. Vorpérian, *Simplified Analysis of PWM Converters using Model of PWM Switch*, IEEE Transactions on Aerospace and Electronic Systems, Vol. 26, NO. 3, 1990

The VM-PWM Switch Model is a Transformer

- The PWM switch large-signal model is a dc "transformer"!

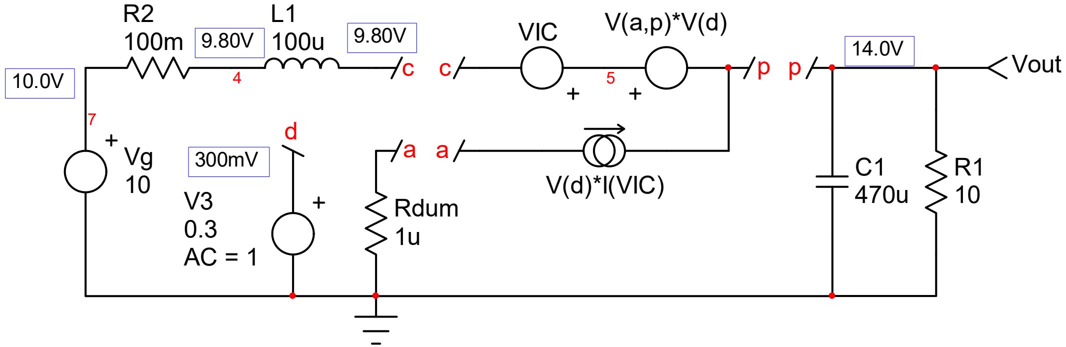


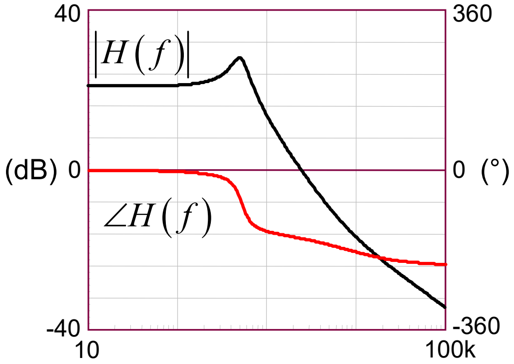
$$I_a = DI_c \qquad I_c = \frac{I_a}{D}$$

$$V_{ap} = \frac{V_{cp}}{D} \qquad V_{cp} = DV_{ap}$$

Immediate results

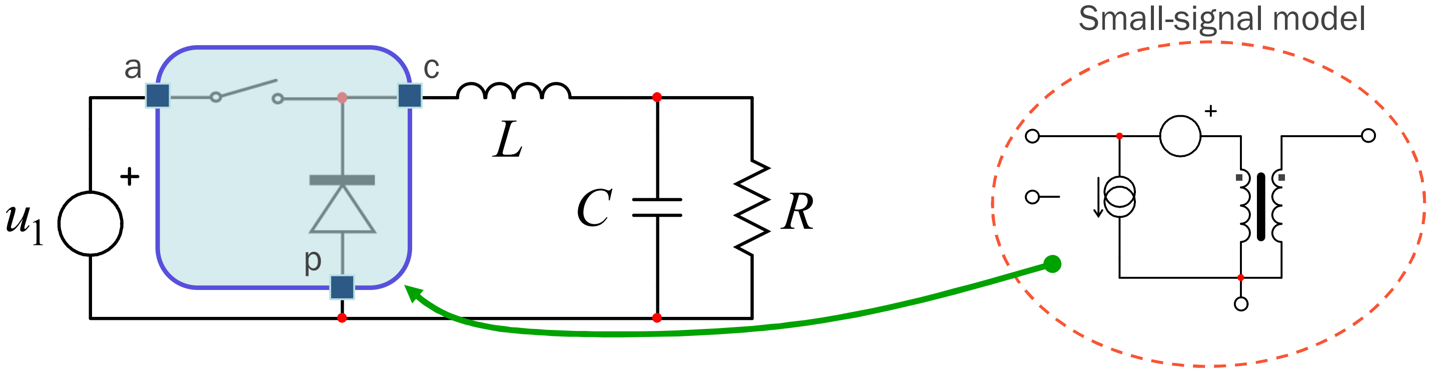
- It can be plugged into any 2-switch CCM converter



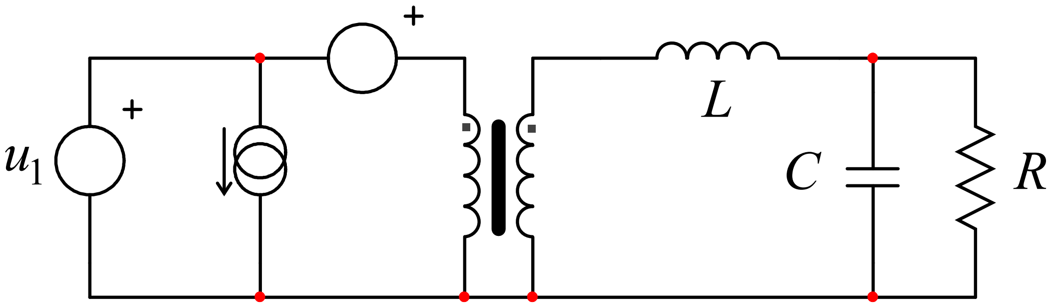


Update the Schematic Diagram with the PWM Switch

- ☐ Like in a bipolar circuit, replace the switching cell...



- ☐ ...and solve a set of linear equations!

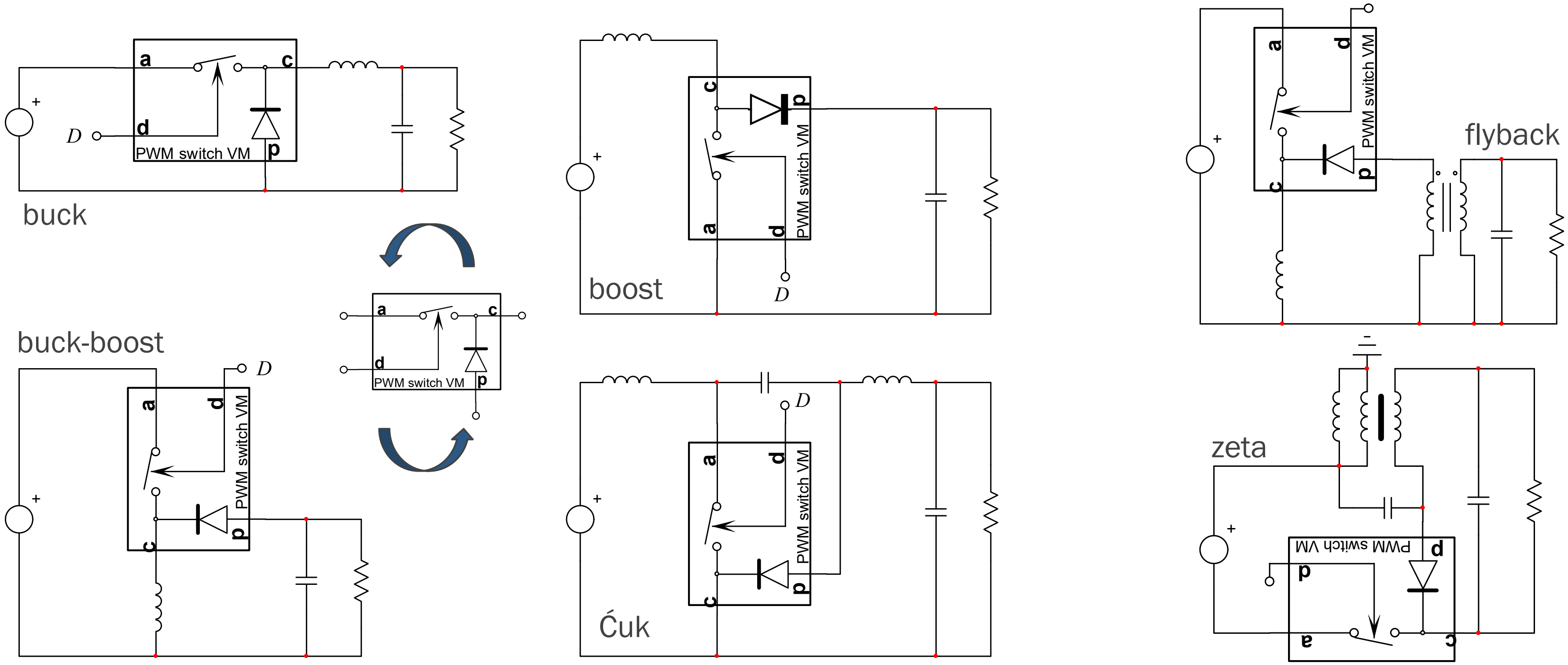


Analyze circuit

$$H(s) = H_0 \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_0}\right)^2}$$

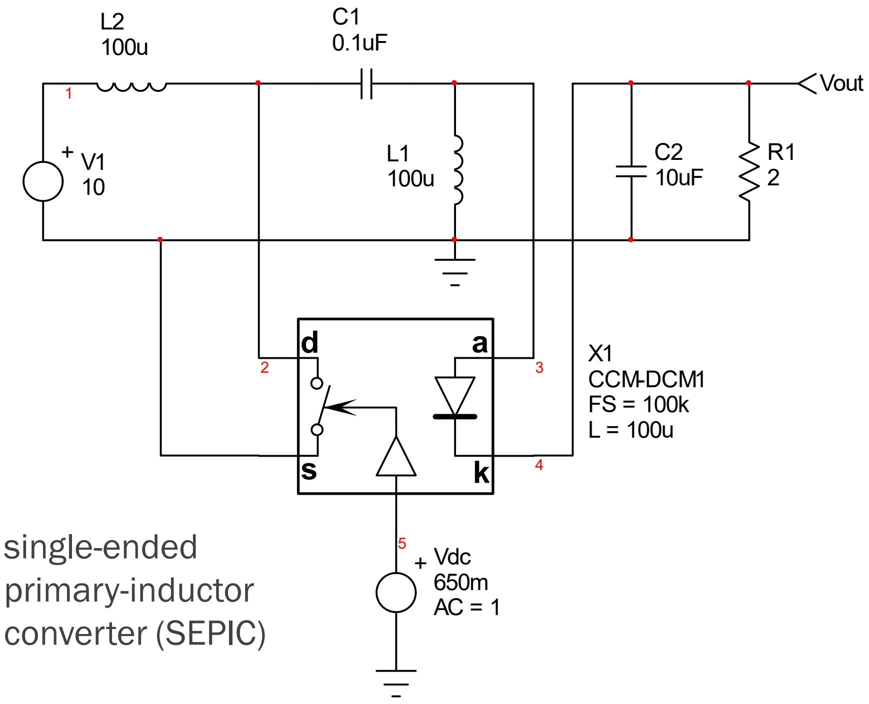
The Model fits all Converters Architectures

- ☐ The switching cell is everywhere: the model is invariant

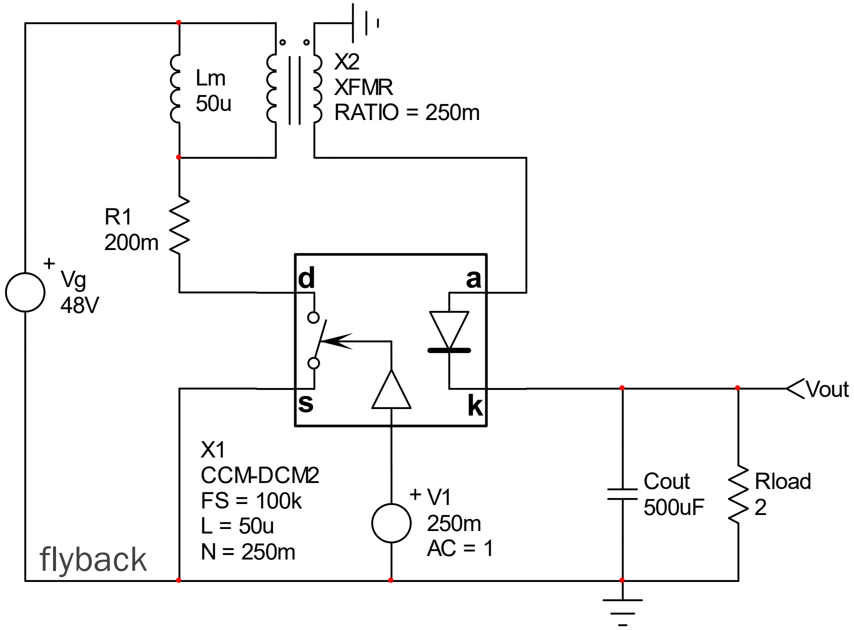


Independent Switch Modeling: Direct Connections

- The switch and diode are individually modeled: easy substitution



single-ended primary-inductor converter (SEPIC)



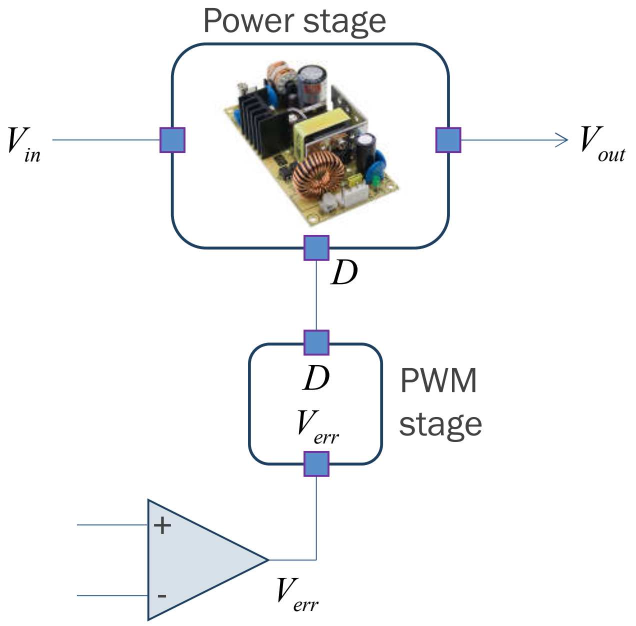
flyback

B. Erickson, D. Maksimovic, *Advances in Averaged Switch Modeling and Simulation*, professional seminar, PESC, Charleston, 1999

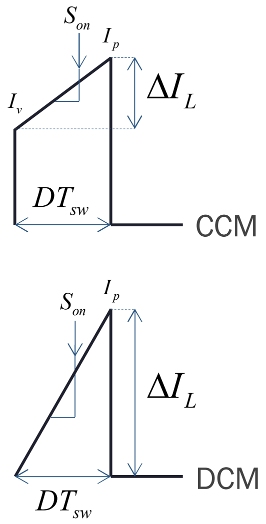
Peak-Current-Mode Control Models

❑ The control voltage V_c adjusts the inductor peak current

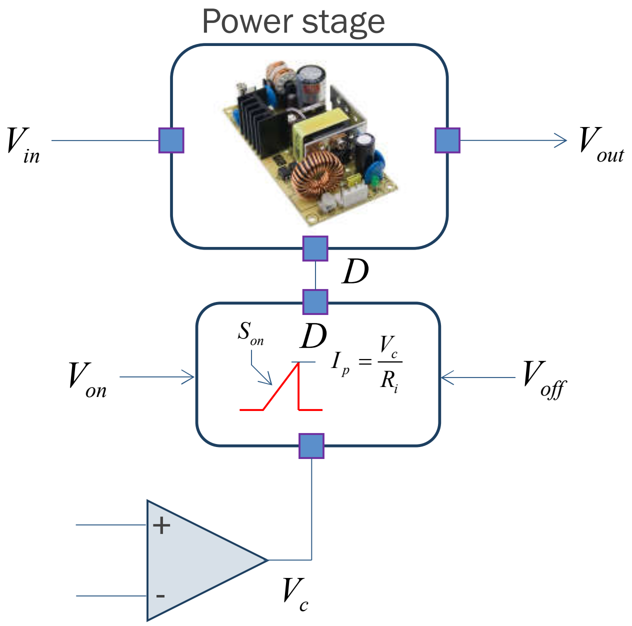
❖ the duty ratio is indirectly controlled by V_c



Direct duty ratio control



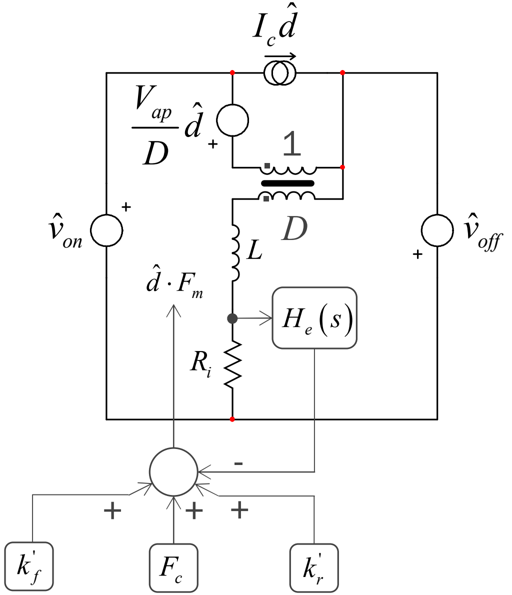
Same static waveforms in VM and CM

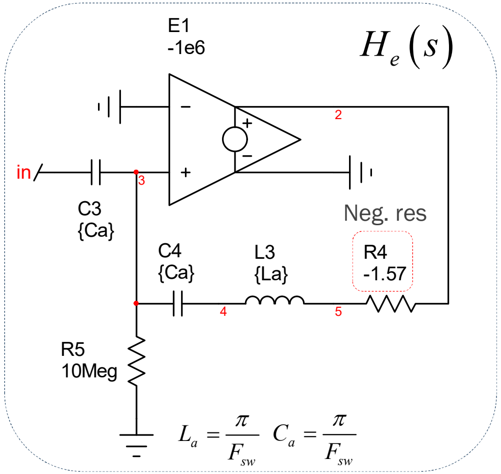


Indirect duty ratio control

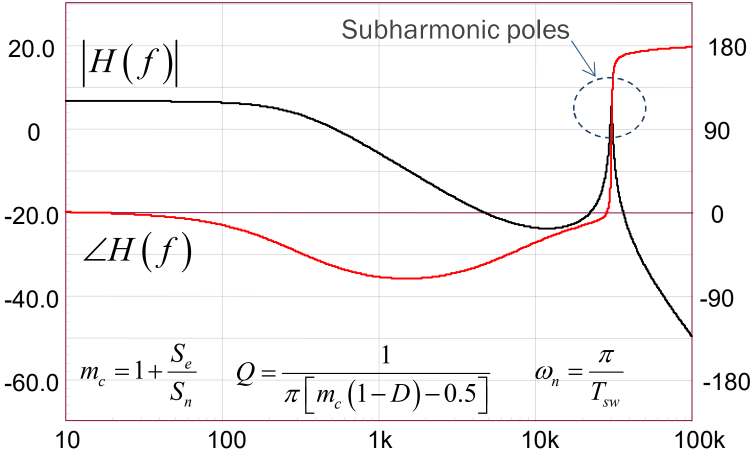
A VM-PWM Switch-Based Small-Signal Model

- ☐ Small-signal model built after sampled-data analysis





Pair of RHP Zeroes

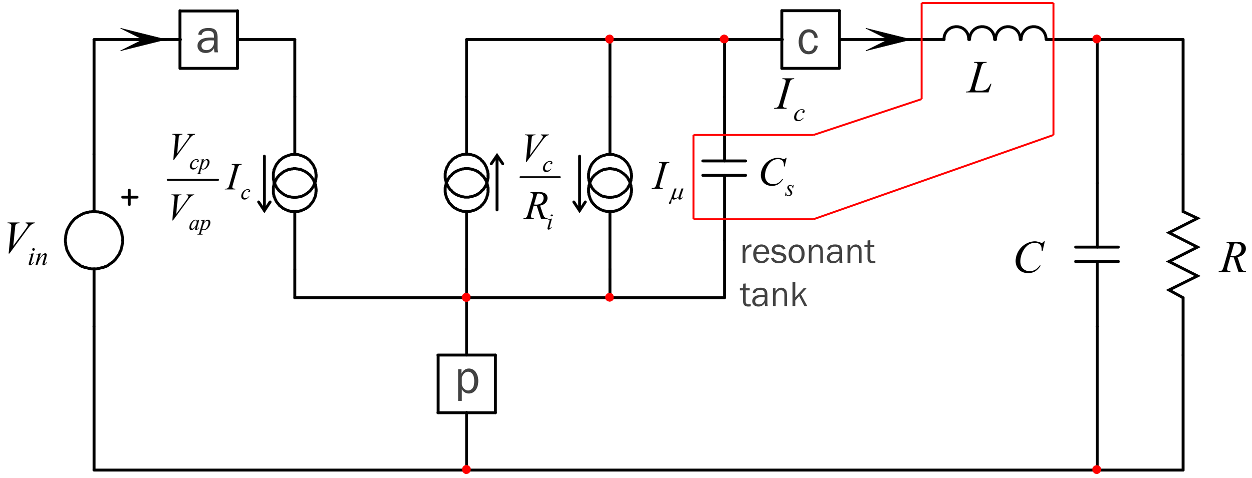


- ☐ Model accurately predicts subharmonic oscillations but is ac only

R. Ridley, *A New Small-Signal Model for Current Mode Control*, Ph. D. dissertation, Virginia Polytechnic Institute and State University, 1990

A Current-Mode PWM Switch Model

- ☐ The large-signal model associates three simple current sources



$$H(s) \approx H_0 \frac{1+\frac{s}{\omega_{z_1}}}{1+\frac{s}{\omega_{p_1}}} \frac{1}{1+\frac{s}{\omega_n Q}+\left(\frac{s}{\omega_n}\right)^2}$$

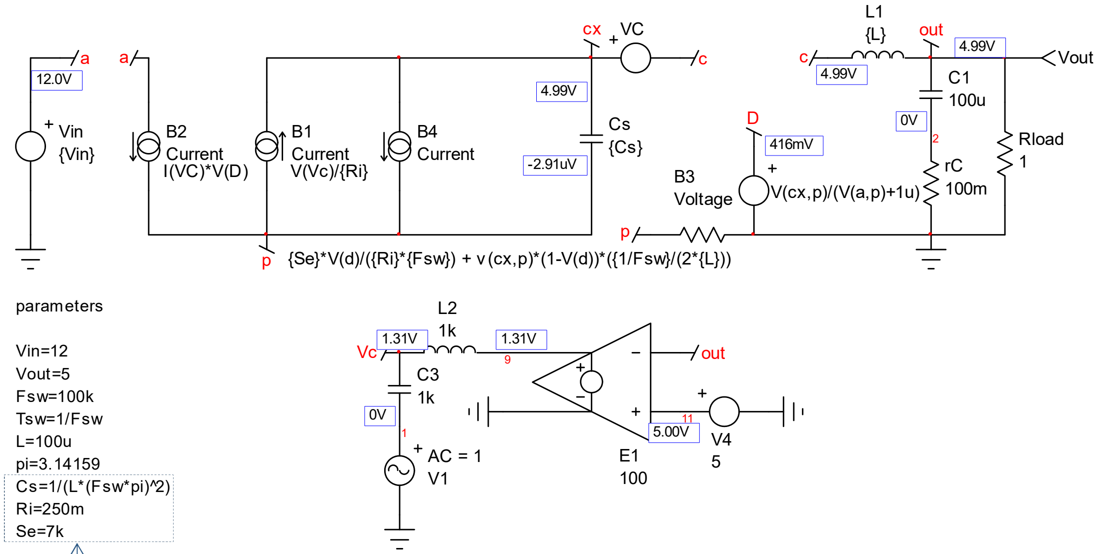
Same result with that obtained with sampled-data analysis.

- ➢ It can compute a bias point and accepts transient simulations
- ➢ The resonant tank reproduces subharmonic oscillations

V. Vorpérian, *Analysis of Current-Controlled PWM Converters using the Model of the PWM Switch*, PCIM Conference, 1990

Implementing the CM-PWM switch Model

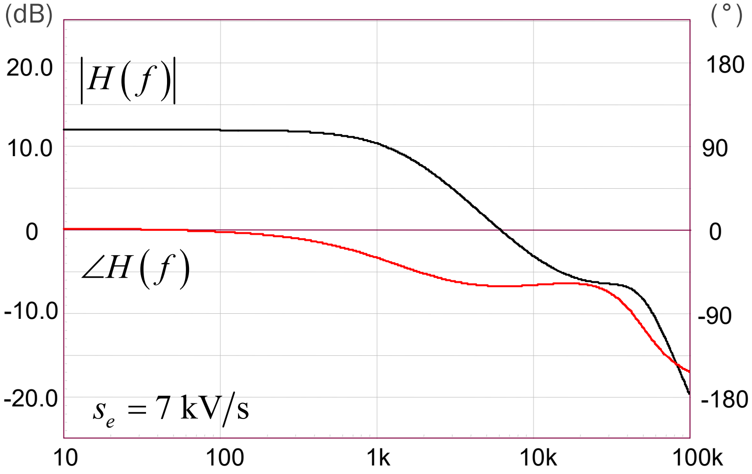
- ❑ You cannot beat the CM-PWM switch model in terms of simplicity



parameters

Vin=12
Vout=5
Fsw=100k
Tsw=1/Fsw
L=100u
pi=3.14159
Cs=1/(L*(Fsw*pi)^2)
Ri=250m
Se=7k

Only these parameters are needed

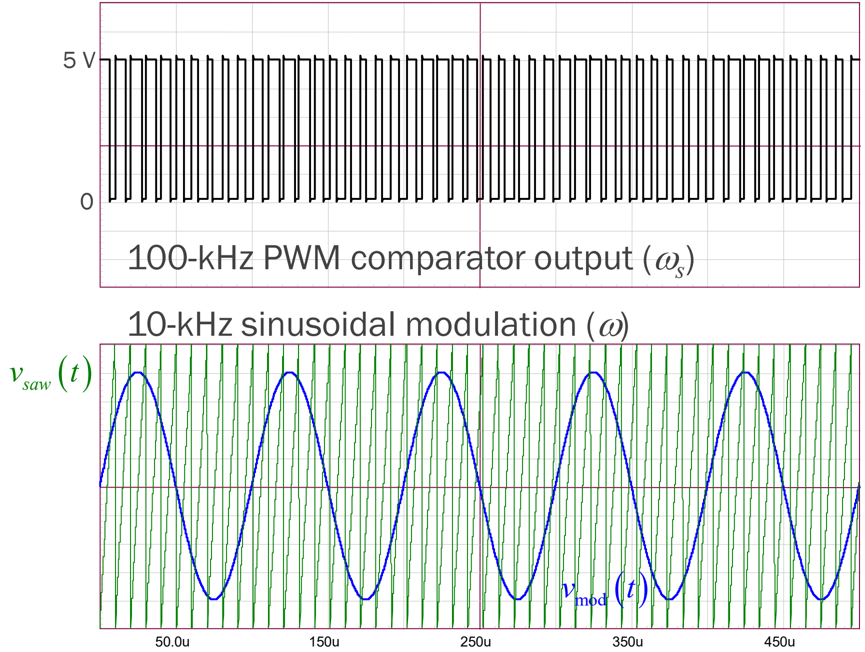


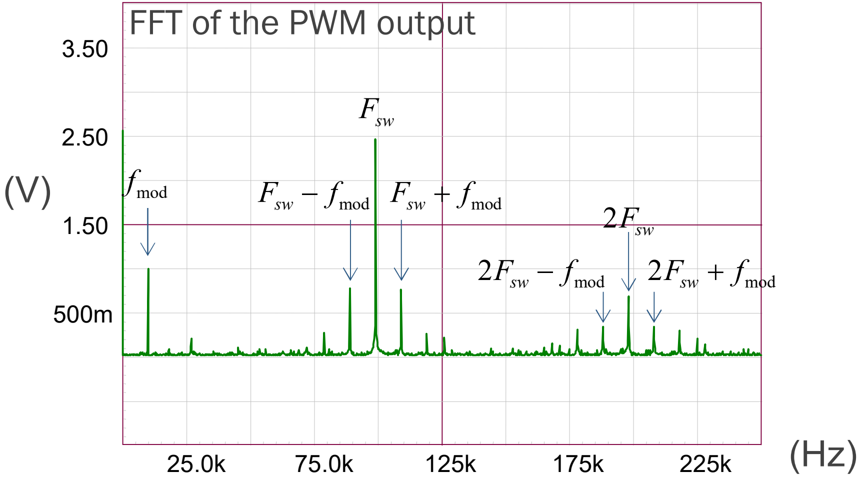
Poles are damped by slope compensation

- ➢ It is a large-signal model which also predicts transient response

Extended Frequency-Range PWM Model

- ❑ The pulse width modulator is a highly nonlinear structure



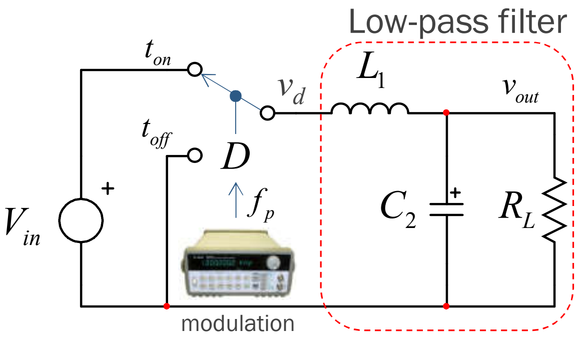


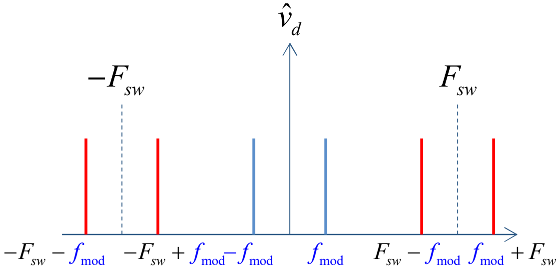
- ❖ Harmonics are fed back through the loop
- ❖ Additional perturbations go through PWM
- ➢ Aliasing effects are observed

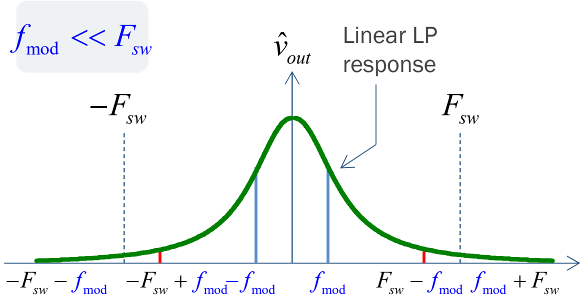
X. Lin and al., *Small-Signal Models with Extended Frequency Range for dc-dc Converters with Large Modulation Ripple*,IEEE Transactions on Power Electronics, VOL. 33, NO. 9, September 2018

Predicting Phase Lag Approaching $F_{sw}/2$

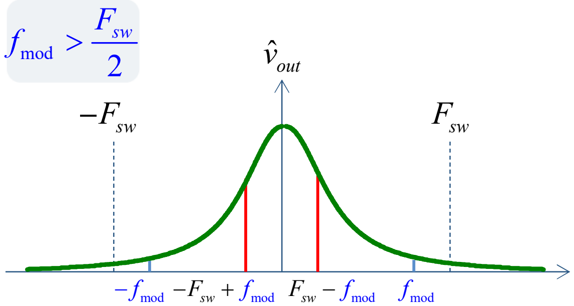
- Modulation and sidebands go through the low-pass filter



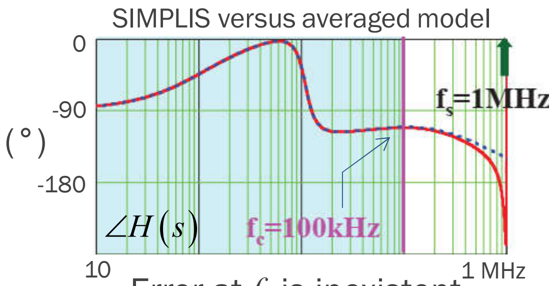




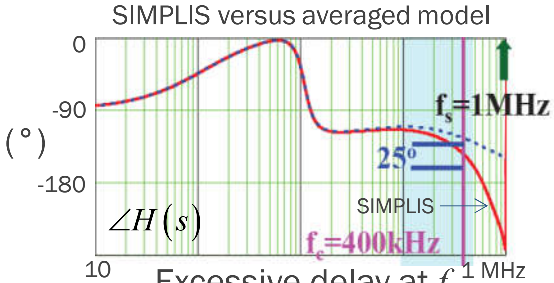
f_{mod} dominates the response



Sideband is no longer negligible



Error at f_c is inexistent

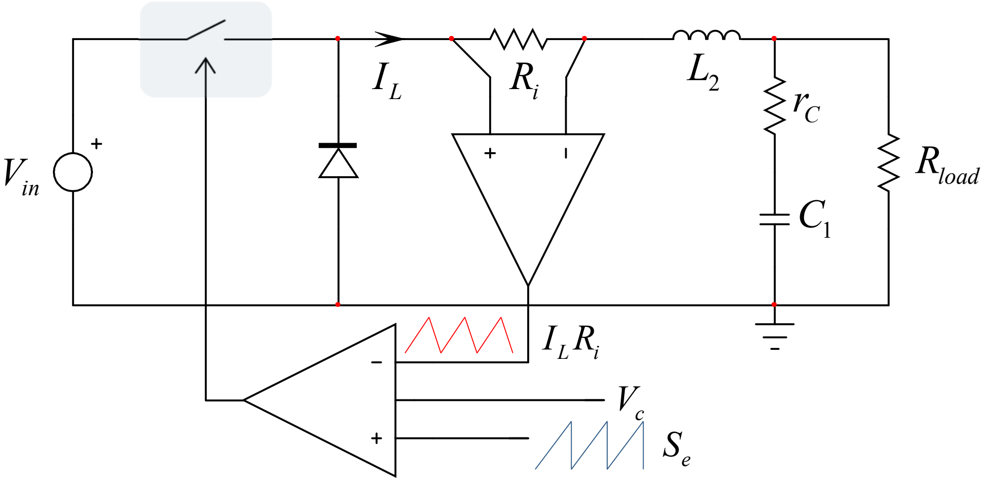


Excessive delay at f_c

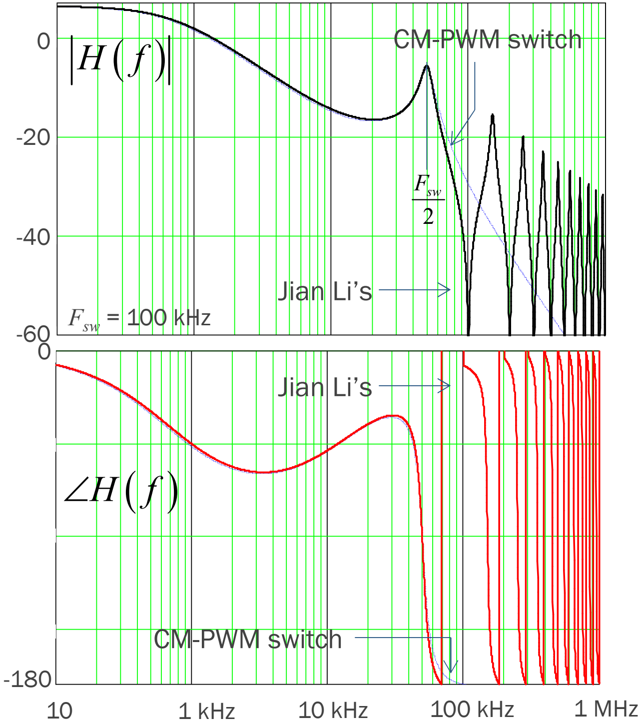
F. Lee, Review of Current-Mode-Control Modeling, APEC 2017 Professional Education Seminars, Tampa, FL

Accounting for Sidebands Effects

- Harmonics are responsible for the phase deviation as f_{mod} approaches $F_{sw}/2$



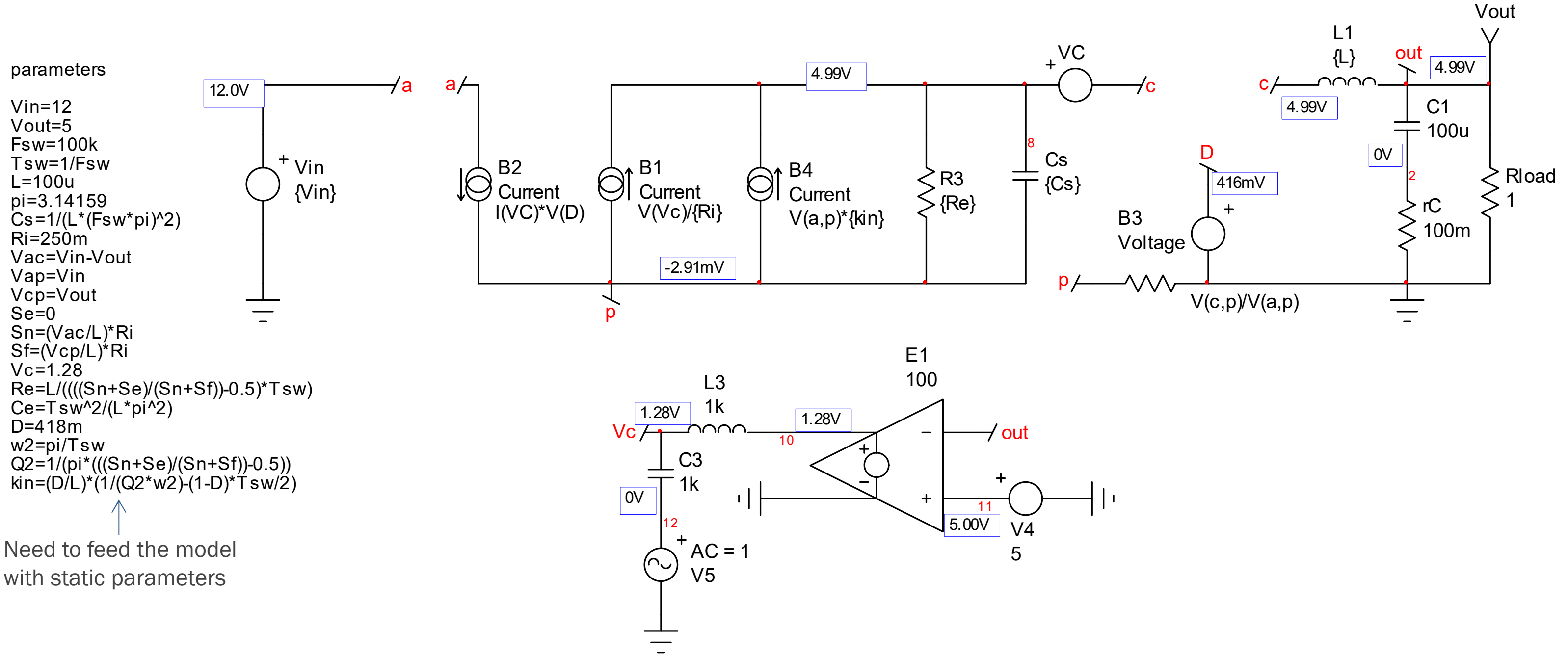
$$\frac{V_{out}(s)}{V_c(s)}=\frac{F_{sw}}{(S_n+S_e)+(S_f-S_e)e^{-sT_{sw}}}\frac{R_{load}V_{in}}{sL}\frac{1+\frac{s}{\omega_z}}{1+\frac{s}{\omega_p}}\left(1-e^{-sT_{sw}}\right)$$



J. Li, F.C. Lee, New Modeling Approach and Equivalent Circuit Representation for CM Control, Power Electronics IEEE Transactions, VOL. 25, NO. 5, pp. 1218-1230, May 2010

Simplifying The Model towards a Universal Subcircuit

- ☐ Yingyi Yan's model is a universal subcircuit covering various operating schemes



```
parameters

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Vout=5
Fsw=100k
Tsw=1/Fsw
L=100u
pi=3.14159
Cs=1/(L*(Fsw*pi)^2)
Ri=250m
Vac=Vin-Vout
Vap=Vin
Vcp=Vout
Se=0
Sn=(Vac/L)*Ri
Sf=(Vcp/L)*Ri
Vc=1.28
Re=L/((((Sn+Se)/(Sn+Sf))-0.5)*Tsw)
Ce=Tsw^2/(L*pi^2)
D=418m
w2=pi/Tsw
Q2=1/(pi*(((Sn+Se)/(Sn+Sf))-0.5))
kin=(D/L)*(1/(Q2*w2)-(1-D)*Tsw/2)
```

Need to feed the model with static parameters

- ☐ Same performance as CM-PWM switch but with a more complex implementation

Y. Yan, F. Lee, P. Mattavelli, *Unified 3-Terminal Switch Model for Current Mode Controls*, IEEE Transactions on Power Electronics, VOL. 27, NO. 9, Sept. 2012

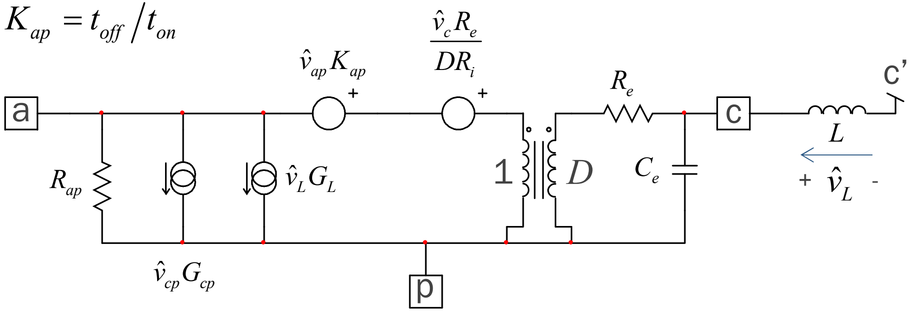
Five Different Control Schemes can be Analyzed

- ❑ Replace some parameters with new expressions and simulate

Constant On-Time:

$$C_e = t_{on}^2 \Big/ \left(L\pi^2\right) \qquad R_e = 2L \big/ t_{on}$$

$$K_{ap} = t_{off} \big/ t_{on}$$



Peak-Current Mode Control:

$$C_e = T_{sw}^2 \Big/ \left(L\pi^2\right) \qquad R_e = L \Bigg/ \left[T_{sw}\left(\frac{S_n + S_e}{S_n + S_f} - 0.5\right)\right]$$

$$K_{ap} = -\left(t_{off} / 2L\right) R_e$$

Constant Off-Time:

$$C_e = t_{off}^2 \Big/ \left(L\pi^2\right) \qquad R_e = 2L \big/ t_{off}$$

$$K_{ap} = -1$$

Charge Control:

$$C_e = T_{sw}^2 \Big/ \left(L\pi^2\right) \qquad R_e = L \Bigg/ \left[T_{sw}\left(\frac{LI_c}{V_{cp} T_{sw}} - \frac{D}{2} + \frac{S_e}{S_f}\frac{C_T}{T_{sw}}\right)\right]$$

$$K_{ap} = \left(t_{off} / 2L\right) R_e$$

Valley-Current Mode Control:

$$C_e = T_{sw}^2 \Big/ \left(L\pi^2\right) \qquad R_e = L \Bigg/ \left[T_{sw}\left(\frac{S_n + S_e}{S_n + S_f} - 0.5\right)\right]$$

$$K_{ap} = \left(t_{on} / 2L\right) R_e$$

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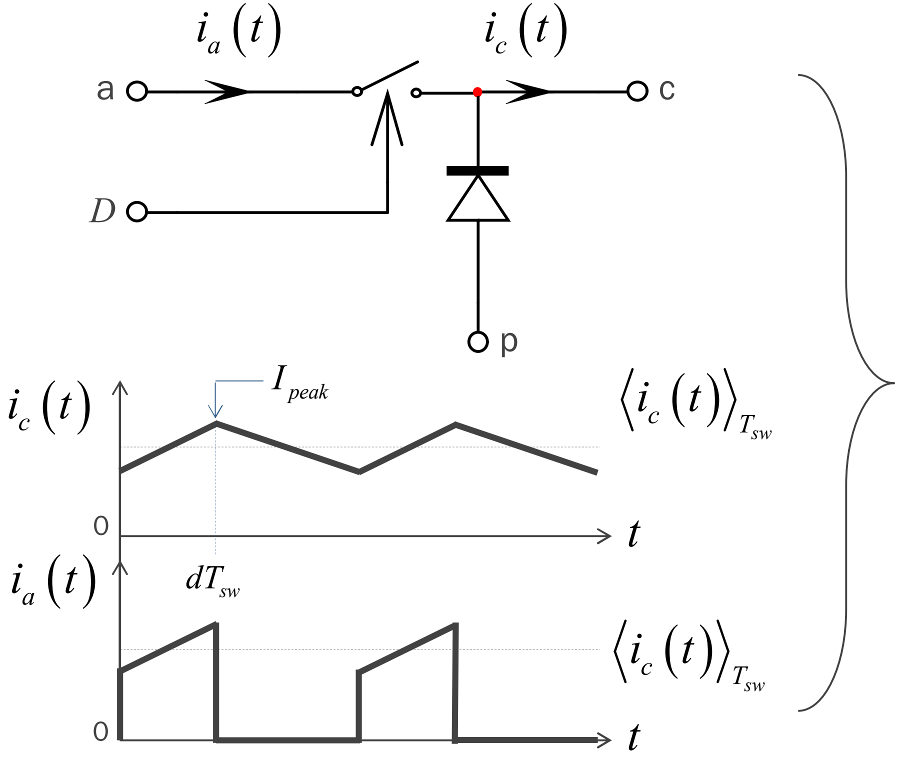
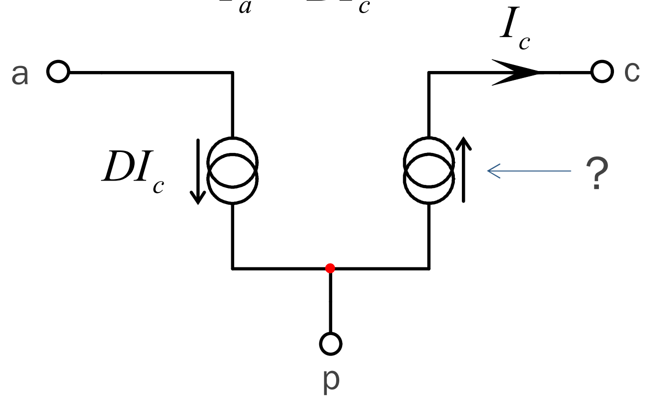
The PWM Switch in Current-Mode Control

☐ Determine a time-continuous equation linking variables



$$\langle i_a(t)\rangle_{T_{sw}} = I_a = \frac{1}{T_{sw}}\int_0^{dT_{sw}} i_a(t)\,dt = D\langle i_c(t)\rangle_{T_{sw}} = DI_c$$

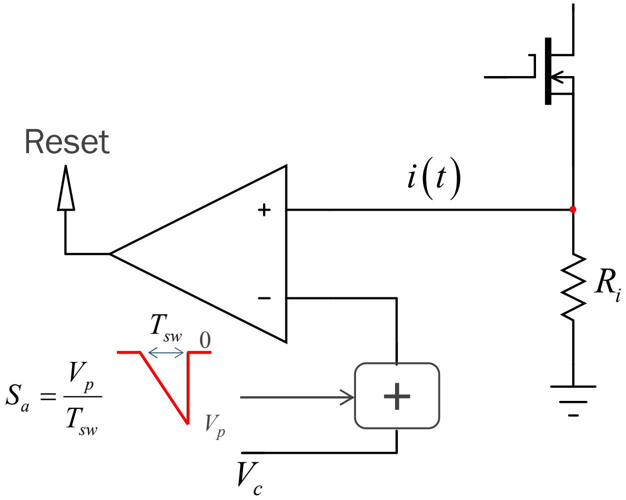
$$I_a = DI_c$$

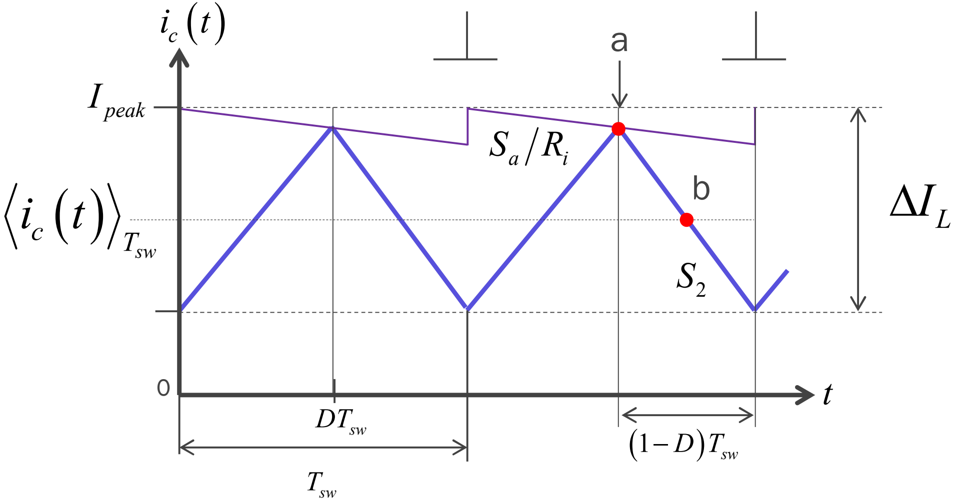


CCM

Accounting for Compensation Ramp Effects

- The compensation ramp reduces the effective peak current



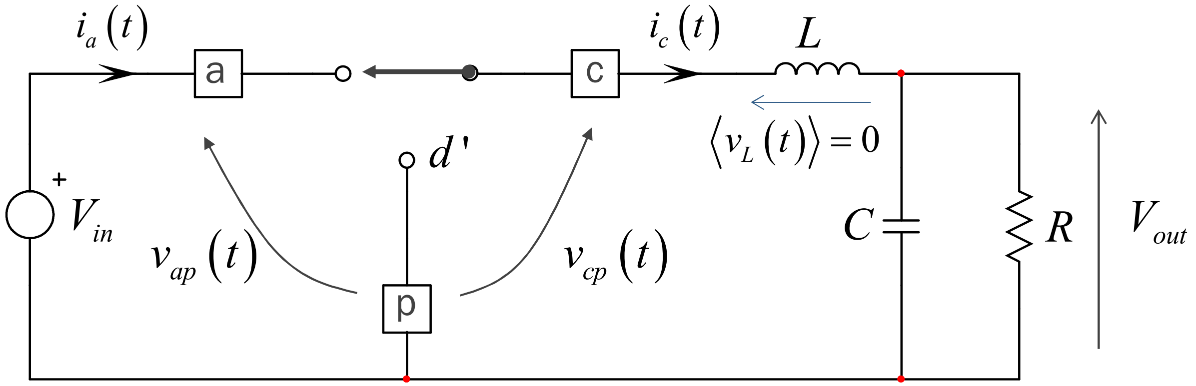


- Current at point ***b*** is that of ***a*** minus half the inductor ripple

$$\left\langle i_c(t)\right\rangle_{T_{sw}} = \boxed{\frac{V_c}{R_i} - \frac{S_a}{R_i}DT_{sw}} - \boxed{\frac{S_2 D' T_{sw}}{2}}$$

Invariant Properties for the PWM Switch Model

- The inductor downslope S_2 in a buck is defined by $S_2 = V_{out}/L$



Topology-dependent

Invariant expression

- The downslope depends on the output voltage V_{out}: $S_2 = \frac{V_{out}}{L}$

- The inductor average voltage is 0 V at steady-state: $V_{cp} = V_{out}$

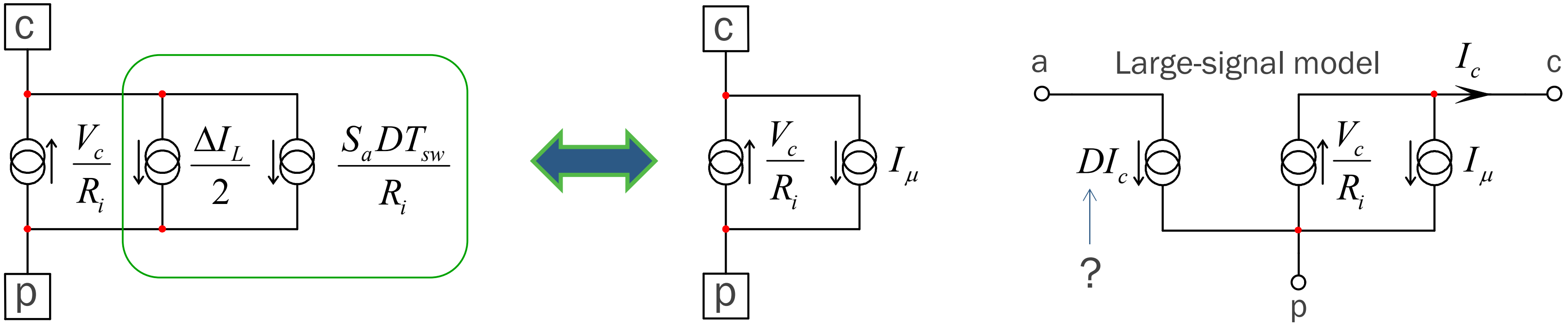
$$\boxed{S_2 = \frac{V_{cp}}{L}}$$

Associating Current Sources

- Update the previous equation to obtain the final definition

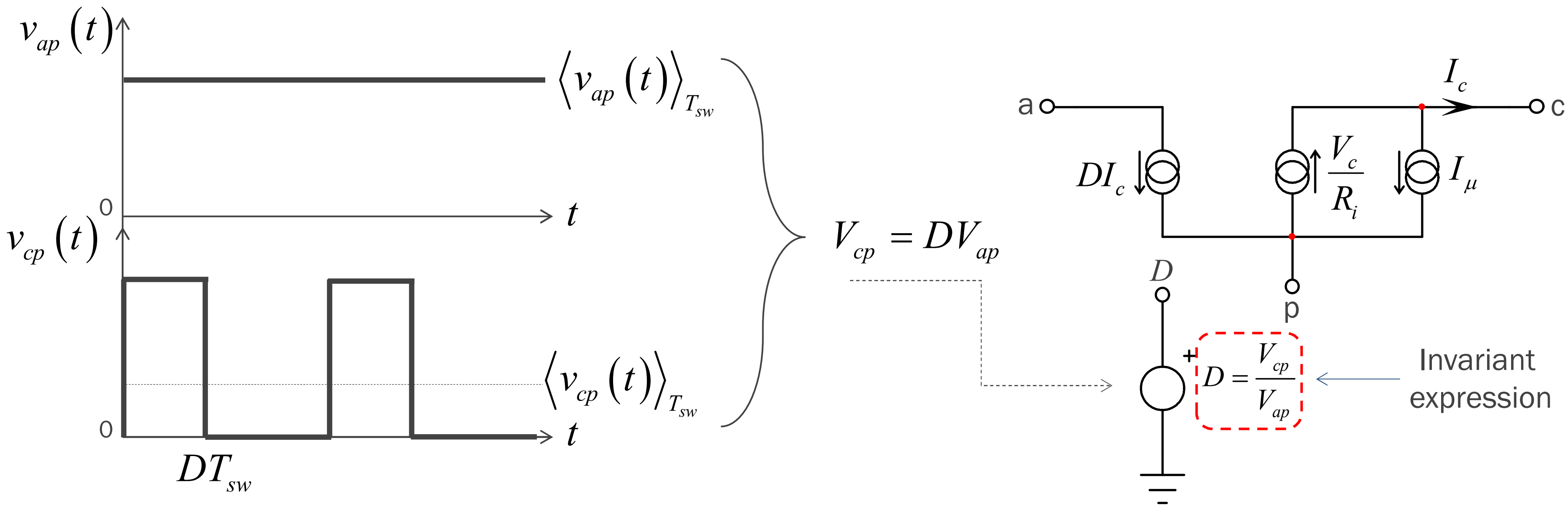
$$I_c = \underbrace{\boxed{\frac{V_c}{R_i}}}_{\text{Peak current setpoint}} - \underbrace{\boxed{V_{cp}\left(1-D\right)\frac{T_{sw}}{2L}}}_{\text{Half inductor ripple}} - \underbrace{\boxed{\frac{S_a}{R_i}DT_{sw}}}_{\text{Compensation ramp}} \xrightarrow[\text{Group 2}^{\text{nd}}\text{ and 3}^{\text{rd}}\text{ terms}]{} I_\mu = V_{cp}\left(1-D\right)\frac{T_{sw}}{2L} + \frac{S_a}{R_i}DT_{sw}$$

- Inductor ripple and compensation ramp alter peak value



Determining the Duty Ratio

- Voltage relationship between v_{ap} and v_{cp} holds in current mode



$$V_{cp} = DV_{ap}$$

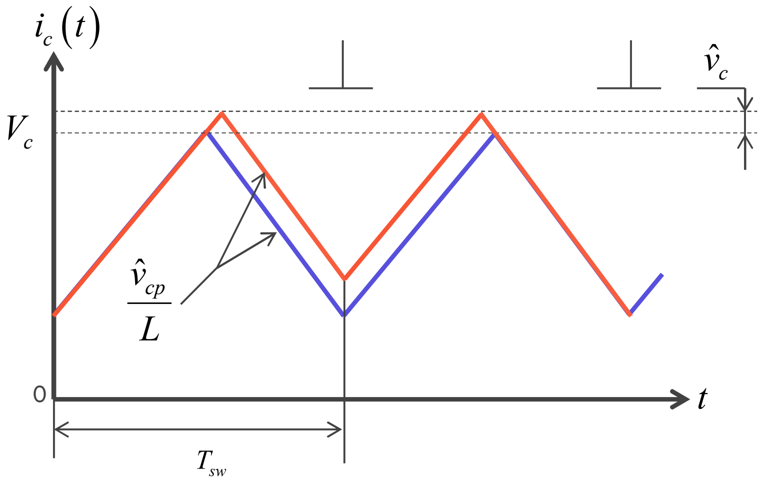
$$D = \frac{V_{cp}}{V_{ap}}$$

Invariant expression

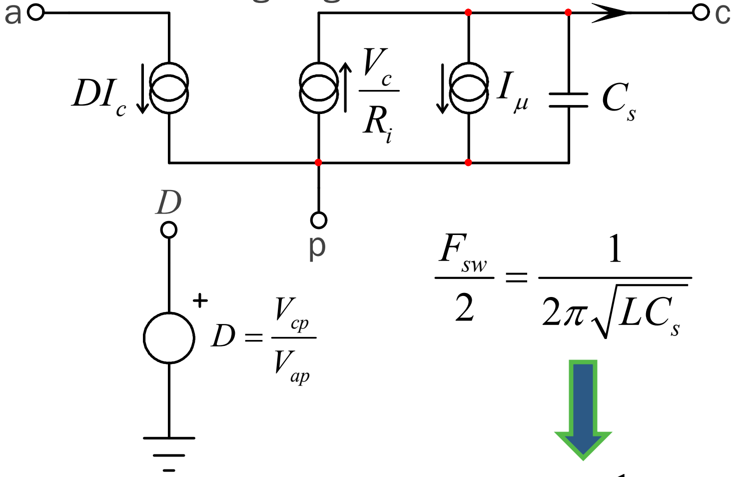
$$\langle v_{cp}(t)\rangle_{T_{sw}} = V_{cp} = \frac{1}{T_{sw}}\int_0^{dT_{sw}} v_{cp}(t)\,dt = D\langle v_{ap}(t)\rangle_{T_{sw}} = DV_{ap}$$

Predicting Sub-Harmonic Oscillations

- The model, as it is, cannot predict instabilities
  - Let's observe a small-signal perturbation in v_c



Final large-signal model



$$\frac{F_{sw}}{2} = \frac{1}{2\pi\sqrt{LC_s}}$$

$$C_s = \frac{1}{L\left(F_{sw}\pi\right)^2}$$

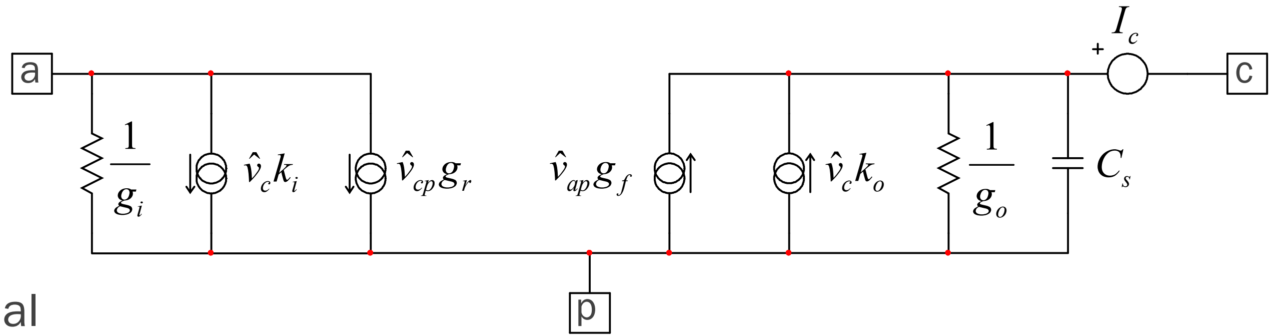
- The off-slope does not change as $\hat{v}_{cp}$ keeps constant
- This "memory" effect is modeled with a capacitor C_s

Small-Signal Model of the CM-PWM Switch

- ☐ You can perturb…

$$I_c + \hat{i}_c \qquad V_c + \hat{v}_c \qquad V_{cp} + \hat{v}_{cp} \qquad V_{ap} + \hat{v}_{ap}$$

$$I_c = \frac{V_c}{R_i} - V_{cp}\left(1-D\right)\frac{T_{sw}}{2L} - \frac{S_a}{R_i}DT_{sw}$$



…or apply partial differentiation

$$\left.\begin{aligned}
\hat{i}_c &= \frac{\partial I_c}{\partial V_c}\hat{v}_c + \frac{\partial I_c}{\partial V_{ap}}\hat{v}_{ap} + \frac{\partial I_c}{\partial V_{cp}}\hat{v}_{cp} \\
\hat{i}_a &= \frac{\partial I_a}{\partial V_c}\hat{v}_c + \frac{\partial I_a}{\partial V_{ap}}\hat{v}_{ap} + \frac{\partial I_a}{\partial V_{cp}}\hat{v}_{cp}
\end{aligned}\right\}$$

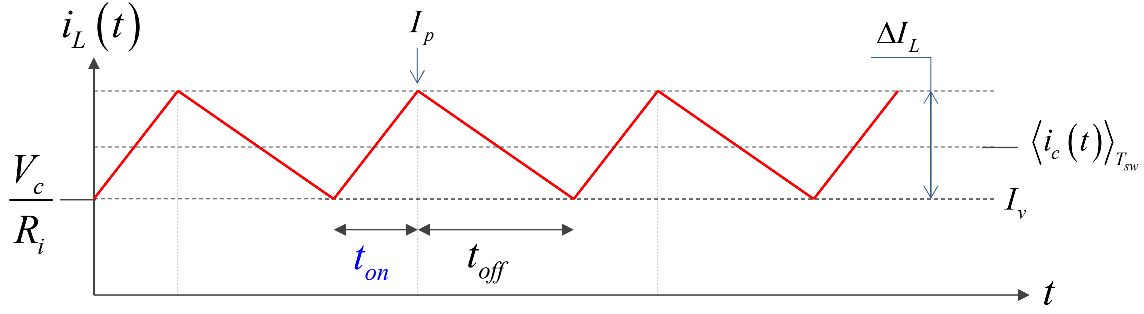
$$g_o = \frac{T_{sw}}{L}\left(D'\frac{S_a}{S_n} + \frac{1}{2} - D\right) \qquad k_o = \frac{1}{R_i} \qquad g_f = Dg_o - \frac{DD'T_{sw}}{2L}$$

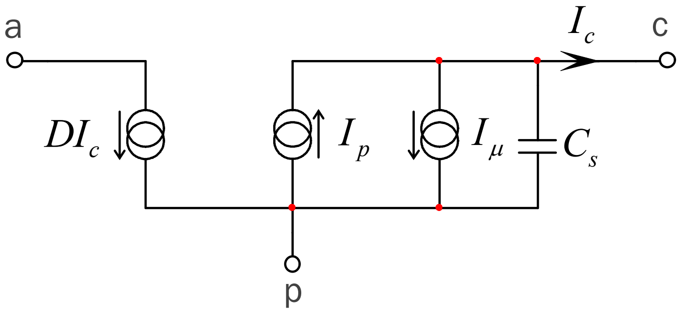
$$g_i = D\left(g_f - \frac{I_c}{V_{ap}}\right) \qquad k_i = \frac{D}{R_i} \qquad g_r = \frac{I_c}{V_{ap}} - g_o D$$

- ☐ Laplace Transform now holds on this linear network

The PWM Switch Model in a COT Converter

- In a constant on-time circuit, the loop controls the *valley* current





Same structure as CM model

$$I_p = \frac{V_c}{R_i} + \frac{V_{ac}}{L} t_{on} \qquad I_c = I_p - \frac{\Delta I_L}{2} = \left(\frac{V_c}{R_i} + \frac{V_{ac}}{L} t_{on}\right) - \frac{V_{cp}}{2L}\left(1 - \frac{t_{on}}{T_{sw}}\right) T_{sw}$$

$$\frac{V_c}{R_i} = I_p - S_{off} t_{off} = I_p - \frac{V_{cp}}{L} t_{off} \qquad t_{off} = \left(I_p - \frac{V_c}{R_i}\right)\frac{L}{V_{cp}} \rightarrow t_{off} = \left(\frac{V_c}{R_i} + \frac{V_{ac}}{L} t_{on} - \frac{V_c}{R_i}\right)\frac{L}{V_{cp}} = \frac{V_{ac}}{V_{cp}} t_{on}$$

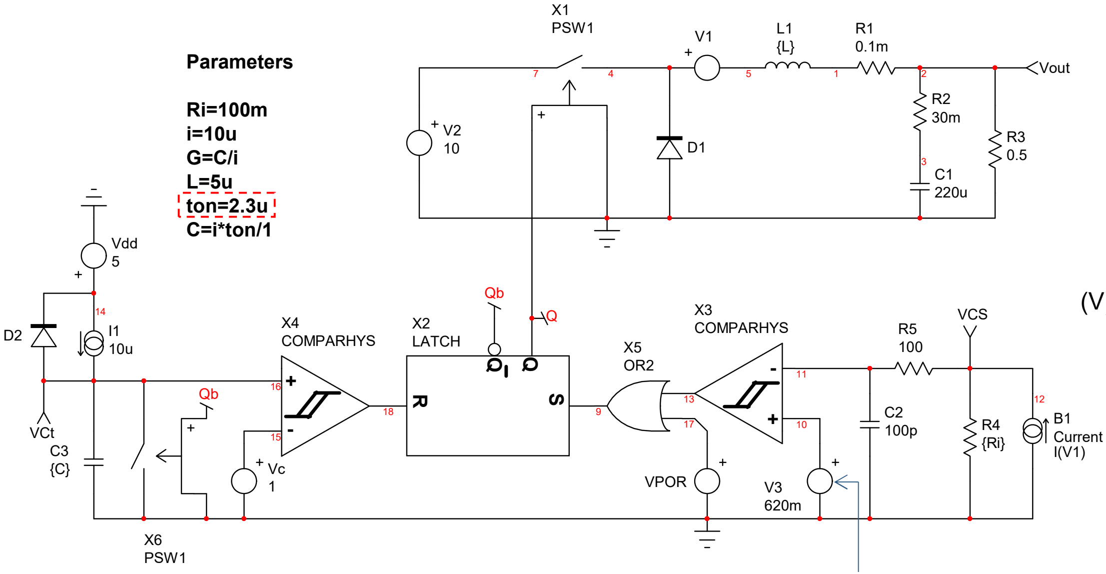
Valley current

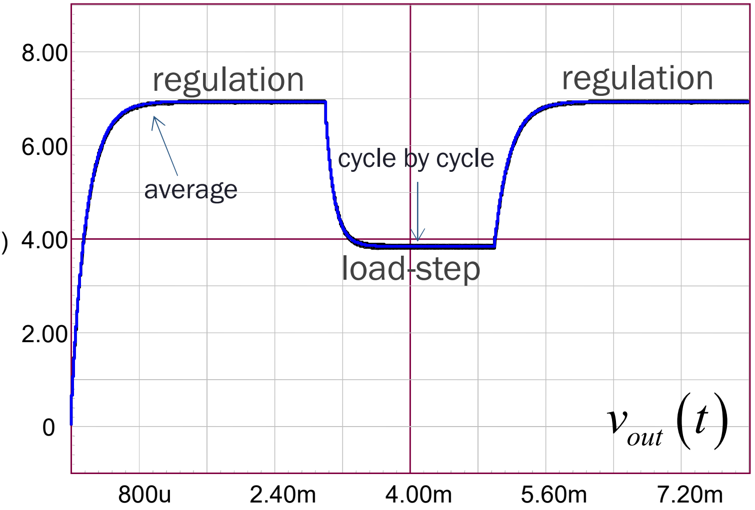
Fixed value

$$I_\mu = \frac{V_{cp}}{2L}\left(1 - \frac{t_{on}}{T_{sw}}\right) T_{sw} \qquad T_{sw} = t_{on} + t_{off} \qquad D = \frac{t_{on}}{T_{sw}}$$

Testing the COT Model in Transient Analysis

- ☐ We compare the cycle-by-cycle response with that of the averaged model





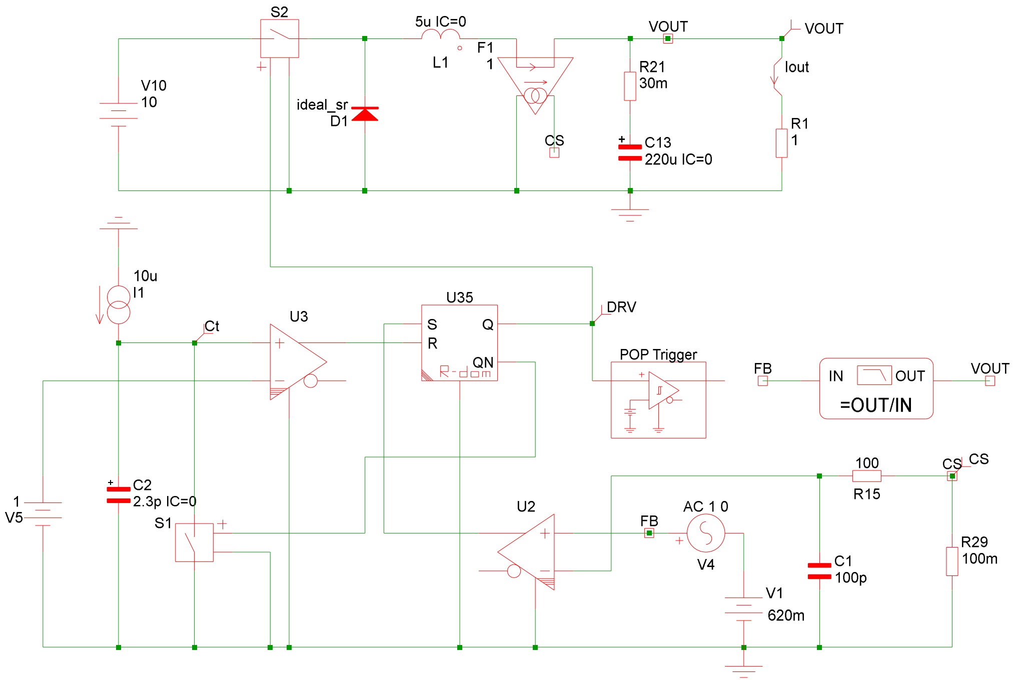
Keeps the switch closed for a fixed t_{on}.

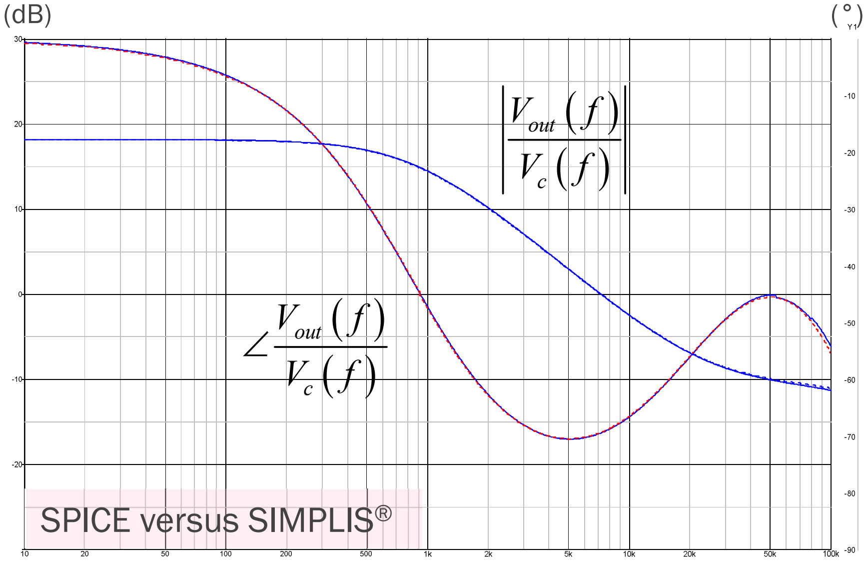
Sets the latch back high when valley current set by V_c is reached.

Transient responses are identical

Small-Signal Response: SPICE versus SIMPLIS®

- ❑ SIMPLIS lets us obtain the dynamic response in a few seconds

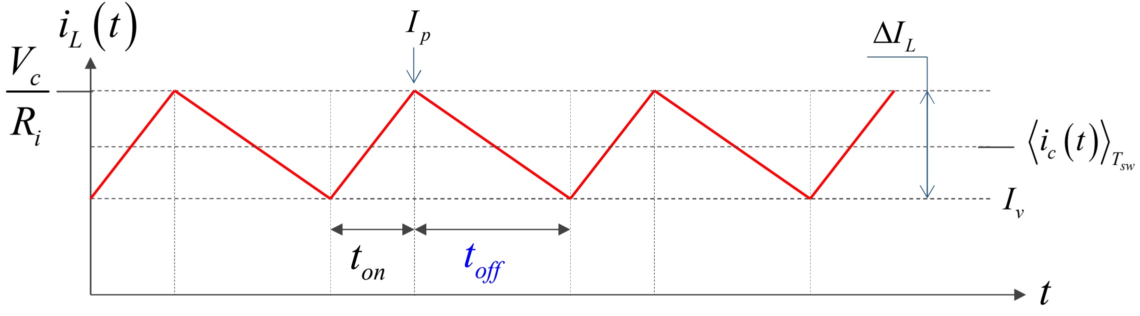


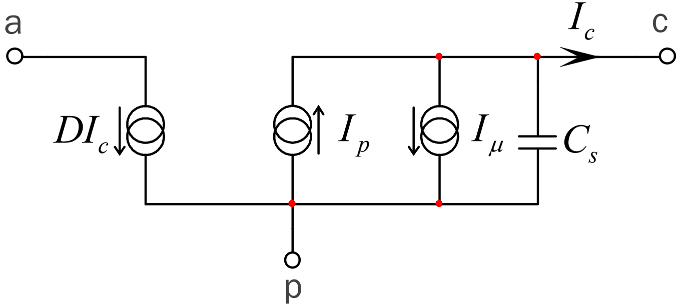


Dynamic responses are identical

The PWM Switch Model in a FOT Converter

- ☐ In a fixed-off-time circuit, the loop controls the *peak* current



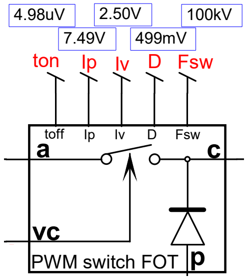


Same structure as CM model

$$I_c = \frac{V_c}{R_i} - \boxed{\frac{\Delta I_L}{2}} = \frac{V_c}{R_i} - \frac{V_{ac}}{2L} t_{on} \longrightarrow t_{on} = \frac{\left(\frac{V_c}{R_i} - I_c\right)2L}{V_{ac}}$$

t_{off} is fixed

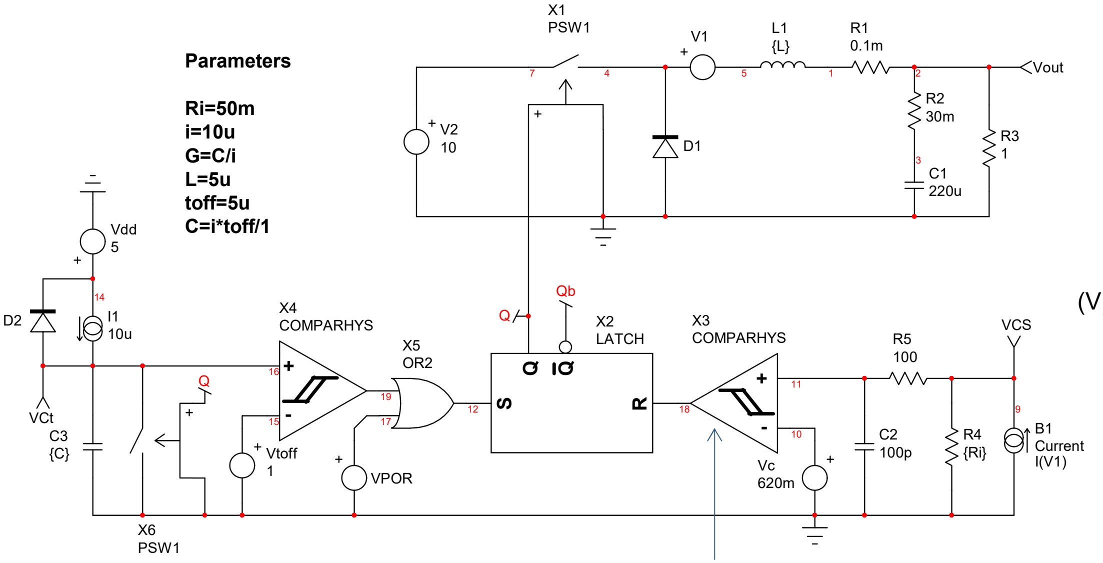
$$I_\mu = \frac{V_{cp}}{2L} t_{off} \qquad T_{sw} = t_{on} + t_{off} \qquad I_C = \frac{V_c}{R_i} - I_\mu \qquad D = \frac{t_{on}}{T_{sw}}$$



Encapsulated version

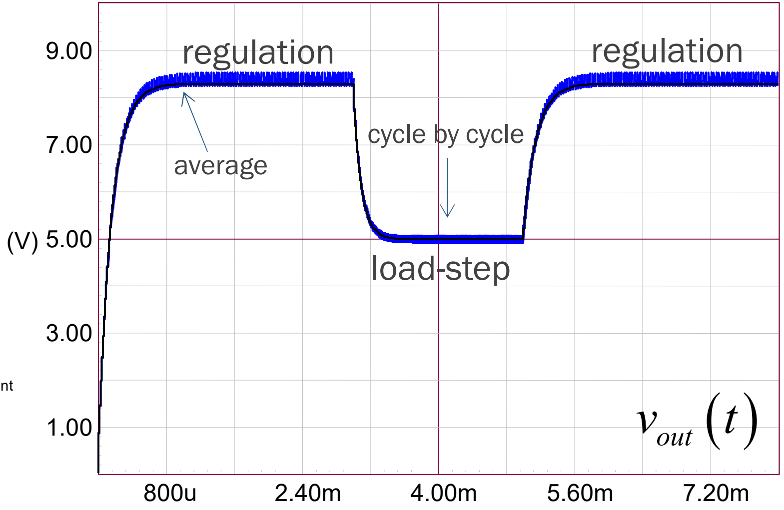
Comparing the Cycle-by-Cycle Transient Waveforms

- ☐ We compare the cycle-by-cycle response with that of the averaged model



Keeps the switch open for a fixed t_{on}.

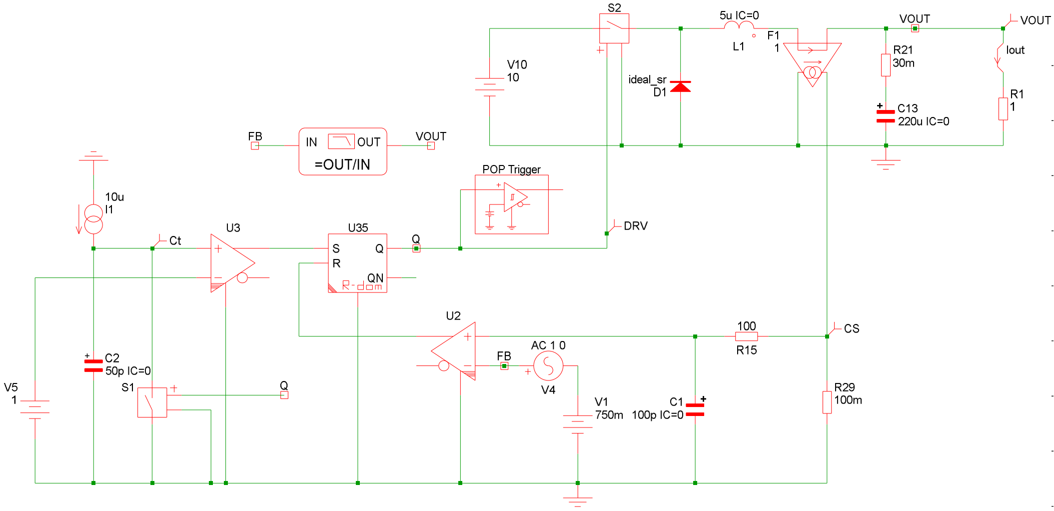
Resets the latch when the peak current set by V_c is reached.



Transient responses are identical

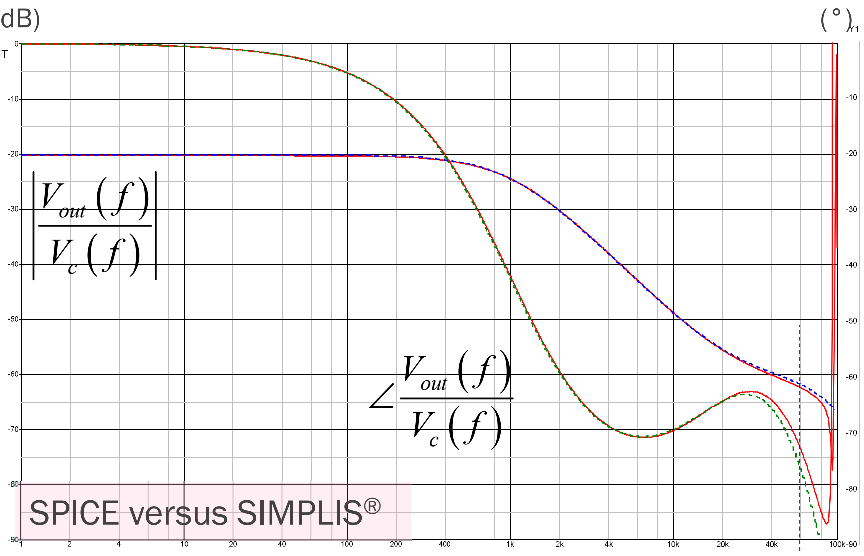
Small-Signal Response: SPICE versus SIMPLIS®

- ❑ SIMPLIS® easily simulates the fixed-off-time converter



Fixed-off-time generator

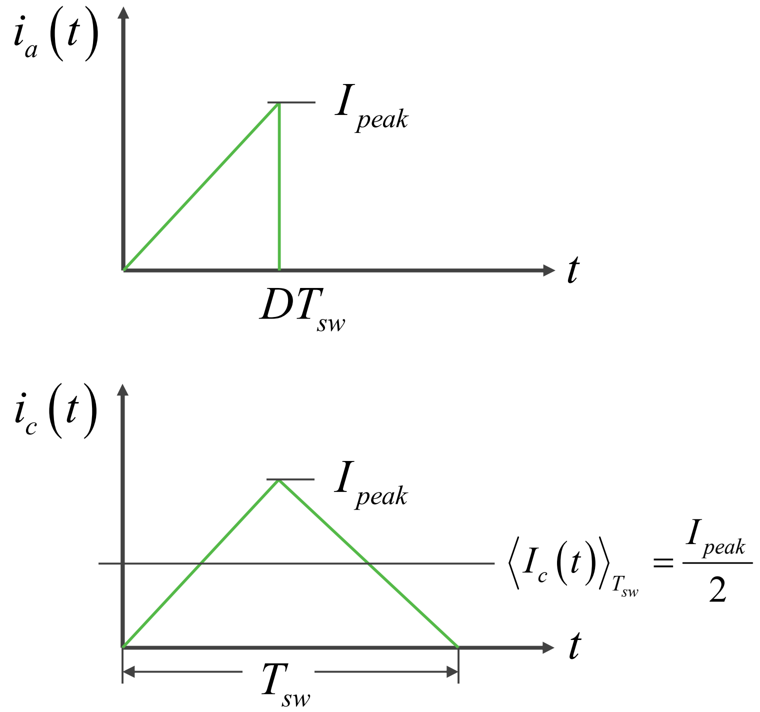
Peak-current control

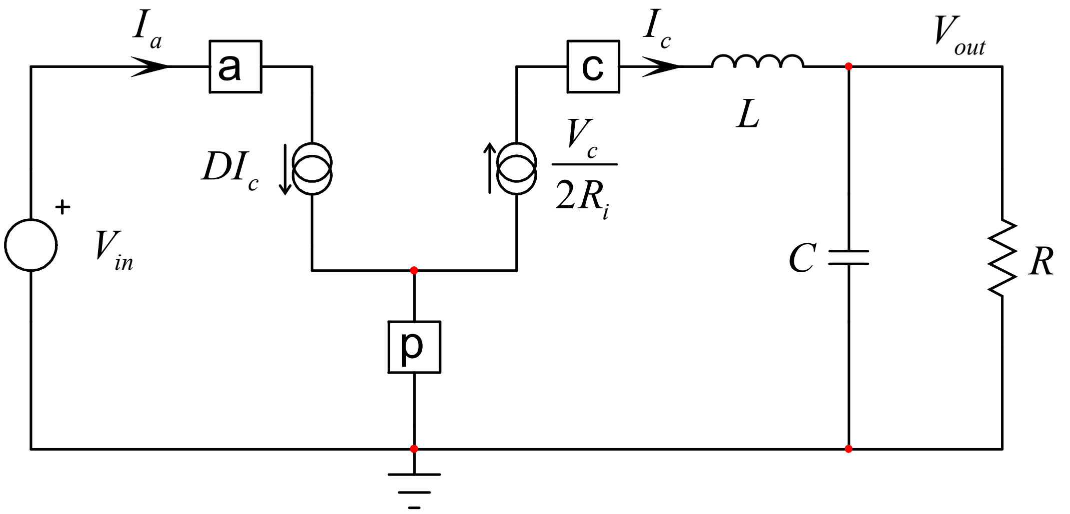


Dynamic responses are very close

Quasi-Square-Wave Converters

- The waveform average values do not include the deadtime





Derive operating points and dc transfer function

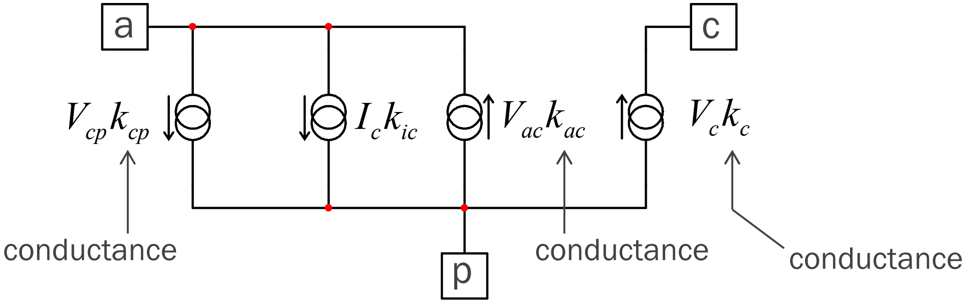
C. Basso, *Small-Signal Modeling of Power Converters,* Professional Education APEC seminar 2013.

From Large- to Small-Signal Models

- ☐ Partial differentiation will give small-signal coefficients

$$I_c = f\left(V_c\right) \longrightarrow \hat{i}_c = \frac{\partial I_c\left(V_c\right)}{\partial V_c}\hat{v}_c \qquad \hat{i}_c = \hat{v}_c\left(\frac{1}{2R_i}\right) = \hat{v}_c k_c \qquad k_c = \frac{1}{2R_i}$$

$$\hat{i}_a = \left.\frac{\partial I_a\left(V_{cp},V_{ac},I_c\right)}{\partial V_{cp}}\right|_{I_c,V_{ac}}\hat{v}_{cp} + \left.\frac{\partial I_a\left(V_{cp},V_{ac},I_c\right)}{\partial I_c}\right|_{V_{ap},V_{ac}}\hat{i}_c + \left.\frac{\partial I_a\left(V_{cp},V_{ac},I_c\right)}{\partial V_{ac}}\right|_{I_c,V_{cp}}\hat{v}_{ac} \longrightarrow \hat{i}_a = \hat{v}_{cp}k_{cp} + k_{ic}\hat{i}_c - \hat{v}_{ac}k_{ac}$$



Small-signal model

$$k_{cp} = \frac{I_{c0}V_{ac0}}{\left(V_{ac0}+V_{cp0}\right)^2}$$

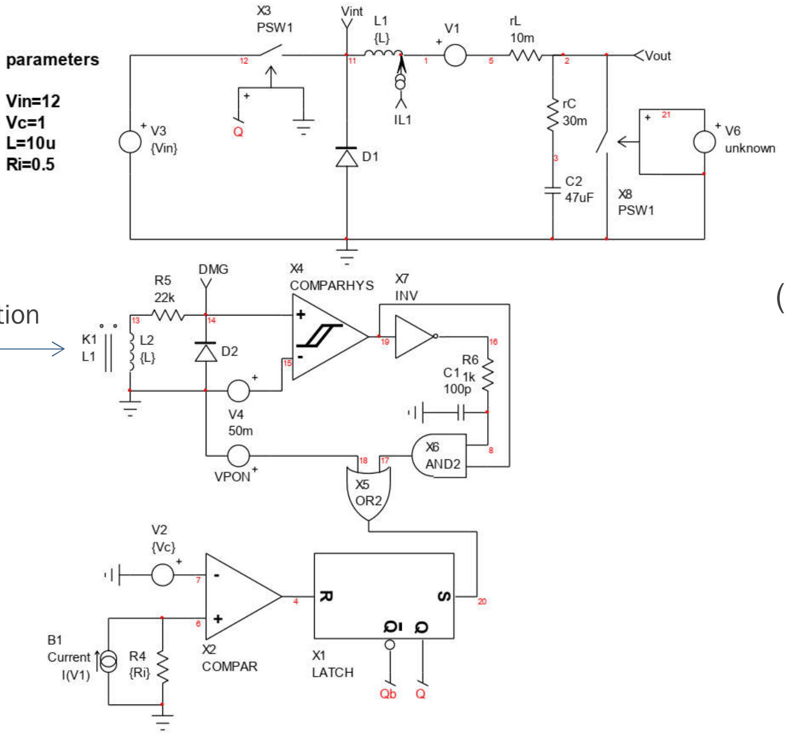
$$k_{ic} = \frac{V_{cp0}}{V_{cp0}+V_{ac0}}$$

$$k_{ac} = \frac{V_{cp0}I_{c0}}{\left(V_{ac0}+V_{cp0}\right)^2}$$

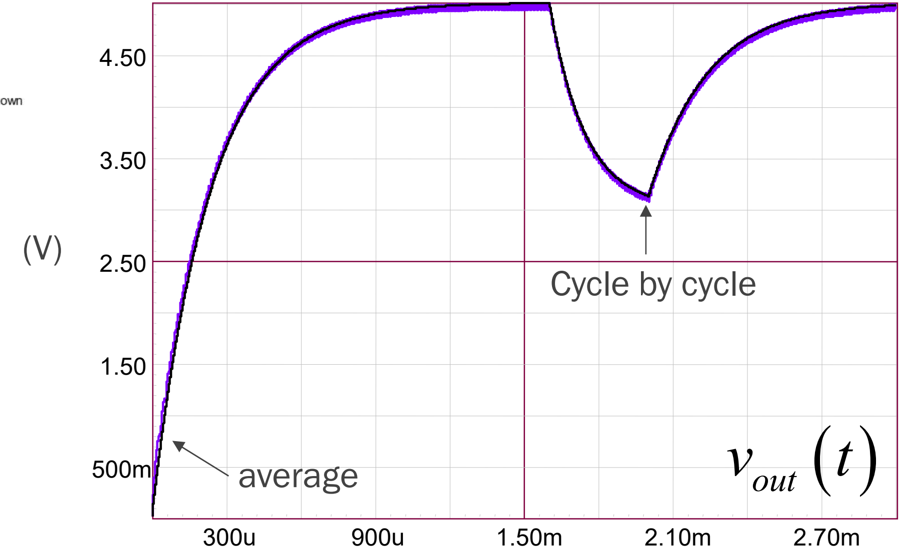
Static coefficients

Cycle-by-Cycle and Averaged Responses

- ☐ The cycle-by-cycle circuit requires an extra winding to detect core reset



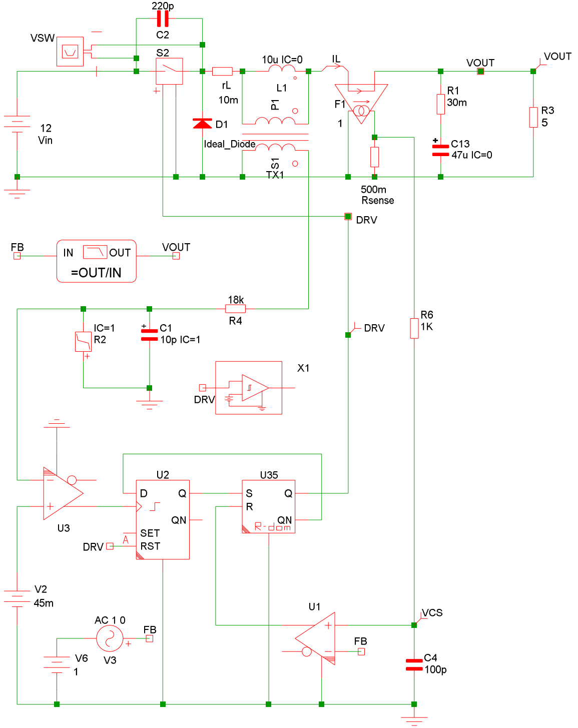
Demagnetization detection

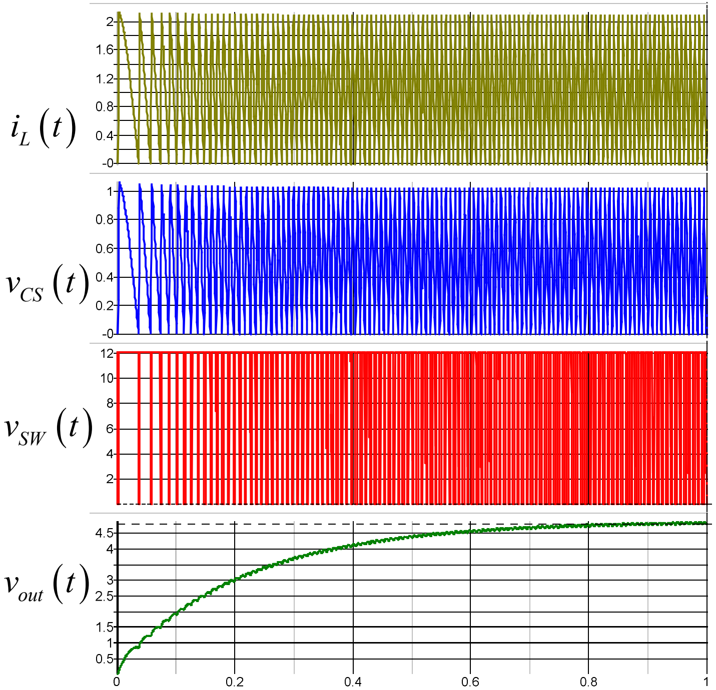


Transient responses are identical

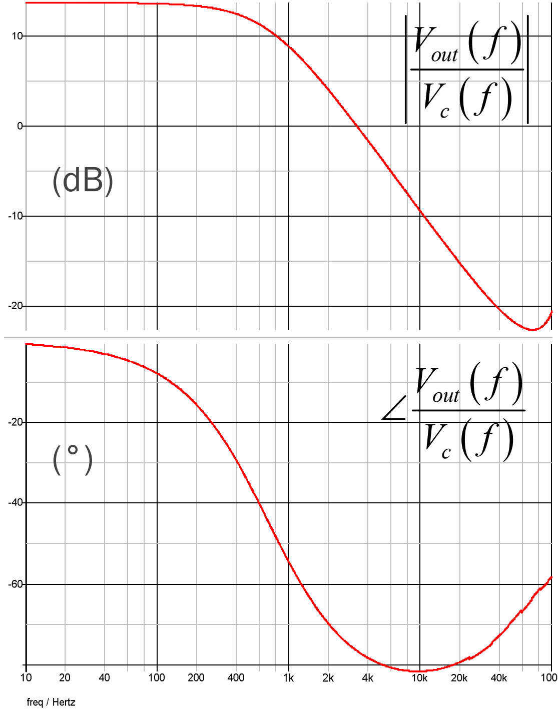
SIMPLIS® Delivers the Full Picture: Transient and Ac

- ☐ The switching frequency varies with line and load conditions





Cycle-by-cycle results



1^{st}-order dynamic response

Course Agenda

- ☐ The Buck Converter
- ☐ Control Schemes
- ☐ Introduction to Modeling
- ☐ The PWM Switch in Current-Mode Control
- ☐ Introduction to the Fast Analytical Techniques
- ☐ The Four Transfer Functions in CCM Current-Mode Control
- ☐ Compensation Strategy
- ☐ Prototype Measurements

The Fastest Way to Determine a Transfer Function

- We can use the Fast Analytical Circuits Techniques or FACTs

$$\boxed{\frac{V_{out}(s)}{D(s)}} = H_0 \frac{1+a_1 s+a_2 s^2}{1+b_1 s+b_2 s^2} \longrightarrow \left.\begin{array}{l} a_1 \text{ and } b_1\ [\mathrm{s}] \\ a_2 \text{ and } b_2\ [\mathrm{s}^2] \end{array}\right\}$$

Same dimension

$$a_1 = \tau_{1N} + \tau_{2N} \qquad b_1 = \tau_1 + \tau_2$$

Nulled response

Zeroed excitation

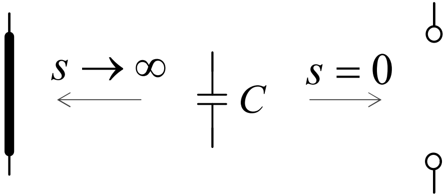
$$a_2 = \tau_{1N}\tau_{2N}^1 \qquad b_2 = \tau_1\tau_2^1$$

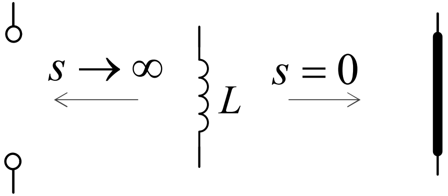
- Energy-storing elements are combined with resistances

$$\tau = \frac{L}{R} \qquad \tau = RC$$

Time constants

- Capacitors and inductors behave differently for $s = 0$ and $s \to \infty$





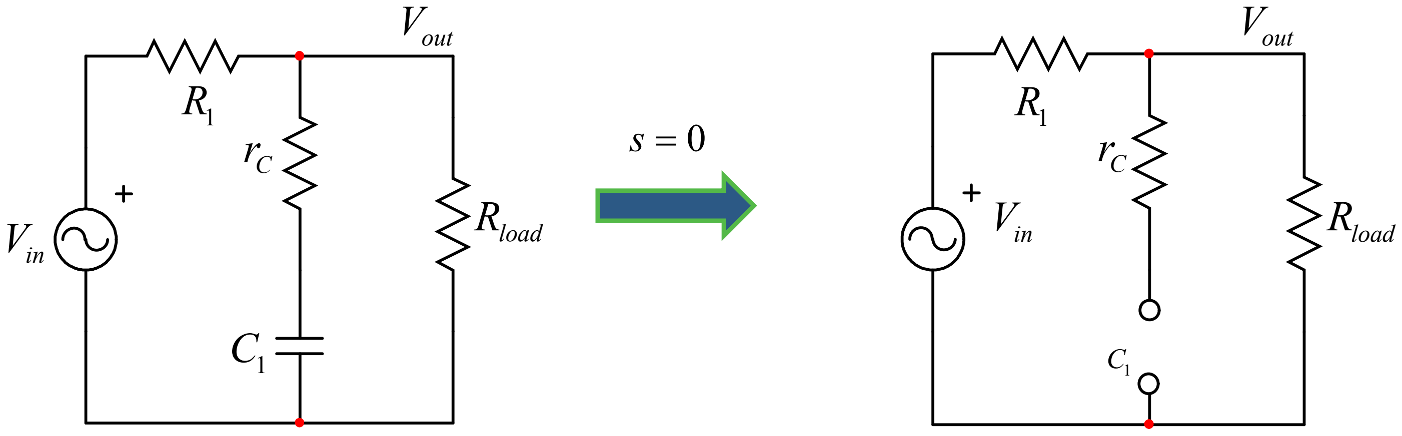
Determine the Dc Gain

❑ Look at the circuit for $s = 0$

➢ Capacitor are open circuited

➢ Inductors are short circuited

SPICE operating point calculation

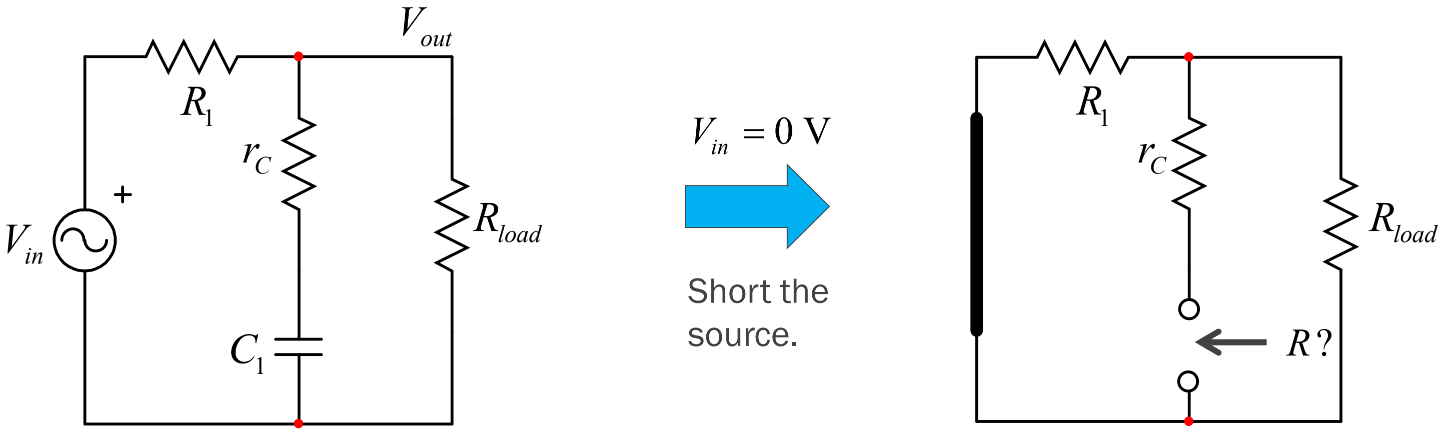


❑ Determine the gain in this condition: $H_0 = \frac{R_{load}}{R_{load} + R_1}$

Determine the Time Constant

❑ Look at the resistance driving the energy-storing element

1. When the excitation is turned off, V_{in} = 0 V



$V_{in} = 0\text{ V}$

Short the source.

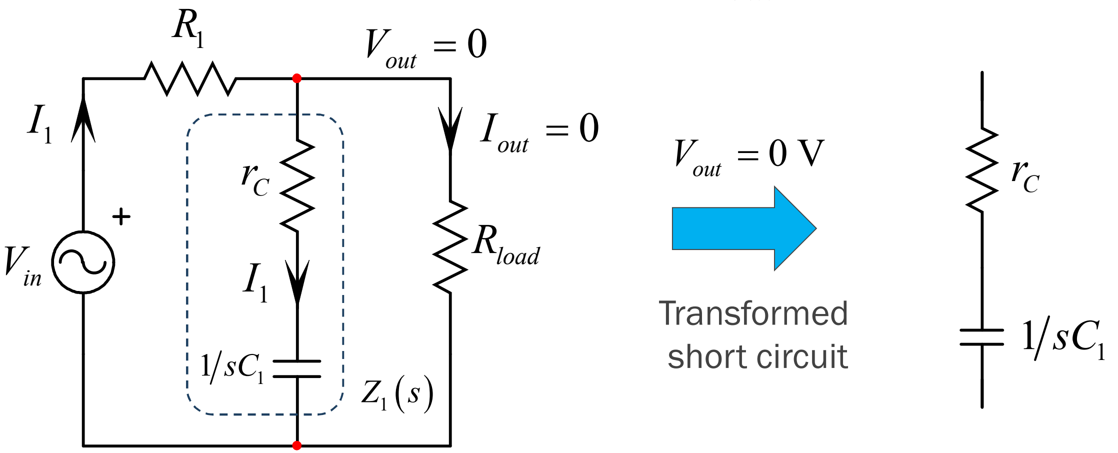
❑ Remove the capacitor and “look” into its terminals

➢ The first time constant is $\tau_1 = \left(r_C + R_1 \,\|\, R_{load}\right)C_1$

Null the Output to Unveil the Zero Location

☐ Bring the excitation back

2. Check the condition bringing V_{out} = 0 V



$$Z_1(s) = r_C + \frac{1}{sC_1} = 0$$

$$s_z = -\frac{1}{r_C C_1}$$

$$\omega_z = \frac{1}{r_C C_1}$$

☐ You can also remove the capacitor and look into its terminals

➢ The second time constant in the null condition is $\tau_2 = r_C C_1$

Combine the Time Constants in a Low-Entropy Form

☐ By combining time constants, we have

$$H(s) = H_0 \frac{1+s\tau_2}{1+s\tau_1} = \frac{R_{load}}{R_{load}+R_1} \frac{1+sr_C C_1}{1+s\left(r_C + R_1 \| R_{load}\right) C_1}$$

☐ Rearrange the equation to unveil a pole and a zero

$$H(s) = H_0 \frac{1+\dfrac{s}{\omega_z}}{1+\dfrac{s}{\omega_p}} \left. \vphantom{\frac{\frac{1}{1}}{\frac{1}{1}}} \right\} \quad \omega_z = \frac{1}{r_C C_1} \qquad H_0 = \frac{R_{load}}{R_{load}+R_1}$$

$$\omega_p = \frac{1}{\left(r_C + R_1 \| R_{load}\right) C_1}$$

☐ This is a *low-entropy* expression

Second-Order Circuits: New Notation

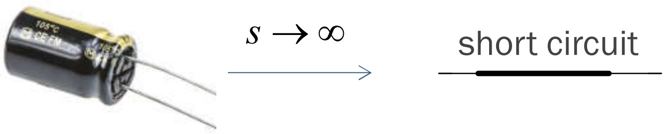
- ☐ Set one reactance into its high-frequency state

$$\tau_2^1 \longrightarrow$$ Reactance 1 is in its high-frequency state

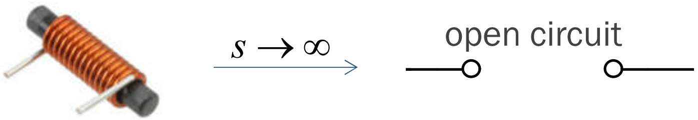
$R? \uparrow$ (on 2) $\longrightarrow$ What resistance drives reactance 2?

$$\tau_1^2 \longrightarrow$$ Reactance 2 is in its high-frequency state

$R? \uparrow$ (on 1) $\longrightarrow$ What resistance drives reactance 1?


$s \to \infty$

short circuit


$s \to \infty$

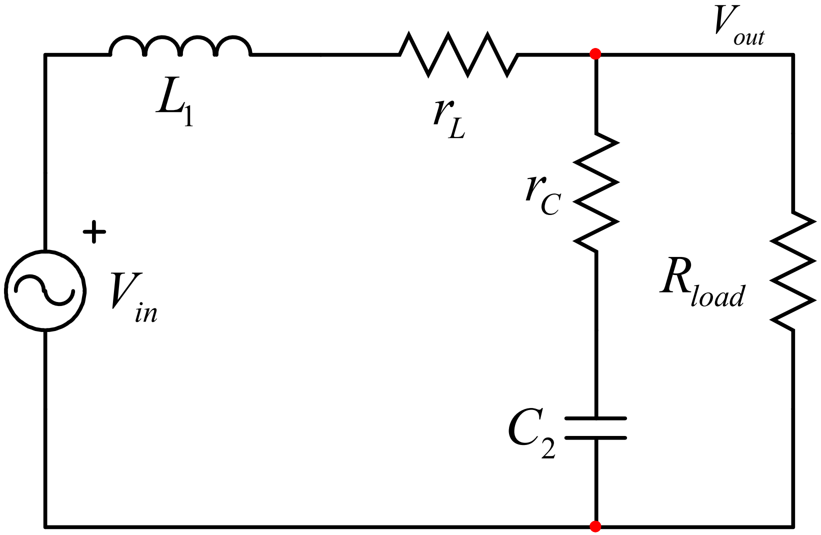
open circuit

- ☐ There is redundancy: pick the simplest result

$$b_2 = \tau_1\tau_2^1 \Longleftrightarrow b_2 = \tau_2\tau_1^2$$

A Simple Low-Pass LC Filter

☐ There are two energy-storing elements: 2nd-order filter



Set s to zero: short inductors, open caps.

↓

Determine H_0

↓

Reduce excitation to 0 V: short V_{in}

↓

Find 1st-order time constants with L_1 and C_2

↓

Find 2nd-order time constants with L_1 and C_2

↓

Null the output and find zero(es)

↓

Assemble results in a low-entropy form

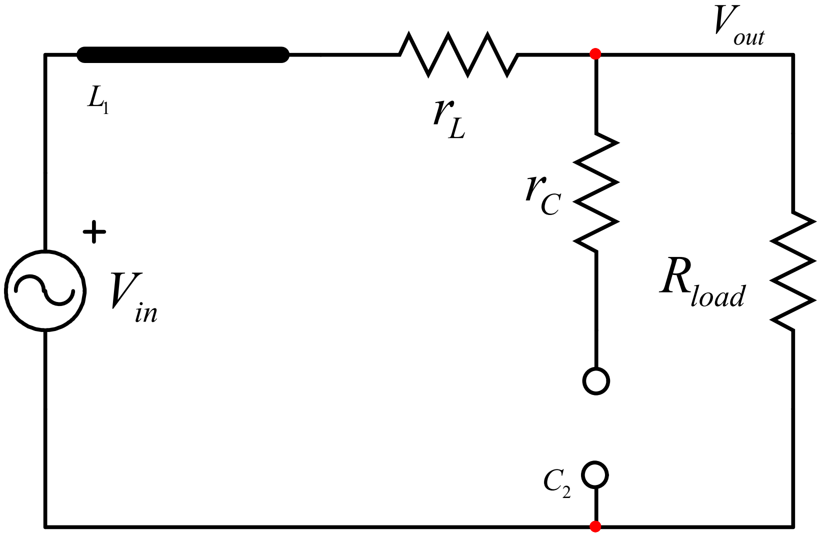
↓

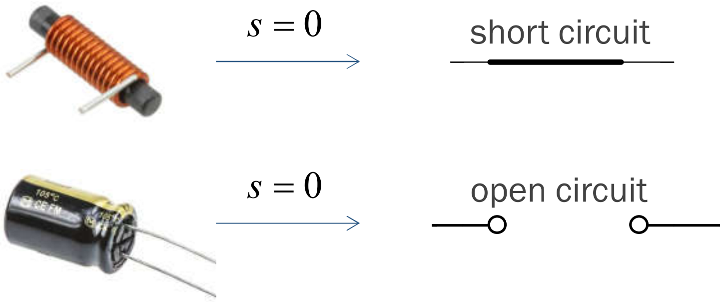
$$H(s) = H_0 \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_0}\right)^2}$$

☐ What transfer function links V_{out} to V_{in}?

Start with the Dc Gain

- ☐ Short the inductor and open the capacitor



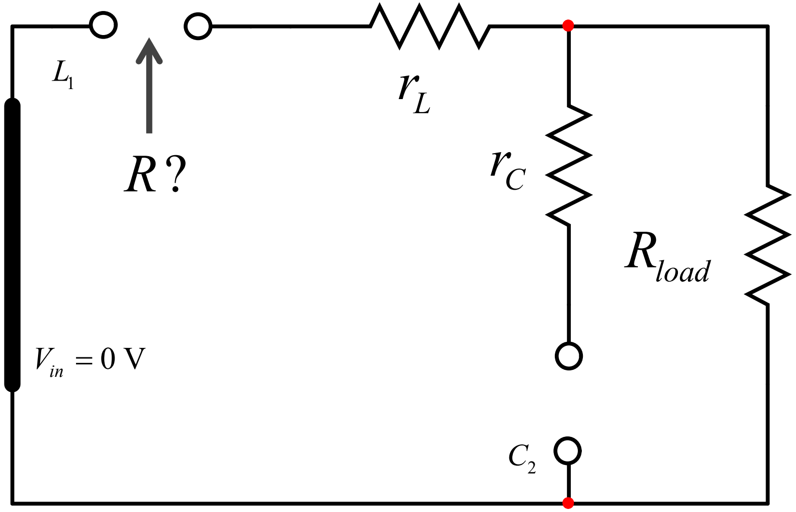


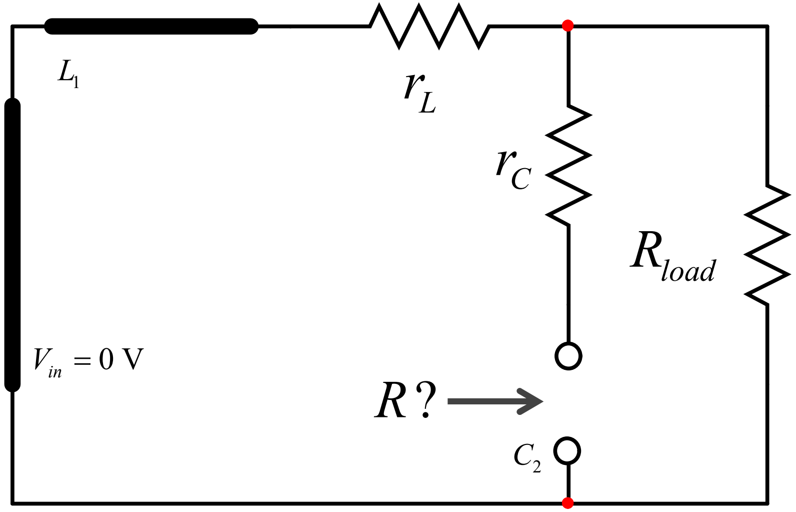
$$H_0 = \frac{R_{load}}{R_{load} + r_L}$$

- ☐ This expression is obtained by inspecting the circuit: no algebra

Reduce the Excitation to 0 V: Short V_{in}

- ☐ Determine the 1st-order time constants





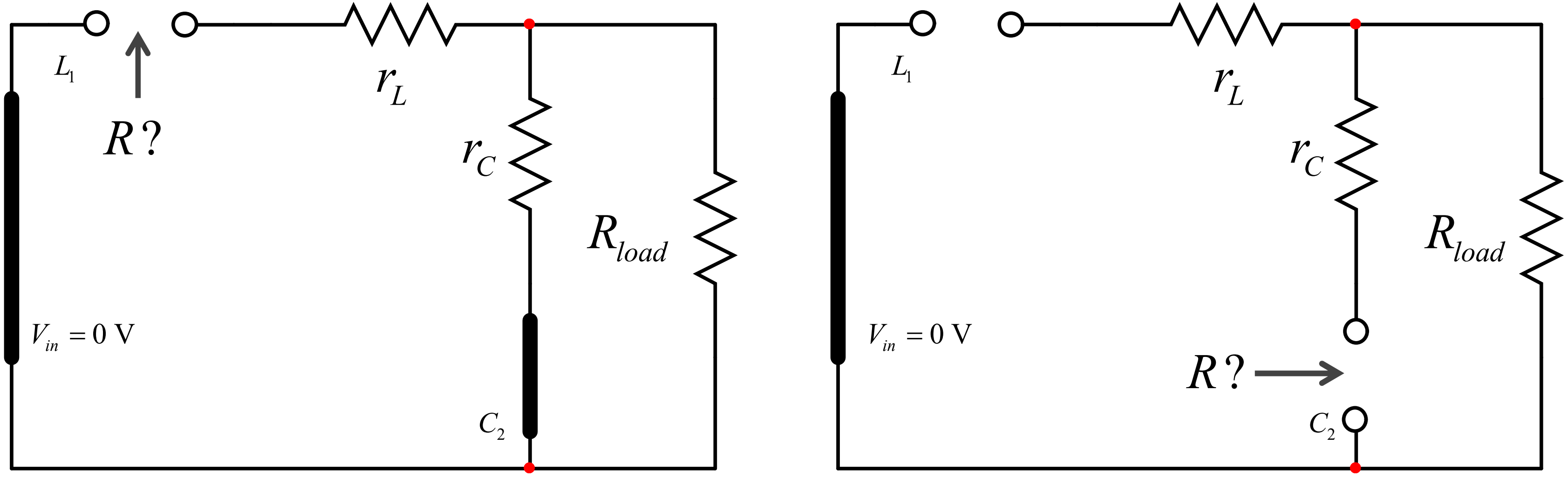
$$\tau_1 = \frac{L_1}{R_{load} + r_L}$$

$$\tau_2 = \left(r_C + \underset{\displaystyle\uparrow}{r_L \parallel R_{load}}\right) C_2$$

Don’t expand!

Pick the Simplest Time Constant Combination

- ☐ Set one of the energy-storing elements in its high-frequency state

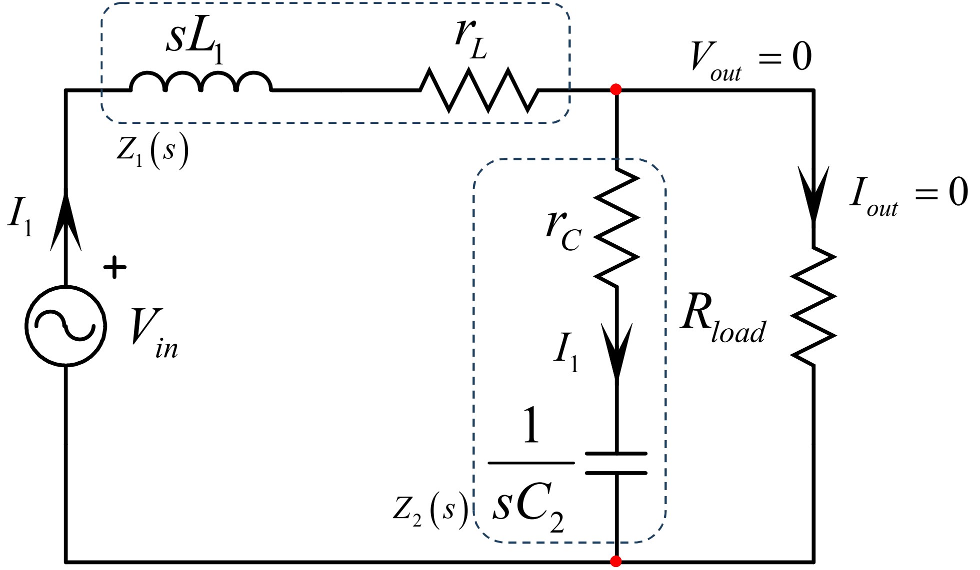


$$\tau_1^2 = \frac{L_1}{r_L + r_C \,||\, R_{load}} \qquad b_2 = \tau_2 \tau_1^2 \Leftrightarrow b_2 = \tau_1 \tau_2^1 \qquad \tau_2^1 = C_2 \left(r_C + R_{load} \right)$$

- ☐ You have to select the simplest combination between the two options

Determine the Zero Instantaneously

- ☐ What would prevent the stimulus (V_{in}) from forming a response (V_{out})?



$$Z_1(s) = sL_1 + r_L \to \infty$$

Only for s approaching infinity

$$Z_2(s) = \frac{1}{sC_2} + r_C = 0$$

$$s_z = -\frac{1}{r_C C_1}$$

$$\omega_z = \frac{1}{r_C C_1}$$

- ☐ There is one single zero contributed by C_2

Assemble all Time Constants to form $H(s)$

- The denominator is obtained by combining the time constants

$$D(s)=1+b_1s+b_2s^2=1+s\left[\frac{L_1}{R_{load}+r_L}+\left(r_C+r_L\,||\,R_{load}\right)C_2\right]+L_1C_2\frac{r_C+R_{load}}{R_{load}+r_L}\cdot s^2$$

- The numerator features one zero only

$$N(s)=1+sr_CC_2$$

- Read gains, poles and zero with the *low-entropy* format

$$\left. H(s)=H_0\frac{1+\dfrac{s}{\omega_z}}{1+\dfrac{s}{\omega_0 Q}+\left(\dfrac{s}{\omega_0}\right)^2}\quad\right\}\quad \begin{aligned}&\omega_z=\frac{1}{r_CC_2}\quad \omega_0=\frac{1}{\sqrt{L_1C_2}}\sqrt{\frac{r_L+R_{load}}{r_C+R_{load}}}\quad H_0=\frac{R_{load}}{r_L+R_{load}}\\ &Q=\frac{r_L+R_{load}}{L_1+C_2\left[r_Cr_L+R_{load}\left(r_C+r_L\right)\right]}\frac{1}{\omega_0}\end{aligned}$$

R. D. Middlebrook, *Methods of Design-Oriented Analysis: Low Entropy Expressions*, New Approaches to Undergraduate Education, July 1992

Check Results and Easily Run Corrections if needed

- ☐ Verify the expression in a Mathcad® sheet and compare responses

$r_L := 0.1\Omega$ $r_C := 1\Omega$ $C_2 := 1\mu F$ $L_1 := 20\mu H$ $R_{load} := 5\Omega$ $\|(x,y) := \frac{x\cdot y}{x+y}$

$$H_0 := \frac{R_{load}}{r_L + R_{load}} = 0.98$$

$$\tau_1 := \frac{L_1}{r_L + R_{load}} = 3.922\mu s \qquad \tau_2 := C_2\cdot\left(r_L \| R_{load} + r_C\right) = 1.098\times 10^3\cdot ns \quad b_1 := \tau_1 + \tau_2 = 5.02\cdot\mu s$$

$$\tau_{12} := C_2\cdot\left(r_C + R_{load}\right) = 6\cdot\mu s \qquad \tau_{21} := \frac{L_1}{r_L + R_{load} \| r_C} = 21.429\mu s \qquad b_2 := \tau_2\cdot\tau_{21} = 23.529\mu s^2$$

$$N_1(s) := 1 + s\cdot r_C\cdot C_2 \qquad D_1(s) := 1 + b_1\cdot s + b_2\cdot s^2 \qquad H_1(s) := H_0\cdot\frac{N_1(s)}{D_1(s)}$$

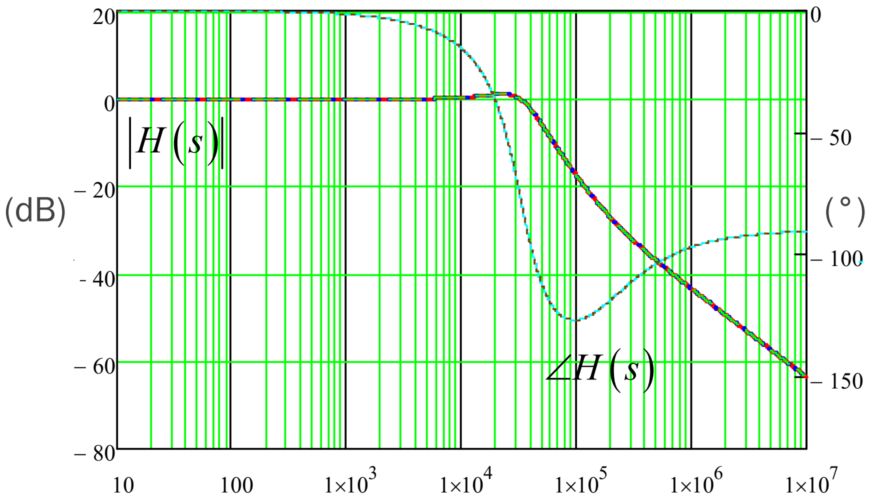
$$\omega_z := \frac{1}{r_C\cdot C_2} \qquad \omega_0 := \frac{1}{\sqrt{L_1\cdot C_2}}\cdot\sqrt{\frac{r_L + R_{load}}{r_C + R_{load}}} \qquad Q := \frac{L_1\cdot C_2\cdot\omega_0\cdot\left(r_C + R_{load}\right)}{L_1 + C_2\cdot\left(r_L\cdot r_C + r_L\cdot R_{load} + r_C\cdot R_{load}\right)}$$

$$H_2(s) := H_0\cdot\frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_0\cdot Q} + \left(\frac{s}{\omega_0}\right)^2}$$

$$Z_1(s) := s\cdot L_1 + r_L \qquad Z_2(s) := \left(\frac{1}{s\cdot C_2} + r_C\right) \| R_{load}$$

$$H_{ref}(s) := \frac{Z_2(s)}{Z_2(s) + Z_1(s)}$$

Brute-force expression



- ☐ If a deviation exists, fix the guilty sketch and don't restart from scratch

Third-Order Transfer Function: more Choice!

- Combine the three low-frequency times constants together

$$\tau_2^{13}$$

→ Reactance 1 and 3 are set in their high-frequency state

→ What resistance drives reactance 2?

$R?$

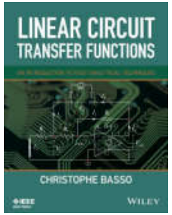
$$\tau_1^{23}$$

→ Reactance 2 and 3 are set in their high-frequency state

→ What resistance drives reactance 1?

$R?$

$$\Rightarrow \quad D(s)=1+s\left(\tau_1+\tau_2+\tau_3\right)+s^2\left(\tau_1\tau_2^1+\tau_1\tau_3^1+\tau_2\tau_3^2\right)+s^3\tau_1\tau_2^1\tau_3^{12}$$



C. Basso, *Linear Circuits Transfer Functions: An Introduction to Fast Analytical Techniques*, Wiley 2016

Course Agenda

- ☐ The Buck Converter
- ☐ Control Schemes
- ☐ Introduction to Modeling
- ☐ The PWM Switch in Current-Mode Control
- ☐ Introduction to the Fast Analytical Techniques
- ☐ **The Four Transfer Functions in CCM Current-Mode Control**
- ☐ Compensation Strategy
- ☐ Prototype Measurements

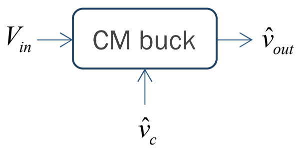
The Four Transfer Functions of the CM Buck Converter

- ☐ There are four open-loop transfer functions of interest:

$$\left.\frac{V_{out}(s)}{V_c(s)}\right|_{\hat{v}_{in}=0}$$

Control-to-output with ac-silent input source

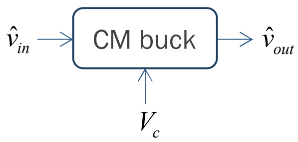
"How does a voltage perturbation on v_c propagate to v_{out}?"



$$\left.\frac{V_{out}(s)}{V_{in}(s)}\right|_{\hat{v}_c=0}$$

Input-to-output with static control V_c

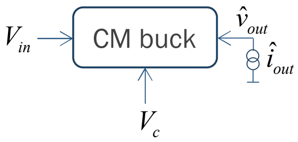
"How does a voltage perturbation on v_{in} propagate to v_{out}?"



$$\left.\frac{V_{out}(s)}{I_{out}(s)}\right|_{\hat{v}_{in}=\hat{v}_c=0}$$

Output impedance with static control V_c

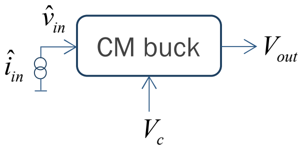
"How does a current perturbation on i_{out} propagate to v_{out}?"



$$\left.\frac{V_{in}(s)}{I_{in}(s)}\right|_{\hat{v}_c=0}$$

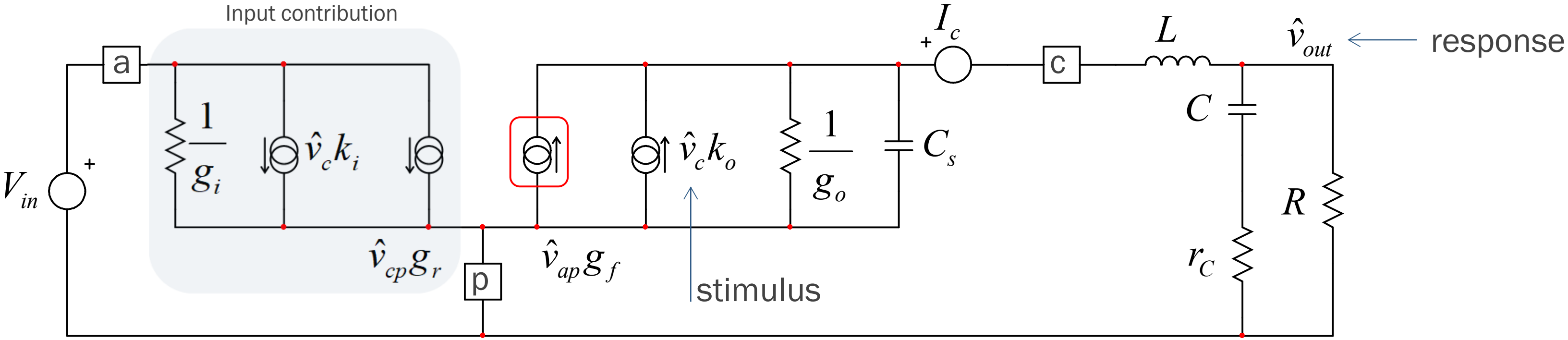
Input impedance with static control V_c

"How does a current perturbation on i_{in} propagate to v_{in}?"



Control-to-Output Transfer Function

- ☐ Install the small-signal CM-PWM switch in the buck converter



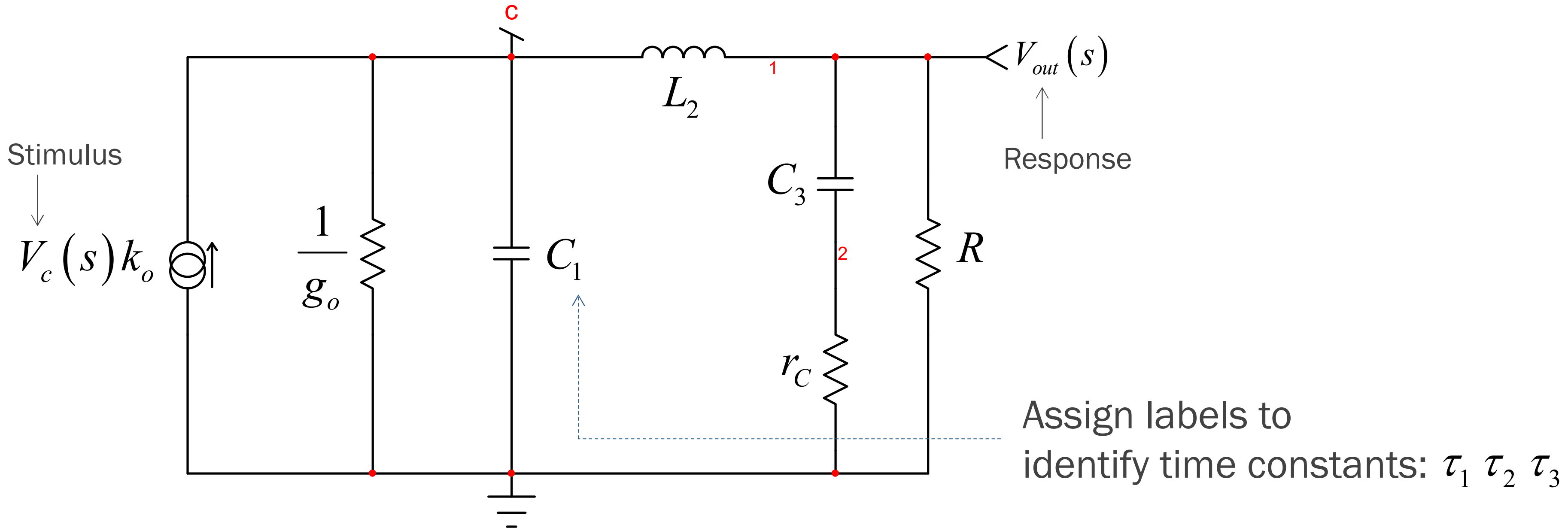
- ☐ We want the control-to-output function, the input source is ac-silent:

$$\hat{v}_{in} = 0 \longrightarrow \hat{v}_{ap} g_f = 0$$

- ☐ The input contribution can also disappear, no interest in Z_{in} for now

Simplify and Rearrange the Circuit

- ☐ The circuit now looks simpler to study



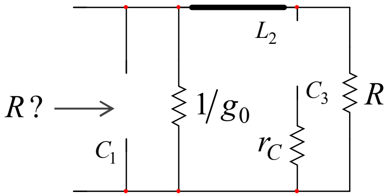
Assign labels to identify time constants: $\tau_1 \ \tau_2 \ \tau_3$

- ☐ There are 3 energy-storing elements: 3rd-order system

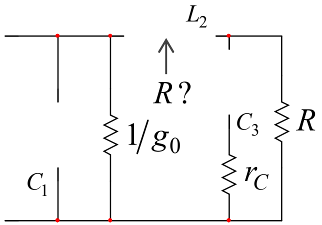
Control to output

Look at each Time Constant when Excitation is Off

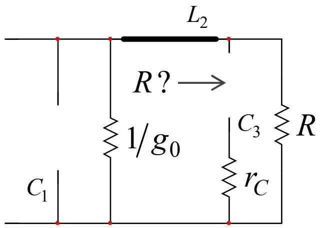
- ☐ The excitation is zero, elements are in their dc states
- ☐ 3 storage elements, 3 time constants, 3 drawings



$$\tau_1 = f\left(C_1\right) \longrightarrow \tau_1 = C_1\left(\frac{1}{g_o}\|R\right)$$



$$\tau_2 = f\left(L_2\right) \longrightarrow \tau_2 = \frac{L_2}{\dfrac{1}{g_0}+R}$$



$$\tau_3 = f\left(C_3\right) \longrightarrow \tau_3 = C_3\left(r_C + \left(\frac{1}{g_0}\|R\right)\right)$$

Sum time constants
Dimension is time [s]

$$b_1 = \tau_1 + \tau_2 + \tau_3$$

$$b_1 = C_1\left(\frac{1}{g_o}\|R\right) + \frac{L_2}{\dfrac{1}{g_0}+R} + C_3\left(r_C + \left(\frac{1}{g_0}\|R\right)\right)$$

Control to output

Determine Second-Order Coefficients

- ☐ One element is now set in its high-frequency state

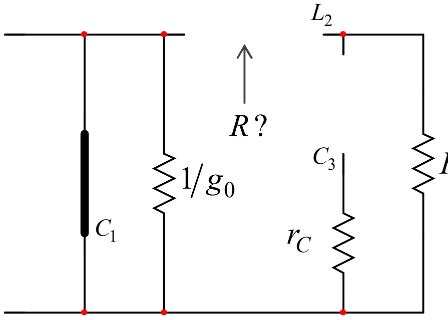
τ_2^1

C_1 (HF)

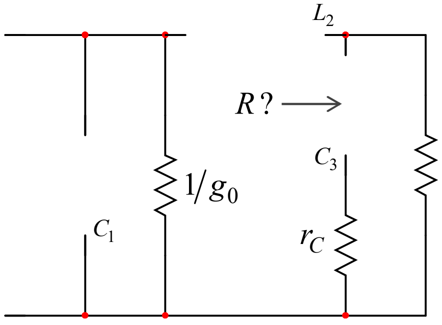
L_2 C_3 (dc)

↑

R?



$$\tau_2^1 = \frac{L_2}{R}$$



$$\tau_3^2 = (r_C + R)C_3$$

τ_3^2

L_2 (HF)

C_3 C_1 (dc)

↑

R?

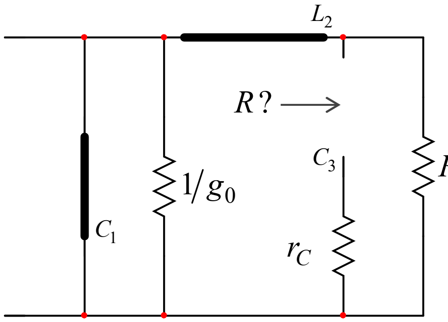
τ_3^1

C_1 (HF)

L_2 (dc) C_3

↑

R?



$$\tau_3^1 = r_C C_3$$

$$b_2 = \tau_1\tau_2^1 + \tau_1\tau_3^1 + \tau_2\tau_3^2 \quad \left[\mathrm{s}^2\right]$$

$$\Downarrow$$

$$b_2 = C_1\left(\frac{1}{g_o}\|R\right)\frac{L_2}{R} + C_1\left(\frac{1}{g_o}\|R\right)r_C C_3 + \frac{L_2}{\frac{1}{g_0}+R}(r_C + R)C_3$$

Control to output

Two Elements are now set in High-Frequency State

- For b_3, we multiply by a third time-constant

$b_3 = \tau_1 \tau_2^1 \tau_3^{12}$ ⟶ Dimension is time³

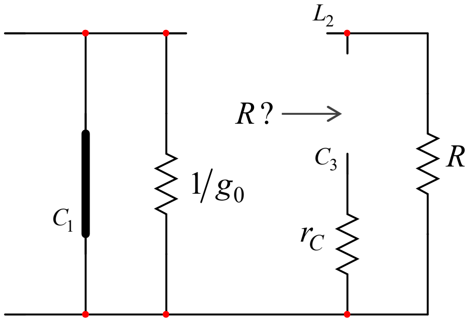
- What is this new time constant definition?

τ_3^{12} | C_1 (HF) L_2(HF)

C_3 (dc)

↑

$R\,?$



$$\tau_3^{12} = \left(r_C + R\right) C_3$$

- The final coefficient has been identified

$$b_3 = C_1 \left(\frac{1}{g_o} \| R \right) \frac{L_2}{R} \left(r_C + R\right) C_3$$

Assemble Time Constants to Build the Transfer Function

- A Mathcad® sheet can be built to verify these calculations

$$H(s) = G_0 \frac{1+\dfrac{s}{s_{z_1}}}{1+b_1 s+b_2 s^2+b_3 s^3} \qquad G_0 = k_0\left(R \,\middle\|\, \frac{1}{g_0}\right)$$

$$\omega_{z_1} = \frac{1}{r_C C}$$

$$b_1 = C_1\left(\frac{1}{g_o}\|R\right)+\frac{L_2}{\dfrac{1}{g_0}+R}+C_3\left(r_C+\left(\frac{1}{g_0}\|R\right)\right)$$

$$b_2 = C_1\left(\frac{1}{g_o}\|R\right)\frac{L_2}{R}+C_1\left(\frac{1}{g_o}\|R\right)r_C C_3+\frac{L_2}{\dfrac{1}{g_0}+R}\left(r_C+R\right)C_3$$

$$b_3 = C_1\left(\frac{1}{g_o}\|R\right)\frac{L_2}{R}\left(r_C+R\right)C_3$$

5 V/1 A buck

$$V_{in} = 10\,\mathrm{V}, F_{sw} = 100\,\mathrm{kHz}, R_i = 0.25\,\Omega, S_e = 2.5\,\mathrm{kV/s}$$

$$C = 100\,\mu\mathrm{F}, r_C = 0.1\,\Omega, L = 100\,\mu\mathrm{H}, C_s = 101\,\mathrm{nF}, V_c = 1.28\,\mathrm{V}$$

$$I_c = 4.94\,\mathrm{A}$$

$$k_i = 2\,\Omega^{-1} \qquad k_0 = 4\,\Omega^{-1}$$

$$g_0 = 0.01\,\Omega^{-1}$$

$$g_f = -7.5\,\mathrm{m}\Omega^{-1} \qquad g_r = 0.49\,\Omega^{-1}$$

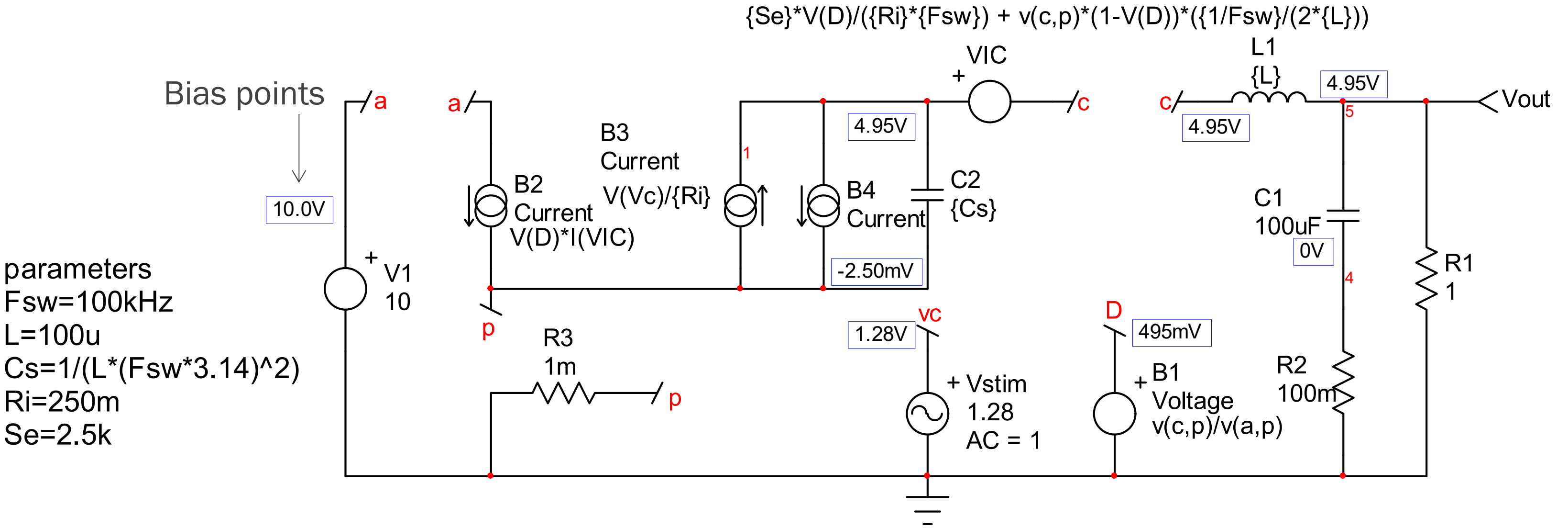
$$g_i = -250\,\mathrm{m}\Omega^{-1}$$

$$G_0 = 12\,\mathrm{dB} \qquad f_{z_1} = 15.9\,\mathrm{kHz}$$

Control to output

Verify the Analysis with a SPICE Simulation

- ❑ We can use the large-signal model to confront our mathematical results

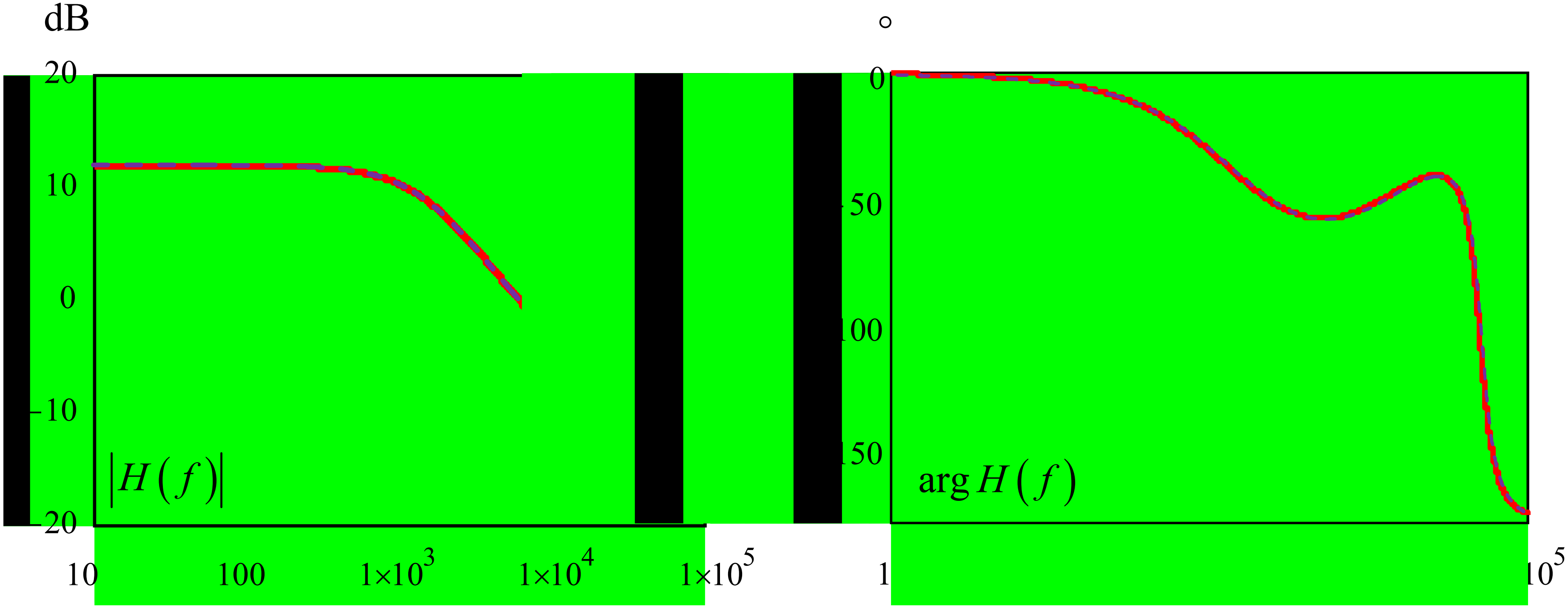


- ❖ Check operating points are correct: V_{out} = 4.95 V - ok

Control to output

Excellent Agreement between SPICE and Equations

- Superimposed curves means transfer functions are identical

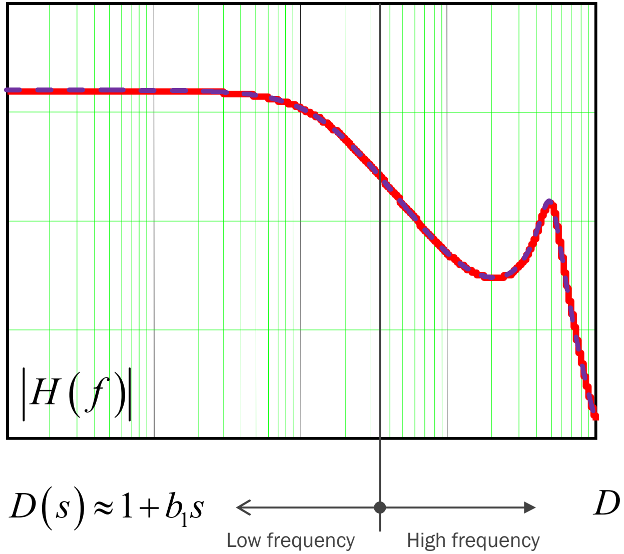


Rearrange the Expression in a *Low-Entropy* Form

- ☐ Proper arrangement is necessary to gain insight in the expression

$$D(s) = 1 + b_1 s + b_2 s^2 + b_3 s^3$$

- ☐ This is a third-order polynomial form that can be factored



$$D(s) \approx 1 + b_1 s$$

$$D(s) \approx 1 + \frac{b_2}{b_1} s + \frac{b_3}{b_1} s^2$$

$$H(s) \approx H_0 \frac{1 + \frac{s}{\omega_{z_1}}}{1 + \frac{s}{\omega_{p_1}}} \frac{1}{1 + \frac{s}{\omega_n Q} + \left(\frac{s}{\omega_n}\right)^2}$$

Control to output

Final Expression shows Poles and Zeroes

- ☐ The transfer function can now unveil peaking and damping

$$H_0 = \frac{R}{R_i}\frac{1}{1+\frac{RT_{sw}}{L_2}\left[m_c\left(1-D\right)-0.5\right]}$$

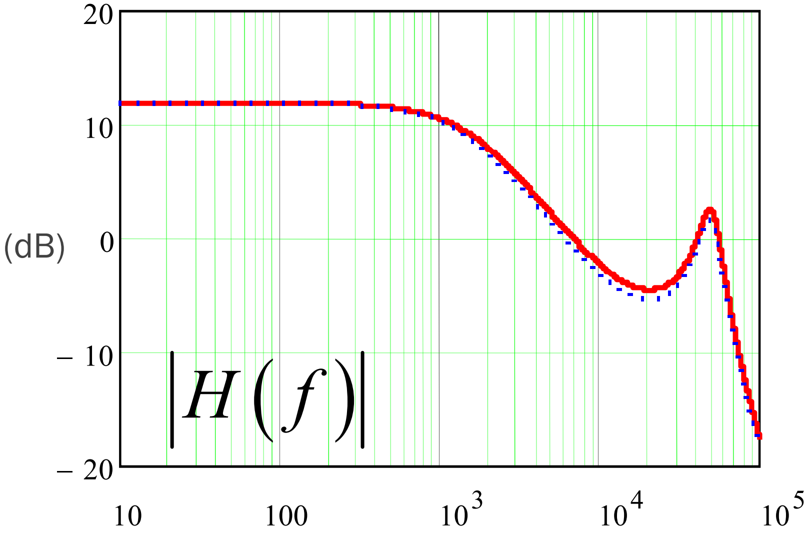
$$\omega_{p_1} = \frac{1}{RC_3}+\frac{T_{sw}}{L_2C_3}\left[m_c\left(1-D\right)-0.5\right] \qquad \omega_{z_1} = \frac{1}{r_C C_3}$$

$$\omega_n = \frac{\pi}{T_{sw}} \qquad Q = \frac{1}{\pi\left[m_c\left(1-D\right)-0.5\right]}$$

$$m_c = 1+\frac{S_e}{S_n}$$

Artificial ramp

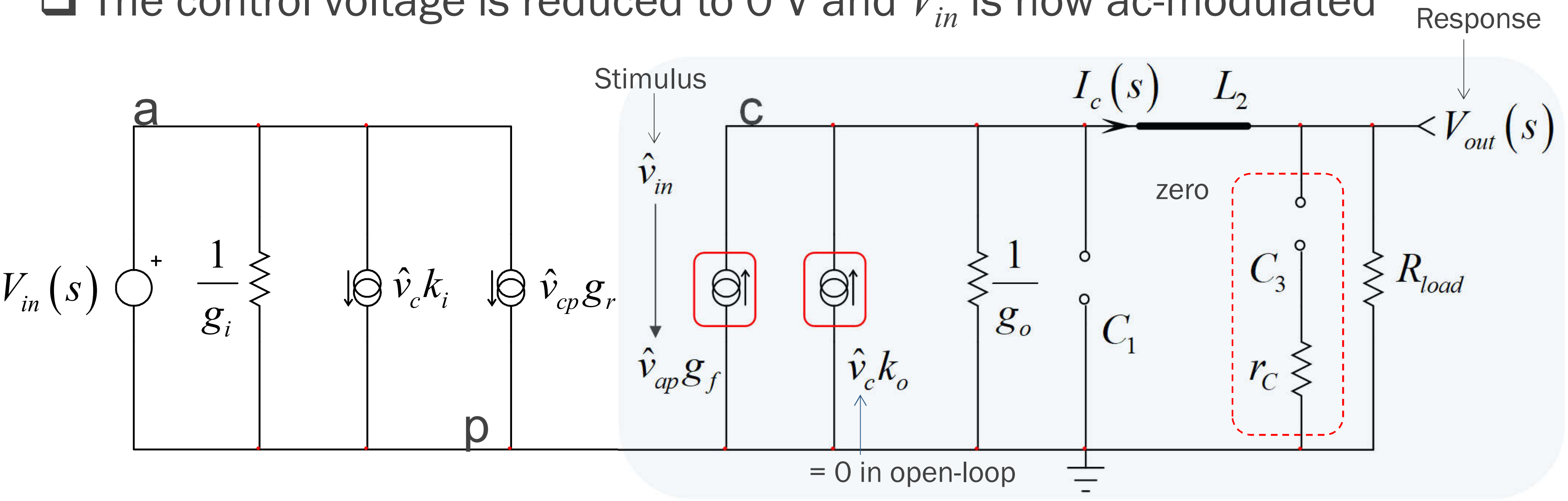
Inductor on-slope



R. B. Ridley, *A new Continuous-Time Model for CM Control*, IEEE Transactions of Power Electronics, Vol. 6, April 1991

Input-to-Output Transfer Function or Audio-Susceptibility

- ☐ The control voltage is reduced to 0 V and V_{in} is now ac-modulated

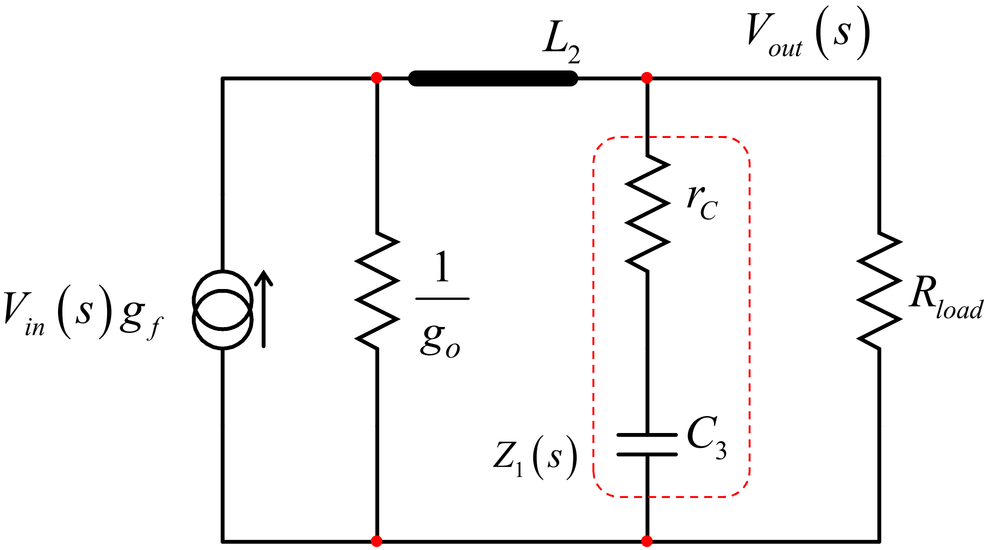


- ☐ We have no interest in the left-side part now as it affects Z_{in} only
- ❖ In open-loop conditions, $\hat{v}_c(s)$ is 0 V and it simplifies the circuit

Input to output

The Dc Gain is Immediate

- ☐ When the excitation is zeroed, the circuit returns to its natural state
- ✓ Reuse the denominator you have already determined



$$H(s) \approx H_0 \frac{1+\frac{s}{\omega_{z_1}}}{1+\frac{s}{\omega_{p_1}}} \frac{1}{1+\frac{s}{\omega_n Q}+\left(\frac{s}{\omega_n}\right)^2}$$

$$H_0 = g_f\left(\frac{1}{g_o} \| R_{load}\right) \qquad \omega_{z_1} = \frac{1}{r_C C_3}$$

- ✓ Current-mode control naturally rejects the input contribution as H_0 is low

Input to output

Rejecting the Input Voltage Contribution

- ☐ Calculate the amount of external ramp to reject V_{in} perturbations

$$H_0 \approx \frac{D\left[m_c\left(1-D\right)-\left(1-\dfrac{D}{2}\right)\right]}{\dfrac{L_2}{R_{load}T_{sw}}+\left[m_c\left(1-D\right)-0.5\right]}$$

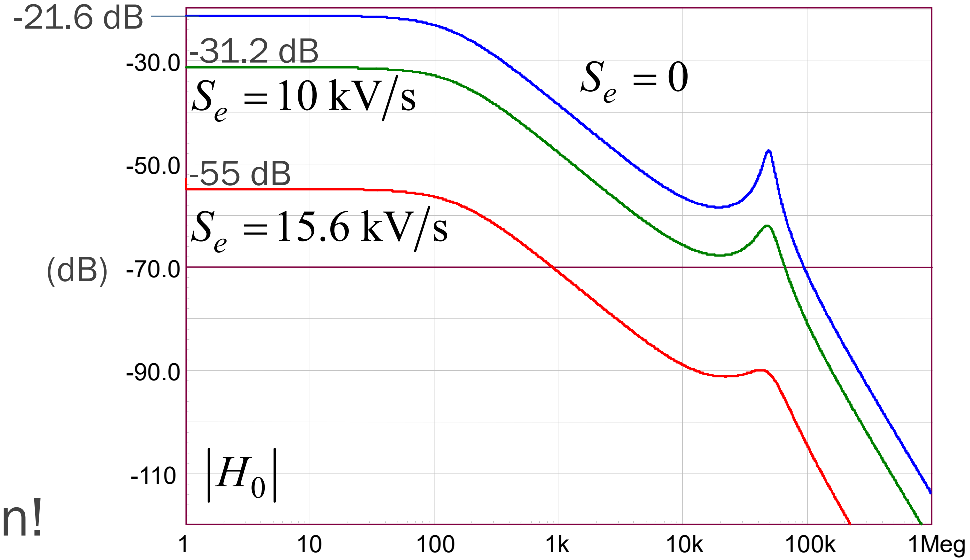
$$D\left[m_c\left(1-D\right)-\left(1-\frac{D}{2}\right)\right]=0$$

$$\Downarrow$$

$$m_c=\frac{D-2}{2D-2} \longrightarrow \begin{array}{l} S_e=50\%\cdot S_n \\ D=50\% \end{array}$$

$D := 56.6\%$ $V_{in} := 9V$ $V_{out} := 5V$ $L_2 := 100\mu H$ $R_L := 5\Omega$ $F_{sw} := 100kHz$ $R_i := 0.6\Omega$

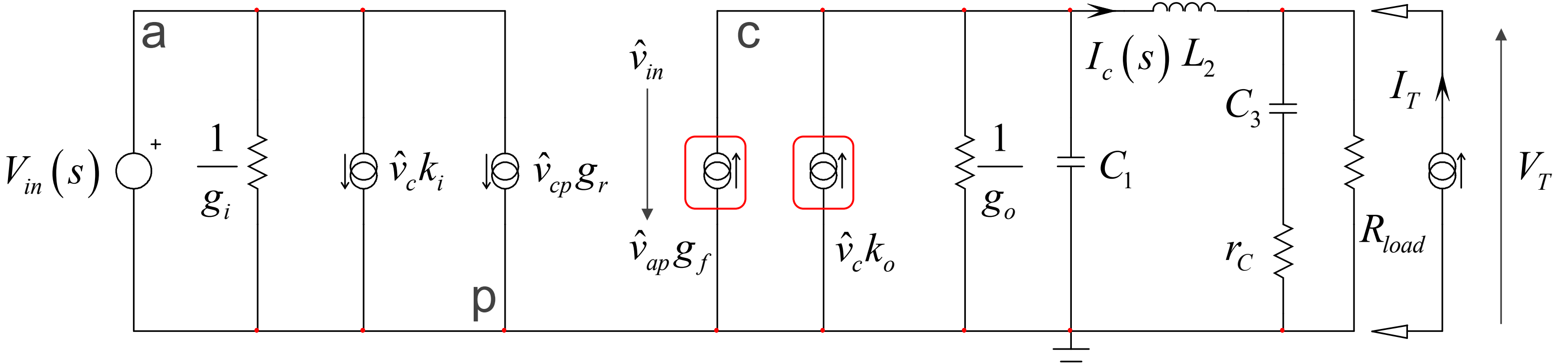
$T_{sw} := \frac{1}{100kHz}$ $S_n := \frac{V_{in}-V_{out}}{L_2}\cdot R_i = 24\frac{kV}{s}$ $S_e := 15.65\frac{kV}{s}$ $m_c := \frac{S_e}{S_n}+1 = 1.652$



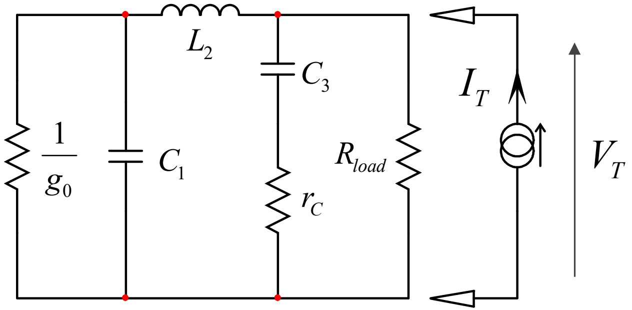
✓ Theoretical infinite input rejection!

Output Impedance Determination

- ☐ Install a current source across the load resistance



- ☐ The two controlled sources are turned off as V_{in} and V_c are 0 V



Draw a simpler sketch

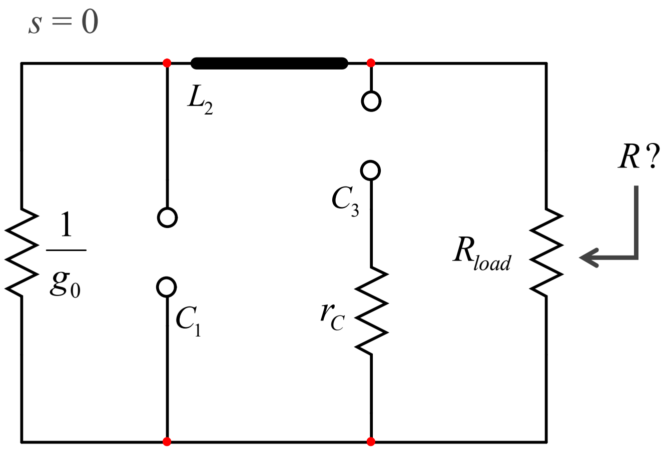
Determine I_T and V_T

$$Z_{out}(s) = \frac{V_T(s)}{I_T(s)}$$

Output impedance

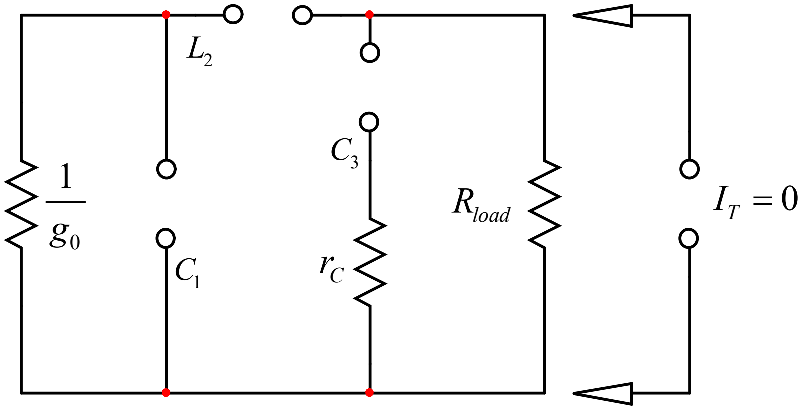
Determine Time Constants by Inspection

- ☐ Short the inductor and open the capacitor for R_0



$$R_0 = R_{load} \,||\, \frac{1}{g_o}$$

Turn excitation off



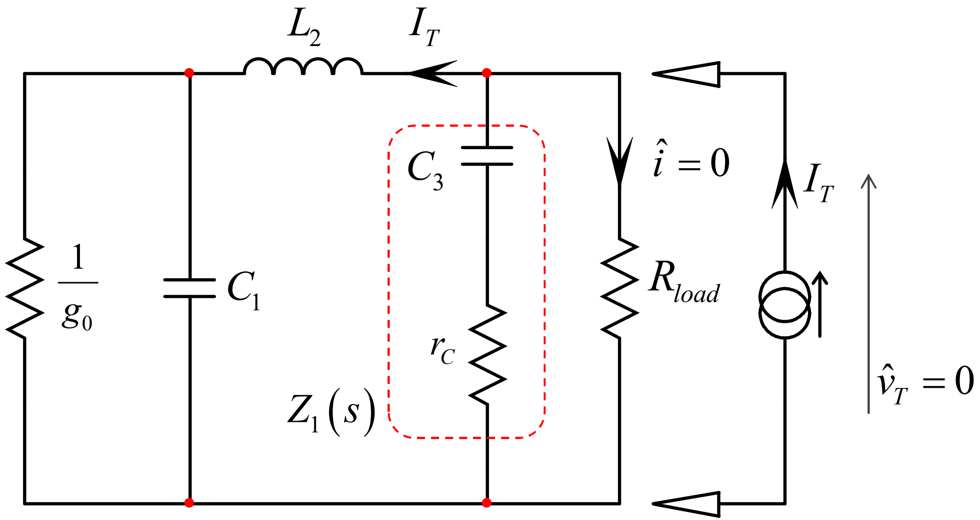
Back to the natural structure
Reuse previous denominator

$$D(s) \approx \left(1+\frac{s}{\omega_{p_1}}\right)\left[1+\frac{s}{\omega_n Q}+\left(\frac{s}{\omega_n}\right)^2\right]$$

Output impedance

Null the Response for the Zeroes

- The response is nulled when no current circulates in R_{load}



1st zero

$$Z_1(s) = r_C + \frac{1}{sC_3} = 0 \qquad \omega_z = \frac{1}{r_C C_3}$$

No current flows in R_{load}

All the I_T current circulates in L_2

The impedance involving L_2, C_1 and the conductance is a transformed short circuit

$$Z_2(s) = sL_2 + \frac{1}{g_o} \| \frac{1}{sC_1} = 0$$

$$Z_2(s) = \frac{1 + sg_oL_2 + s^2L_2C_1}{D(s)} = 0$$

$$N(s) = 1 + \frac{s}{\omega_N Q_N} + \left(\frac{s}{\omega_N}\right)^2$$

Simplify and test the Final Expression

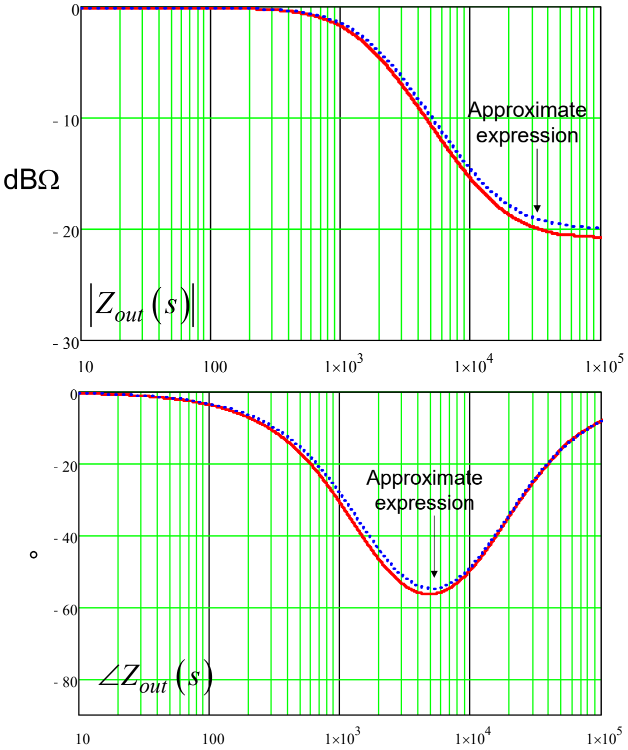
☐ The second-order polynomial form is affected by Q and ω_0

$$\omega_N = \frac{1}{\sqrt{L_2 C_1}} \qquad Q_N = \frac{1}{g_o}\sqrt{\frac{C_1}{L_2}}$$

High-frequency contribution

$$Z_{out}(s) = R_0 \frac{1+\frac{s}{\omega_z}}{1+\frac{s}{\omega_p}} \frac{1+\frac{s}{\omega_N Q_N}+\left(\frac{s}{\omega_N}\right)^2}{1+\frac{s}{\omega_0 Q}+\left(\frac{s}{\omega_0}\right)^2}$$

$$Z_{out}(s) \approx \frac{R_{load}}{1+\frac{R_{load}}{L_2} T_{sw}\left[m_c(1-D)-0.5\right]} \frac{1+\frac{s}{\omega_z}}{1+\frac{s}{\omega_p}}$$



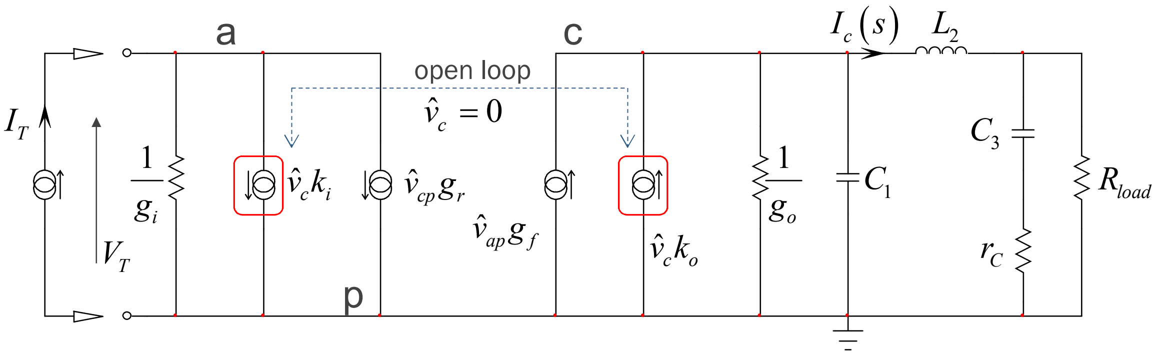
The impedance is dominated by R_{load} in low frequency

Need high open-loop gain to lower Z_{out}

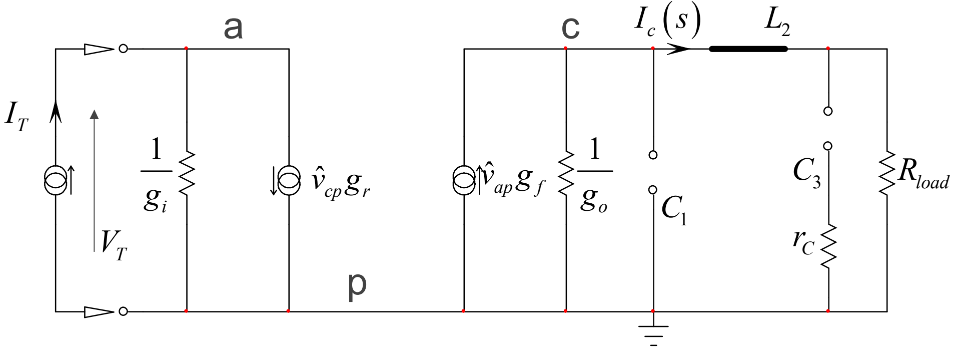
Output impedance

Input Impedance Determination

- ☐ The excitation source now biases the input port



- ☐ Determine the incremental resistance R_0 for $s = 0$



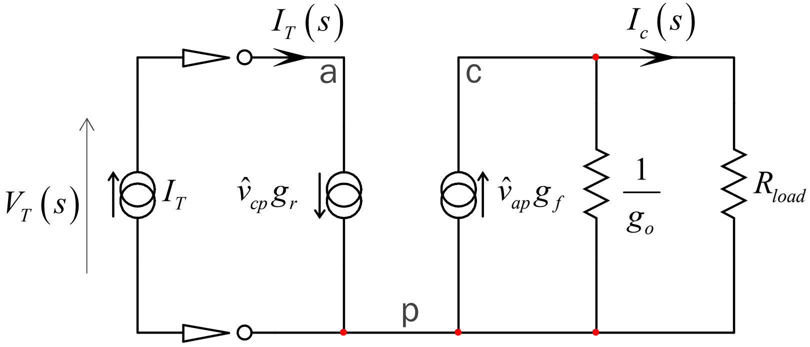
Temporarily disconnect $\frac{1}{g_i}$

$$R_0 = R_{\text{int}} \parallel \frac{1}{g_i}$$

Intermediate result without $\frac{1}{g_i}$

A Negative Open-Loop Incremental Resistance

- ☐ The input current is the test current I_T



$$I_T = \hat{v}_{cp} g_r$$

$$\hat{v}_{cp} = \hat{v}_{ap} g_f \left(\frac{1}{g_o} \| R_{load}\right)$$

$$I_T = \hat{v}_{ap} g_f g_r \left(\frac{1}{g_o} \| R_{load}\right)$$

V_T

$$R_0 = \frac{1}{g_i} \| \frac{1}{g_f g_r \left(\frac{1}{g_o} \| R_{load}\right)}$$

- ☐ Replace coefficients by their definition and rearrange

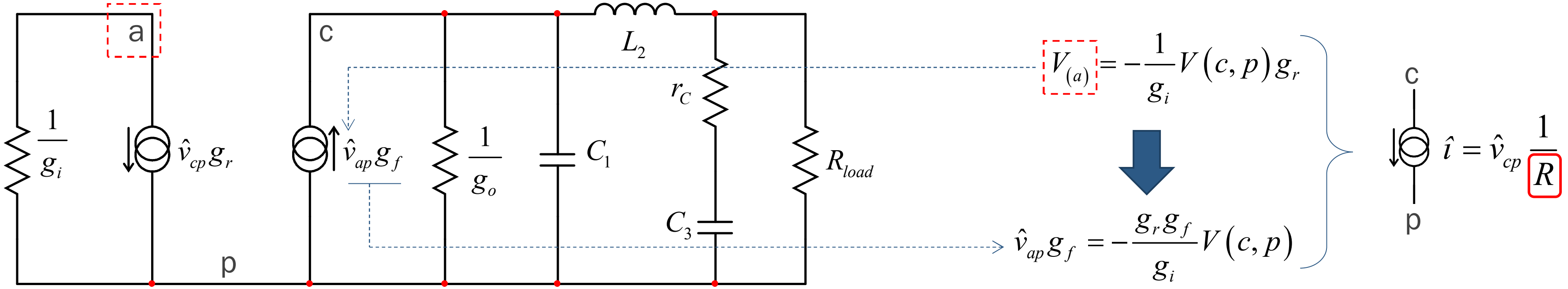
$$R_0 = -\frac{R_1\left[\left(\frac{L_2}{R_{load}T_{sw}} + 0.5 - D\right)S_n + S_e\left(1 - D\right)\right]}{\left[\left(S_e + \frac{S_n}{2}\right)D^3 - \left(S_e + \frac{M}{2}S_n\right)D^2 + \frac{M}{2}S_n D\right] + MDS_n\frac{L_2}{R_{load}T_{sw}}}$$

The incremental resistance of the CCM open-loop buck converter is negative

Input impedance

Turn the Excitation off and Determine Time Constants

- ☐ Get rid of the controlled sources for an easy inspection

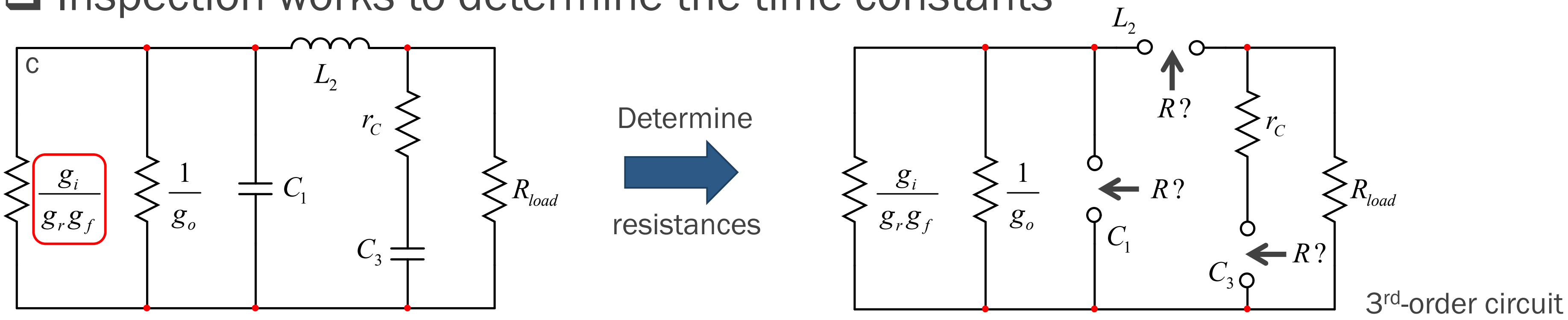


$$\boxed{V_{(a)}} = -\frac{1}{g_i} V(c,p) g_r$$

$$\hat{v}_{ap} g_f = -\frac{g_r g_f}{g_i} V(c,p)$$

$$\hat{\imath} = \hat{v}_{cp} \frac{1}{\boxed{R}}$$

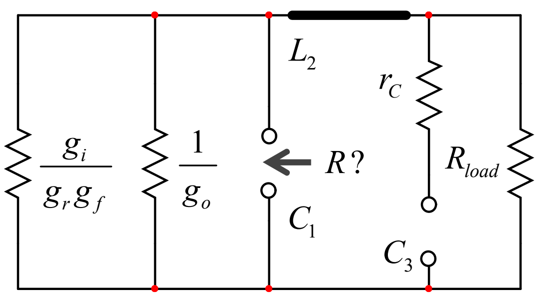
- ☐ Inspection works to determine the time constants



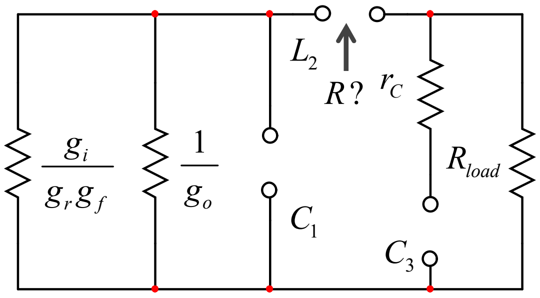
Determine resistances

3rd-order circuit

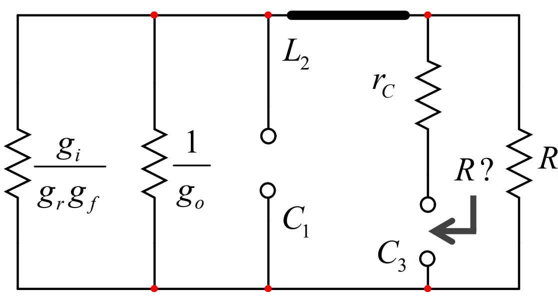
Break the Circuit into Small Individual Sketches



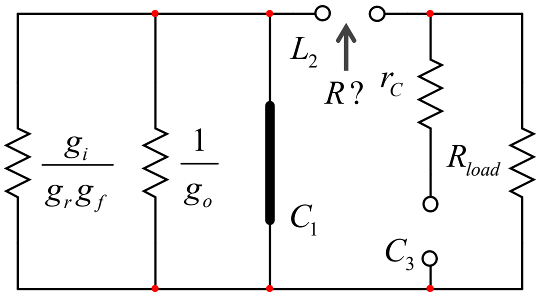
$$\tau_1 = C_1\left(\frac{1}{g_o} \parallel R_{load} \parallel \frac{g_i}{g_r g_f}\right)$$



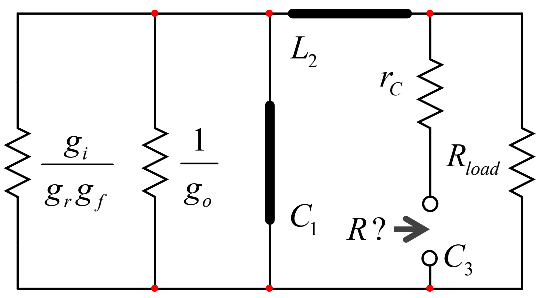
$$\tau_2 = \frac{L_2}{\frac{1}{g_o} \parallel \frac{g_i}{g_r g_f} + R_{load}}$$



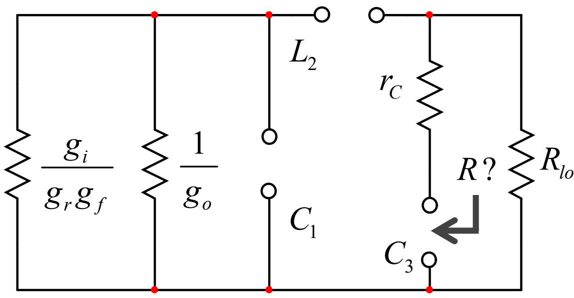
$$\tau_3 = C_3\left(r_C + \frac{1}{g_o} \parallel R_{load} \parallel \frac{g_i}{g_r g_f}\right)$$



$$\tau_2^1 = \frac{L_2}{R_{load}}$$



$$\tau_3^1 = C_3 r_C$$

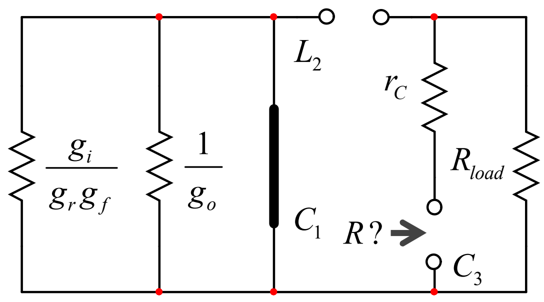


$$\tau_3^2 = C_3\left(r_C + R_{load}\right)$$

Input impedance

Combine the Time Constants

- ☐ Assemble the time constants to form the denominator



$$\tau_3^{12} = C_3\left(r_C + R_{load}\right)$$

$$D(s) = 1 + sb_1 + s^2 b_2 + s^3 b_3$$

$$b_1 = \tau_1 + \tau_2 + \tau_3$$

$$b_2 = \tau_1\tau_2^1 + \tau_1\tau_3^1 + \tau_2\tau_3^2$$

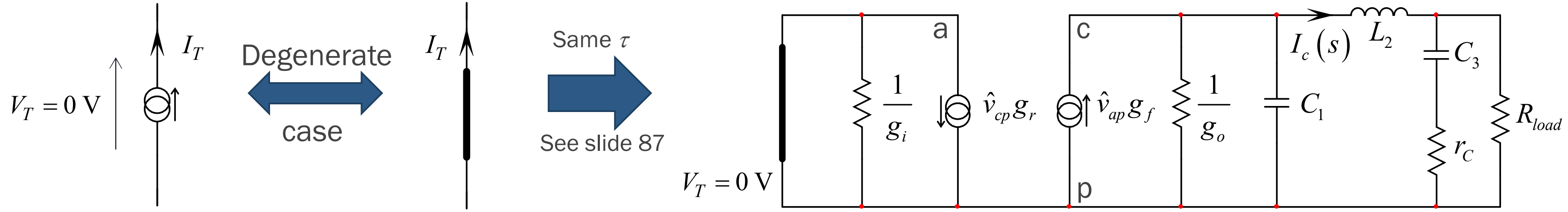
$$b_3 = \tau_1\tau_2^1\tau_3^{12}$$

$$b_1 = C_s\left(\frac{1}{g_o} \| R_{load} \| \frac{g_i}{g_r g_f}\right) + \frac{L_2}{\frac{1}{g_o} \| \frac{g_i}{g_r g_f}} + C_1\left(r_C + \frac{1}{g_o} \| R_{load} \| \frac{g_i}{g_r g_f}\right)$$

$$b_2 = C_s\left(\frac{1}{g_o} \| R_{load} \| \frac{g_i}{g_r g_f}\right)\frac{L_2}{R_{load}} + C_s\left(\frac{1}{g_o} \| R_{load} \| \frac{g_i}{g_r g_f}\right)C_3 r_C$$
$$+ \frac{L_2}{\frac{1}{g_o} \| \frac{g_i}{g_r g_f}} C_3\left(r_C + R_{load}\right)$$

$$b_3 = C_s\left(\frac{1}{g_o} \| R_{load} \| \frac{g_i}{g_r g_f}\right)\frac{L_2}{R_{load}} C_3\left(r_C + R_{load}\right)$$

- ☐ 0-V V_T brings the circuit back to its structure: reuse former $D(s)$ of slide 87

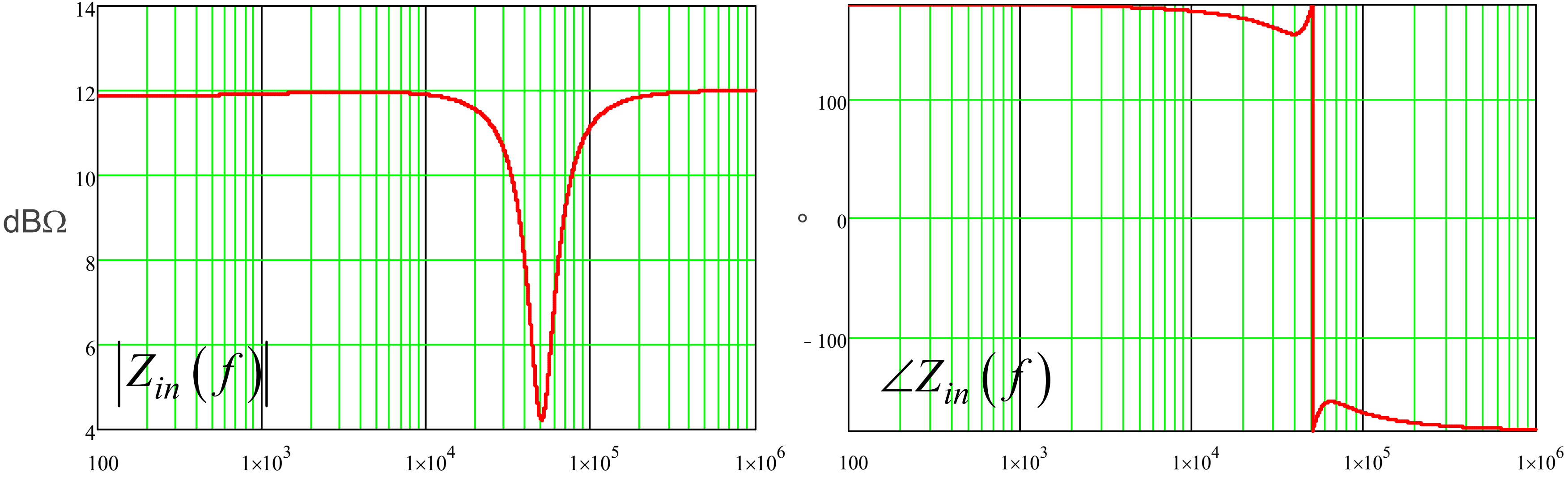


Plot the Open-Loop Input Impedance

- ❑ The input impedance is negative up to high frequencies

$$Z_{in}(s)=R_0\frac{N(s)}{D(s)} \qquad N(s)=1+a_1s+a_2s^2+a_3s^3 \qquad D(s)=1+b_1s+b_2s^2+b_3s^3$$

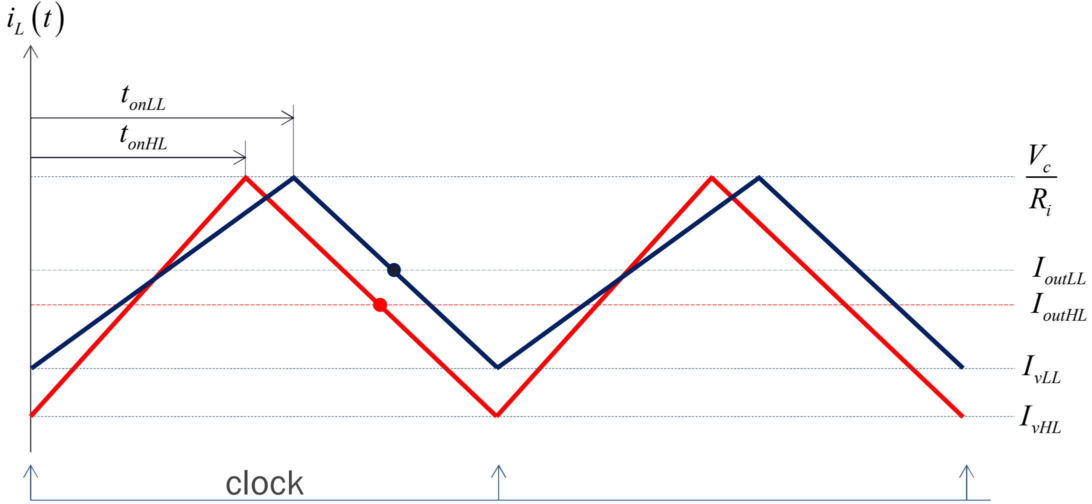
See slide 87

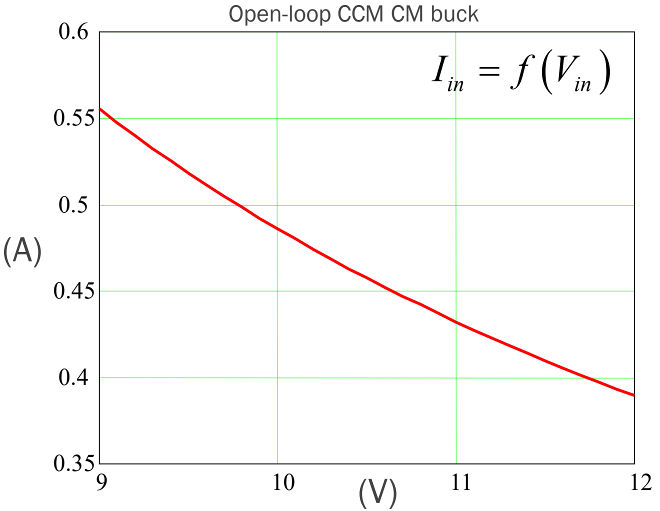


Input impedance

Why is the Incremental Resistance Negative?

- ☐ The smaller on-time at high line offers more demagnetization time
  - ➢ The valley current is lower: I_{out} decreases when V_{in} increases





A voltage increase brings an input current decrease:

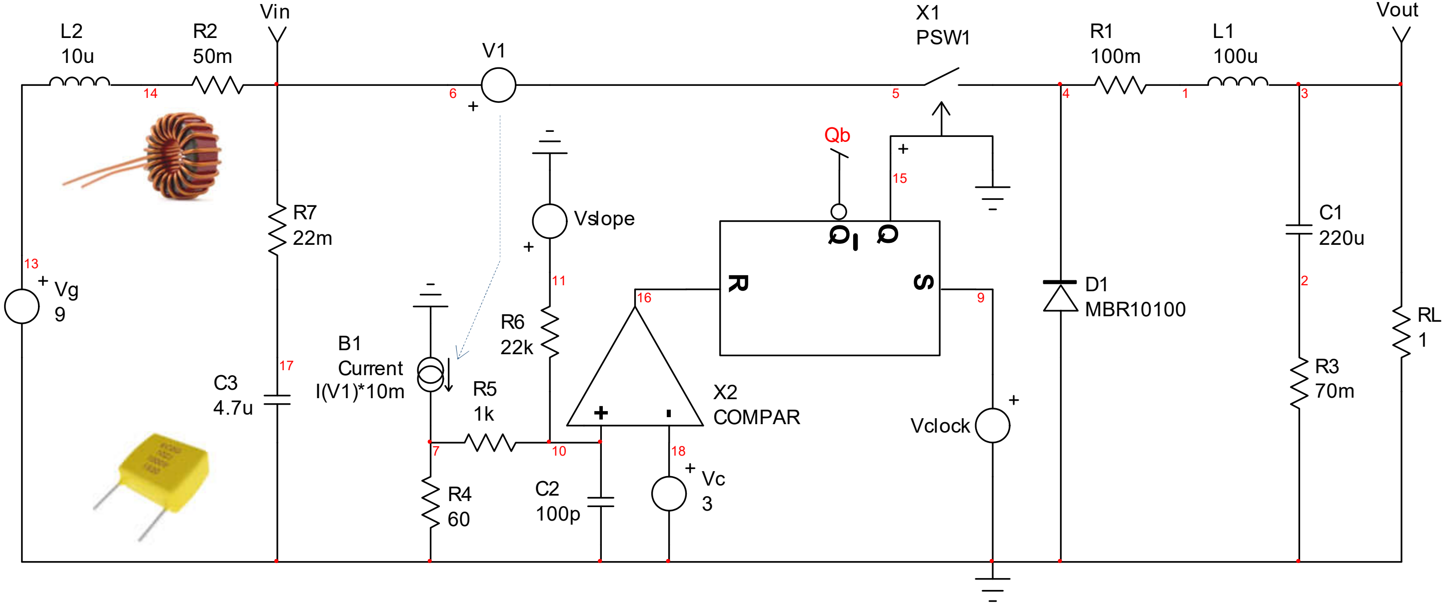
$$R_0 := \frac{V_1 + V_{step} - V_1}{I_{in}(V_1 + V_{step}) - I_{in}(V_1)} = -12.61\Omega$$

5 V/1 A

$V_1 = 9\text{ V}$ $V_{step} = 1\text{ mV}$

Unstable in Open-Loop Conditions with an EMI Filter

- ☐ For many converters, filter-related instabilities occur in closed loop
  - ➢ The CCM CM buck can be unstable in open-loop conditions

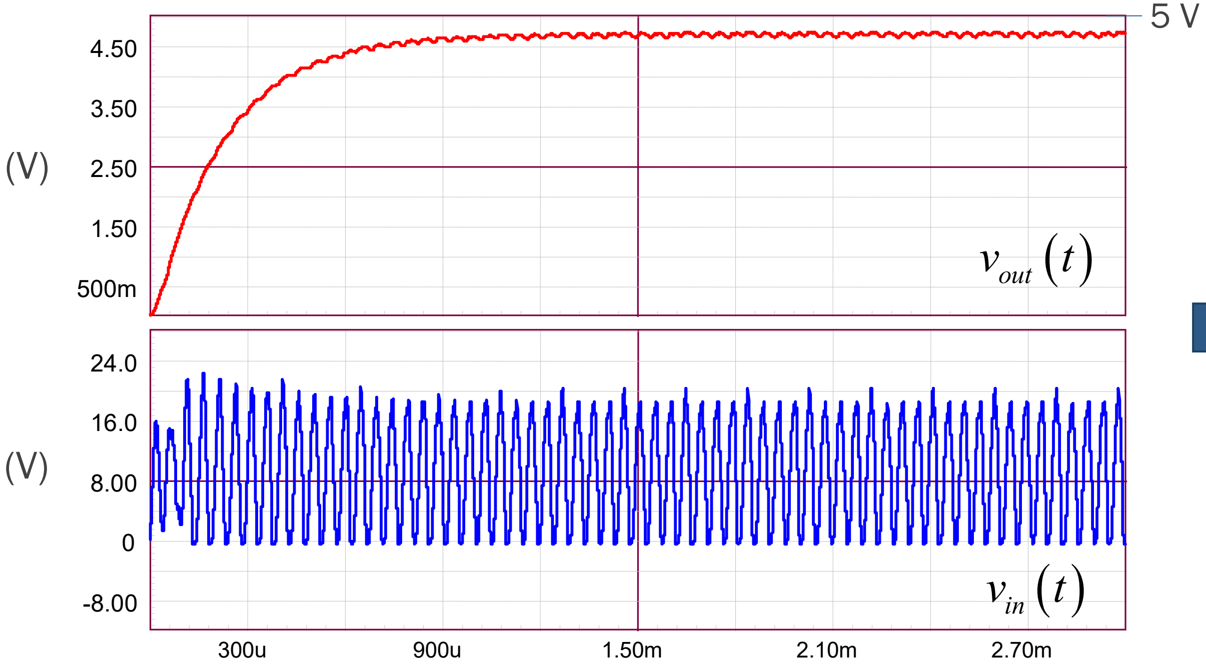


- ☐ It is a simple open-loop configuration, 5 V/5 A.

Input impedance

The Filter Output is Affected by a High Ripple

- ❑ The converter can barely deliver the voltage but its input is highly unstable

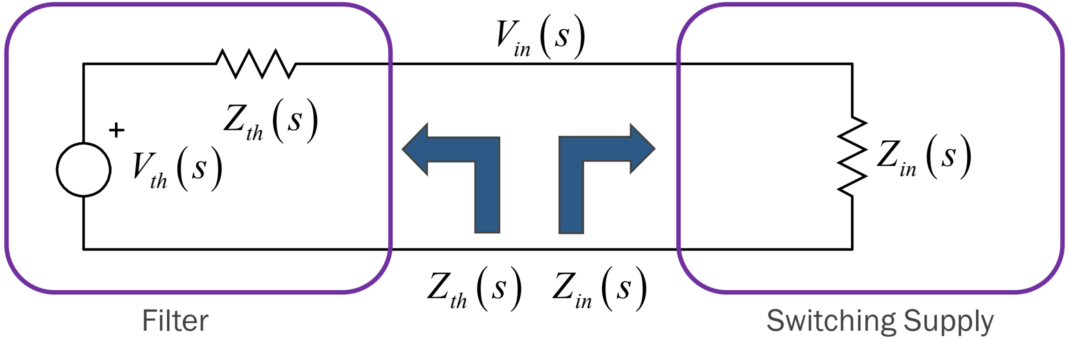


Filter damping is an absolute necessity!

Generate losses

The Filter Output Impedance brings the Problem

- Stability can be at stake when inserting the filter



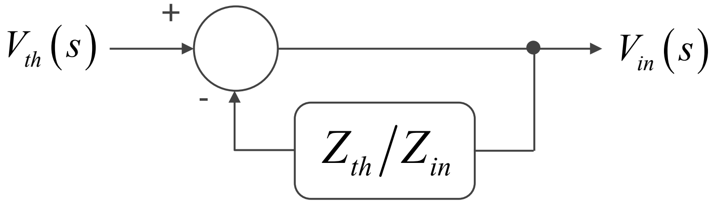
The Nyquist criterion applies

$$\frac{Z_{th}(s)}{Z_{in}(s)} = -1$$ ⚠

$$\left|\frac{Z_{th}(s)}{Z_{in}(s)}\right| = 1 \text{ and } \angle\frac{Z_{th}(s)}{Z_{in}(s)} = -180°$$

Stay away from overlaps

- The circuit can be modeled to reveal a minor loop

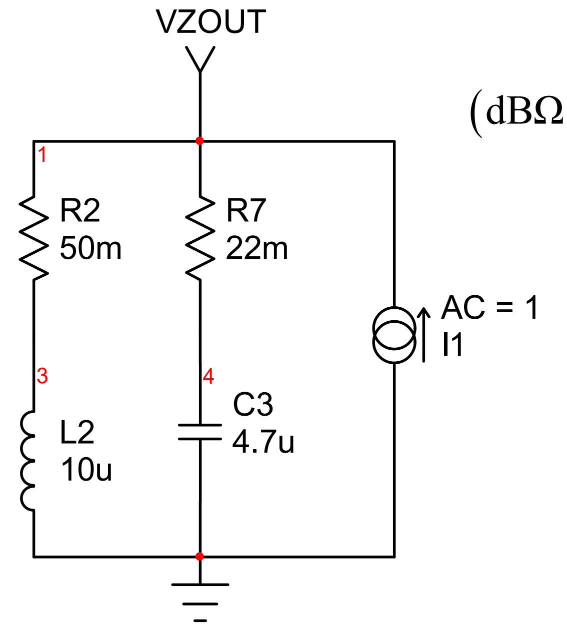


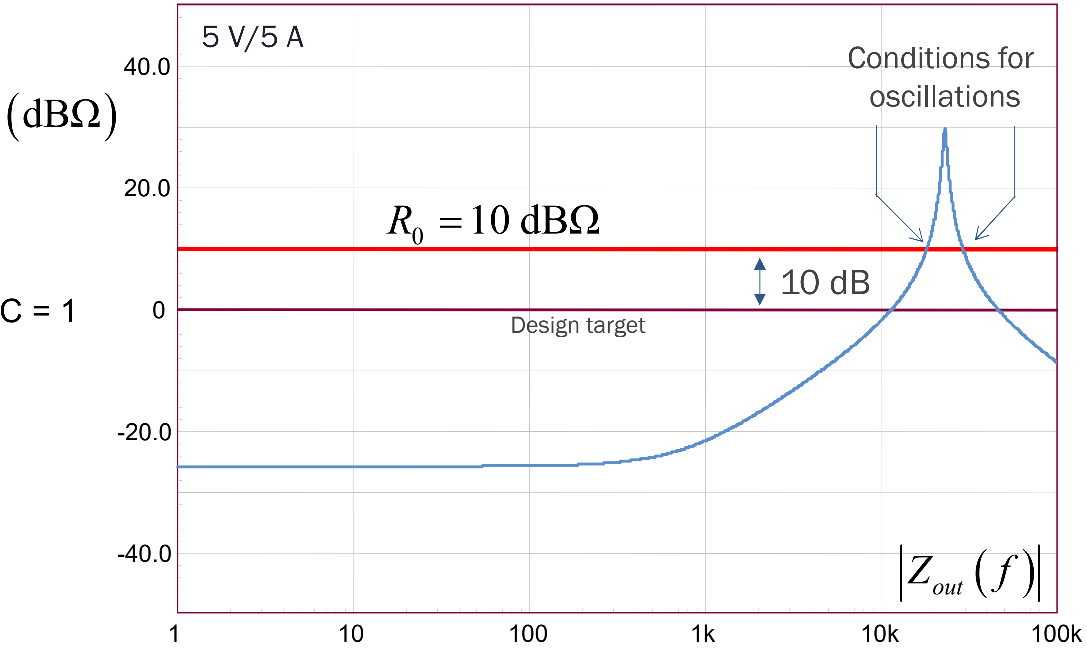
$$V_{in}(s) = V_{th}(s)\frac{1}{1+\dfrac{Z_{th}(s)}{Z_{in}(s)}}$$

R. D. Middlebrook, *Input Filter Considerations in Design and Application of Switching Regulators*, IEEE Proceedings, 1976

Check EMI Filter Output Impedance Peaking

- ☐ Does overlap exist with the negative incremental resistance?
  - ➢ Yes – damp the filter with external components





Input impedance

Optimally Damp the Filter Output Impedance

- ❑ The target is to reduce the filter impedance peak to 0 dBΩ or 1 Ω

$$R_0 = \sqrt{\frac{L_f}{C_f}}$$

$$\frac{\left|Z_{out}\right|_{mm}}{R_0} = \sqrt{\frac{2(2+n)}{n^2}}$$

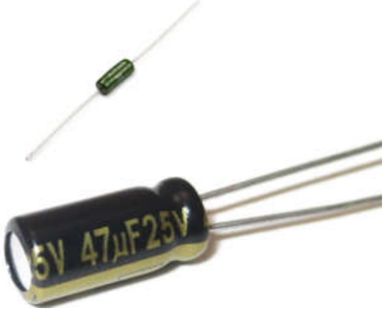
$$n = \frac{R_0\left(R_0 + \sqrt{R_0^{\,2} + 4\left(\left|Z_{out}\right|_{mm}\right)^2}\right)}{\left(\left|Z_{out}\right|_{mm}\right)^2} = 5.738$$

1-Ω
target

$$Q_{opt} = \sqrt{\frac{(4+3n)(2+n)}{2n^2(4+n)}} = 0.506$$

$$R_{damp} = R_0 Q_{opt} = 0.74\ \Omega$$

$$C_{damp} = nC_f = 27\ \mu\text{F}$$

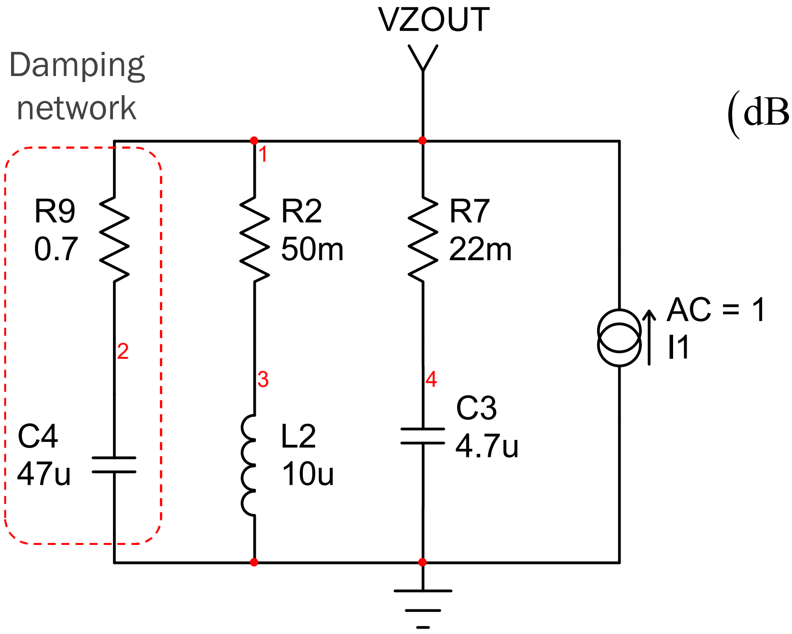


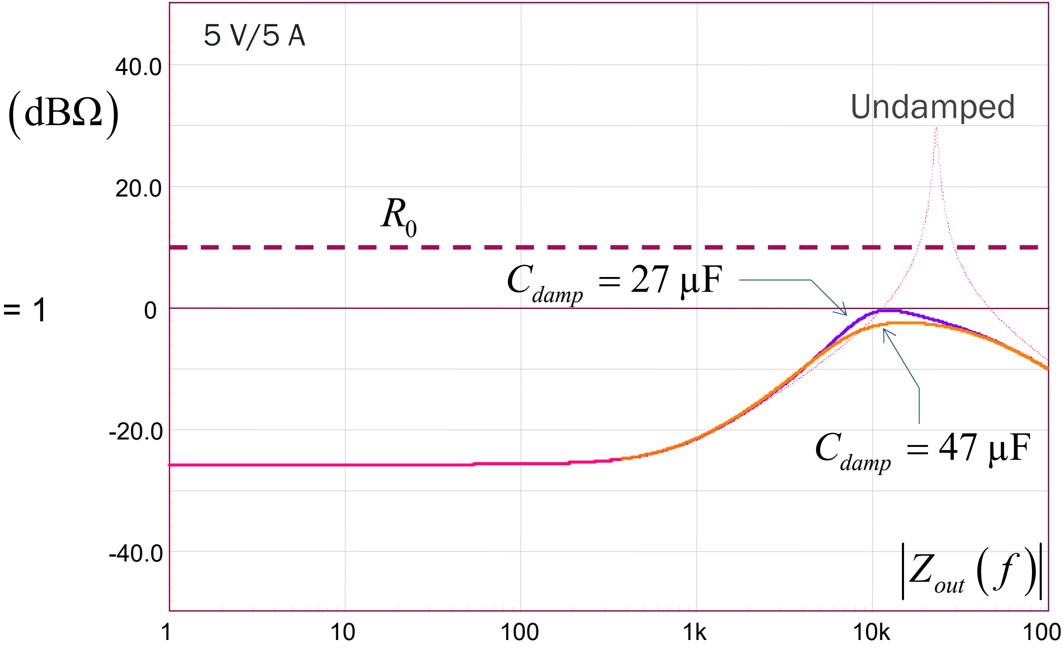
- ❑ This is a rather large capacitance value for a ceramic device
- ➢ A 47-µF electrolytic capacitor and its ESR can do the job
- ❖ Watch for temperature effects as ESR increases at low temp!

C. Basso, *EMI Input Filter Interaction with Switching Converters*, APEC Professional Seminars, 2017

The Peak is Lowered Owing to Extra Losses

- ☐ The added RC network exactly meets the design target

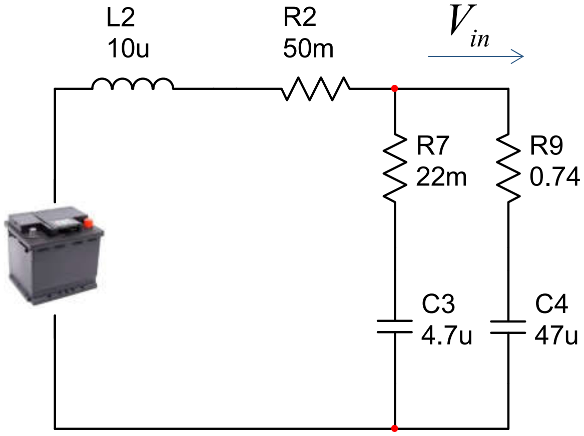


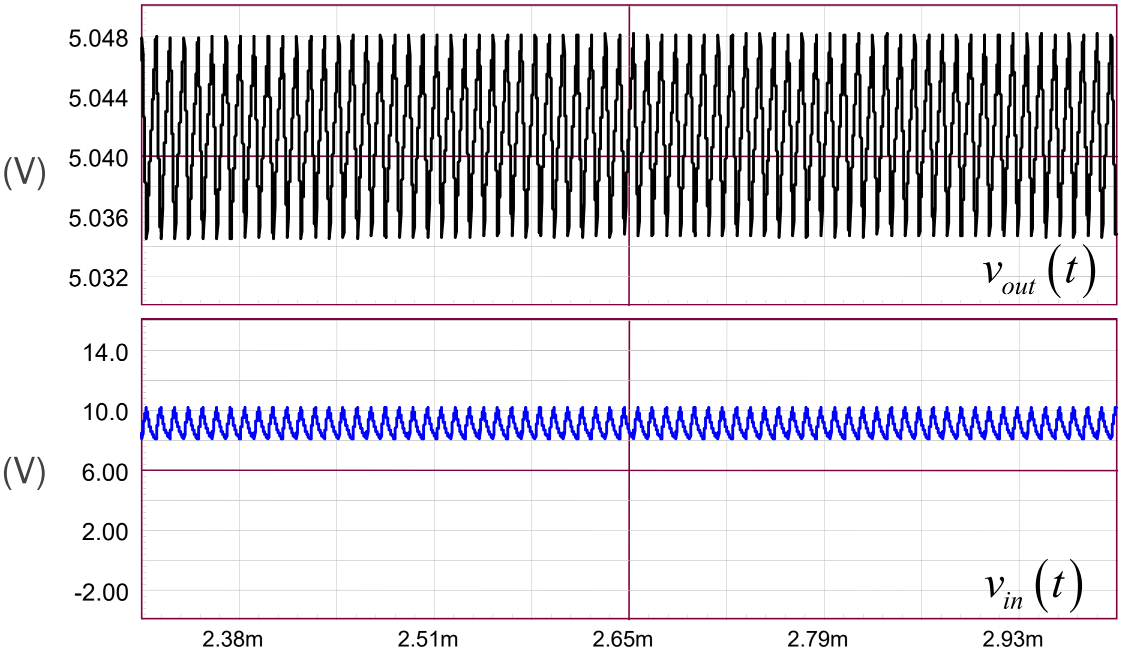


Input impedance

Filter Damping has calmed down Oscillations

- ❑ Once the damper is added, high-frequency ripple remains



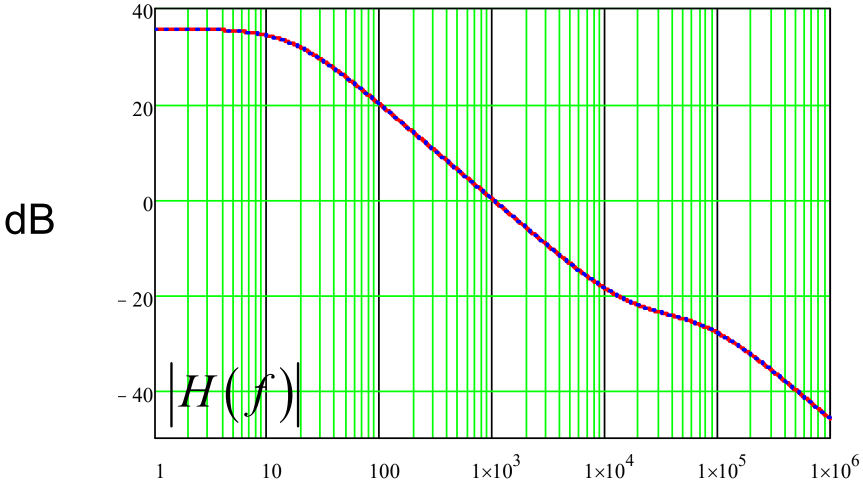


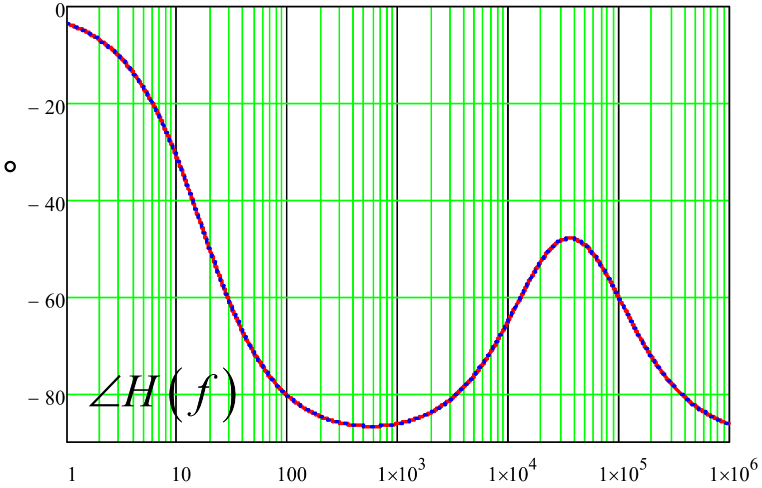
- ❖ Check margins again once the converter is compensated

Reduce the Load Current to enter DCM

- The control-to-output transfer function is still of second order!

$$H(s) \approx H_0 \frac{1+\frac{s}{\omega_z}}{\left(1+\frac{s}{\omega_{p_1}}\right)\left(1+\frac{s}{\omega_{p_2}}\right)} \qquad \omega_{p_1} = \frac{1}{R_{load}C_1}\frac{2m_c-(2+m_c)M}{m_c(1-M)} \qquad \omega_{p_2} = 2F_{sw}\left(\frac{M}{D}\right)^2$$





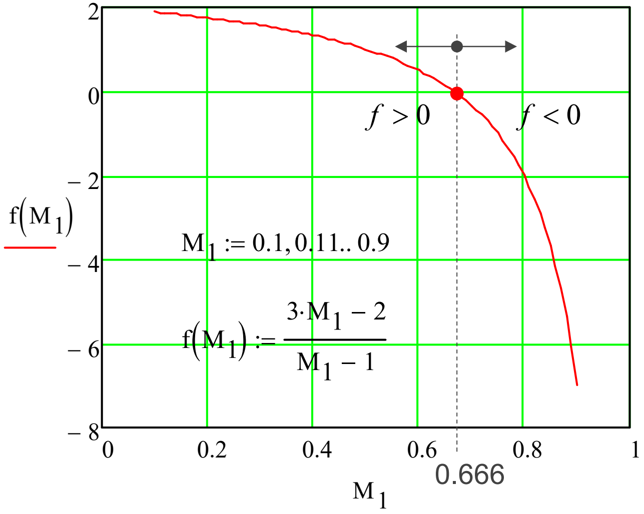
R. B. Ridley, *A new Continuous-Time Model for CM Control*, IEEE Transactions of Power Electronics, Vol. 6, April 1991

Instability in Absence of Compensation Ramp

- ☐ When the ramp reduces to zero, the DCM CM buck can be unstable

$$\omega_{p_1} = \frac{1}{R_{load}C_1}\frac{2m_c-(2+m_c)M}{m_c(1-M)} \xrightarrow{\text{Reduce } m_c \text{ to } 1} \omega_{p_1} = \frac{1}{R_{load}C_1}\frac{2-3M}{1-M}$$

- ☐ Plot the pole position versus M



- As M increases (V_{in} is lowered), the pole approaches the origin
- For M = 0.666, the pole is at the origin
- For M greater than 0.666, the pole jumps in the right half-plane

→ Despite DCM, a compensation ramp is needed

$$M = \frac{V_{out}}{V_{in}}$$

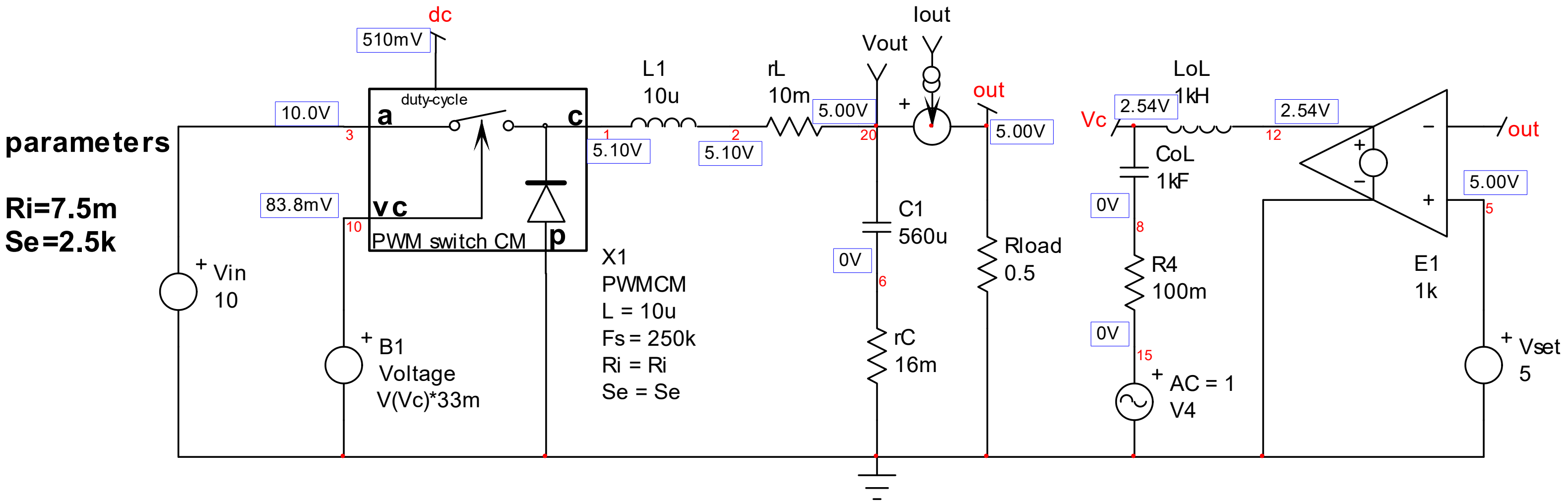
Course Agenda

- ☐ The Buck Converter
- ☐ Control Schemes
- ☐ Introduction to Modeling
- ☐ The PWM Switch in Current-Mode Control
- ☐ Introduction to the Fast Analytical Techniques
- ☐ The Four Transfer Functions in CCM Current-Mode Control
- ☐ **Compensation Strategy**
- ☐ Prototype Measurements

Transient Response and Compensation Strategy

❑ A nonlinear average model is the ideal tool

❖ Auto-bias the output to its operating point, I_{out} = 10 A

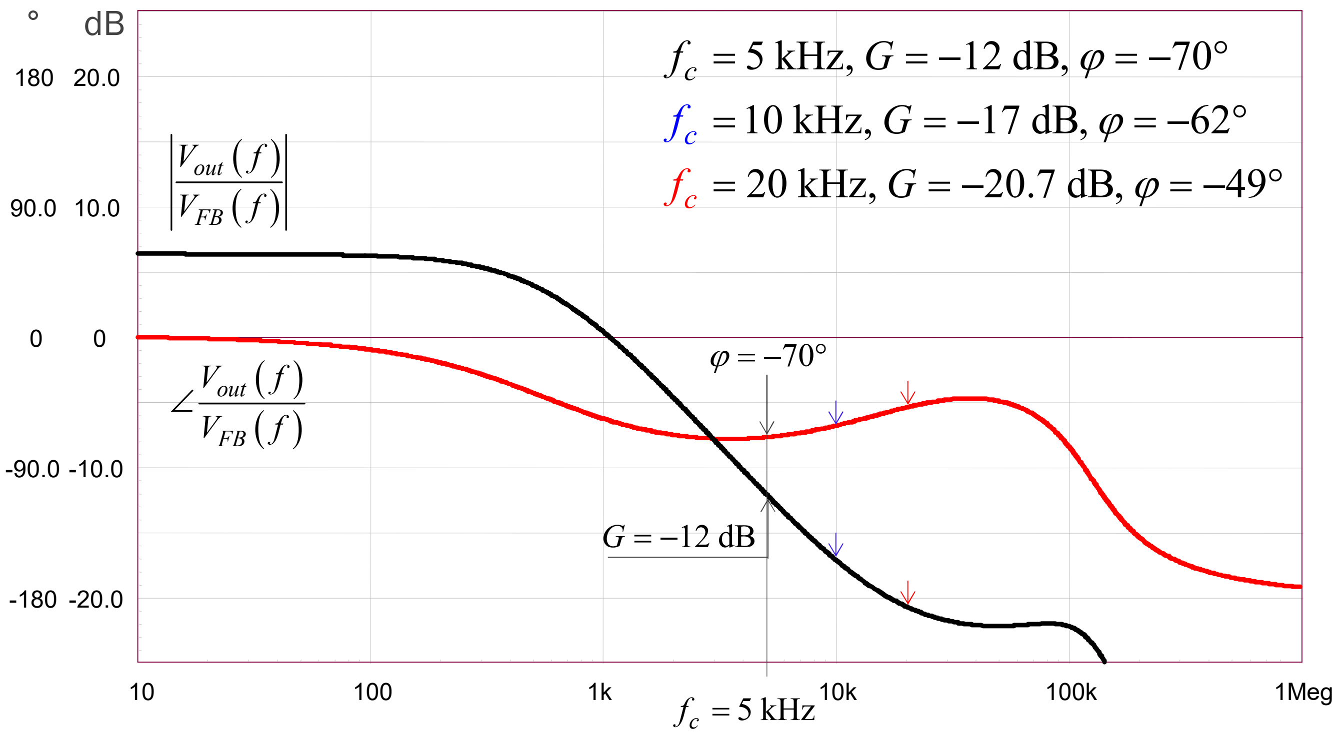


❑ Check all operating points are correct (V_{out} = 5 V)

❑ Extract magnitude and phase information from the Bode plot

Stabilizing the Current-Mode Buck Converter

☐ What you need is the control-to-output transfer function of the power stage



Buck CM

Automate the Compensation Process with k-Factor

- ☐ Use a SPICE simulation to try different compensation scenarios

parameters

Rupper=10k
Rlower=Rupper
fc=20k
pm=60

Gfc=-20.7
pfc=-49

parameters

Ri=7.5m
Se=2.5k

Power stage data

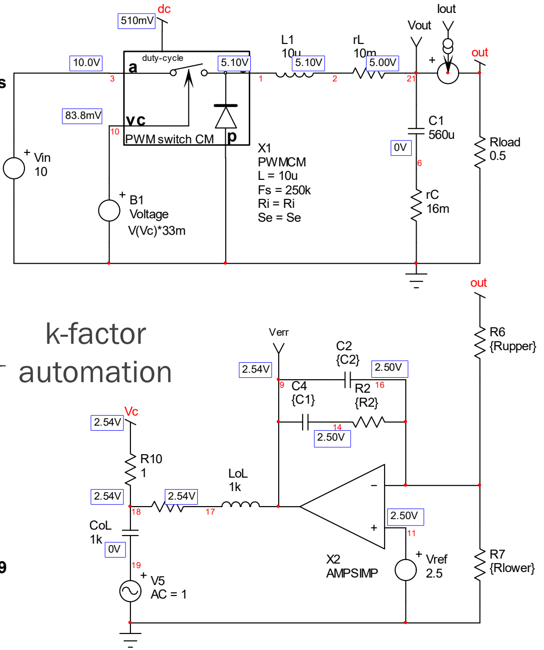
G=10^(-Gfc/20)
boost=pm-(pfc)-90
pi=3.14159
K=tan((boost/2+45)*pi/180)
C2=1/(2*pi*fc*G*k*Rupper)
C1=C2*(K^2-1)
R2=k/(2*pi*fc*C1)

fp1=1/(2*pi*R2*C2)
fz1=1/(2*pi*R2*C1)

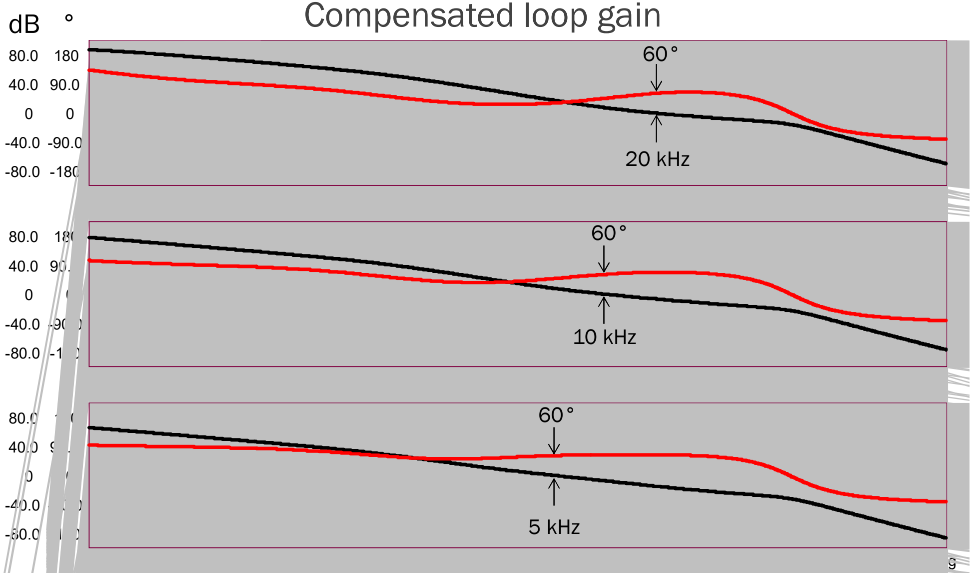
k-factor automation

PWR stage response

fc=5 kHz, Gfc =-12 , pfc =-70
fc=10 kHz, Gfc =-17 , pfc =-62
fc=20 kHz, Gfc =-20.7 , pfc =-49



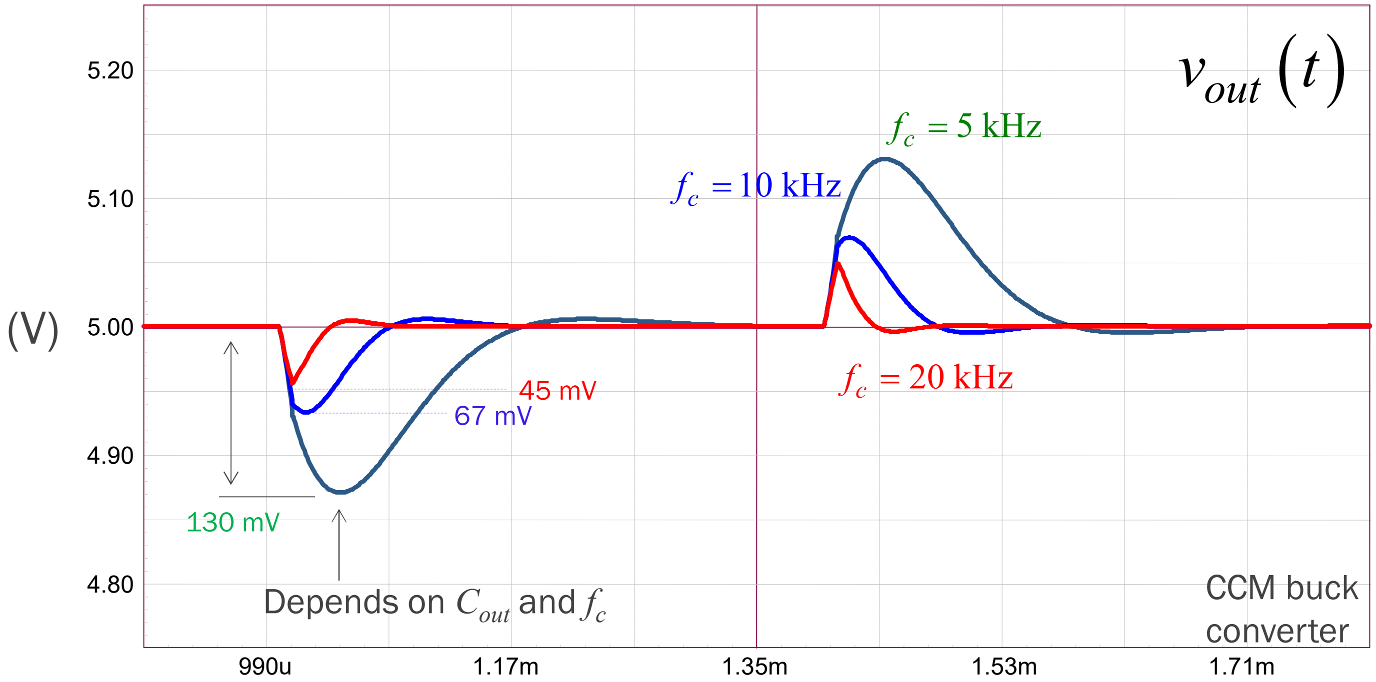
Compensated loop gain

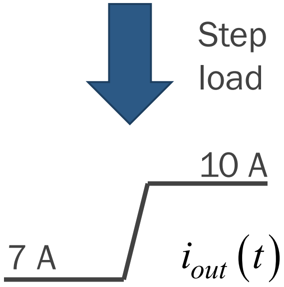


- ➢ Keep phase margin constant and adjust crossover frequency

Step-Load the Output of the Converter in Closed Loop

- ☐ With a constant 60° phase margin, recovery slope is constant

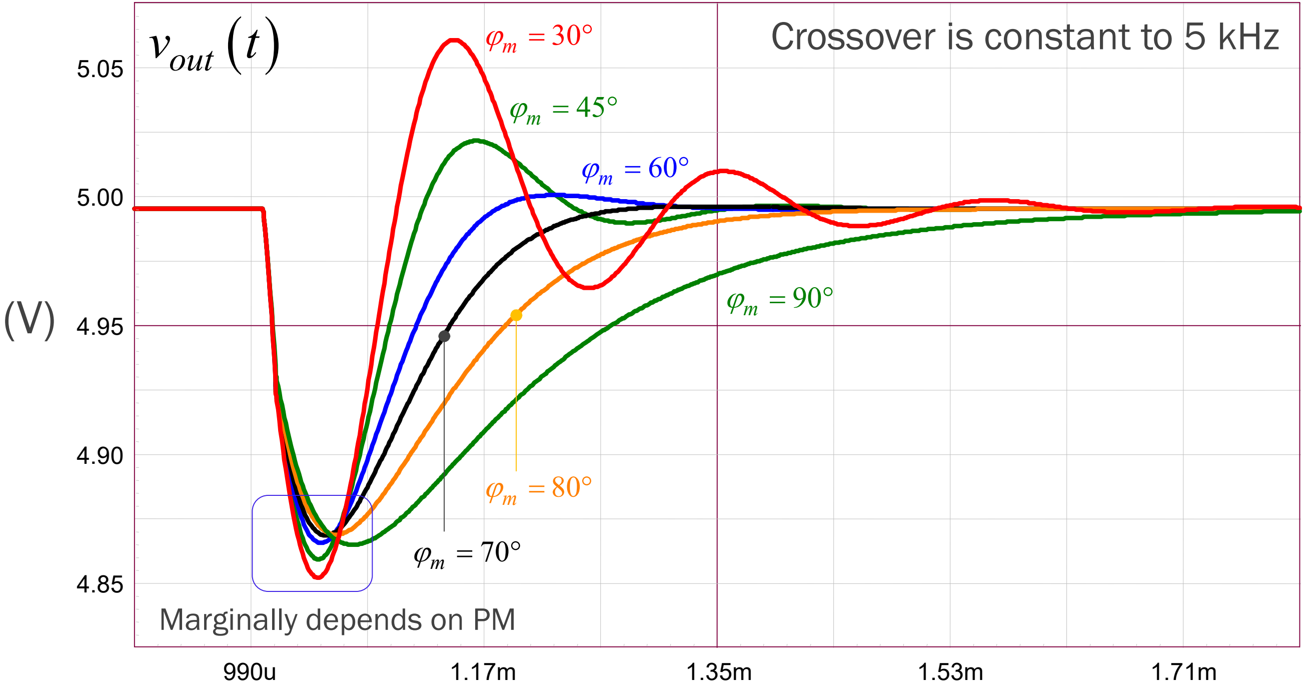




ΔI_{out} = 3 A in 3 µs

Sweep Phase Margin at Constant Crossover

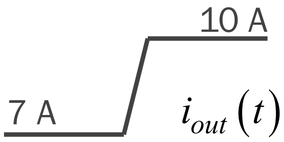
- ☐ The phase margin affects the overshoot and the recovery time



Crossover is constant to 5 kHz

Marginally depends on PM

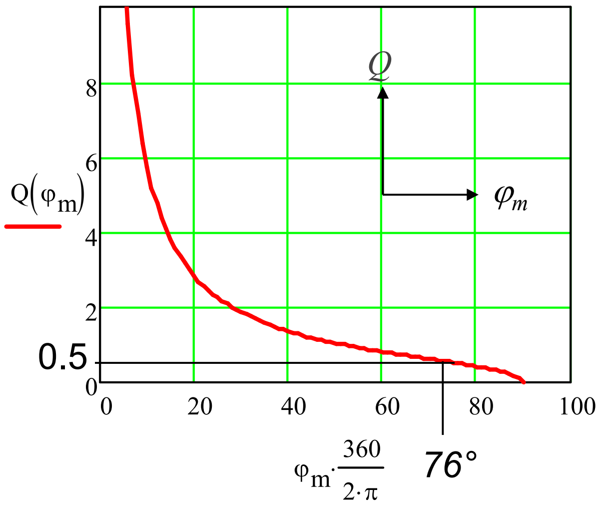
Step load



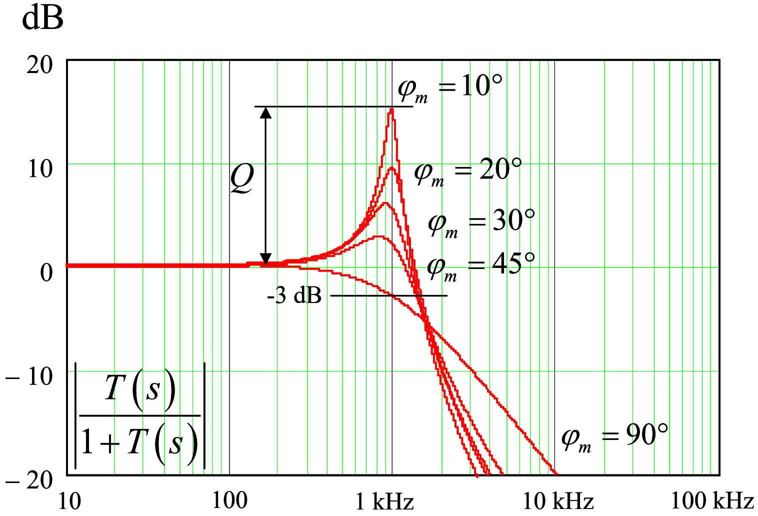
ΔI_{out} = 3 A in 3 µs

Phase Margin and Transient Response

- The phase margin selection depends on the transient response you want



$$\underset{\substack{\uparrow\\ \text{Closed}\\ \text{loop}}}{Q} = \frac{\sqrt{\cos(\varphi_m)}}{\underset{\substack{\uparrow\\ \text{Open}\\ \text{loop}}}{\sin(\varphi_m)}}$$



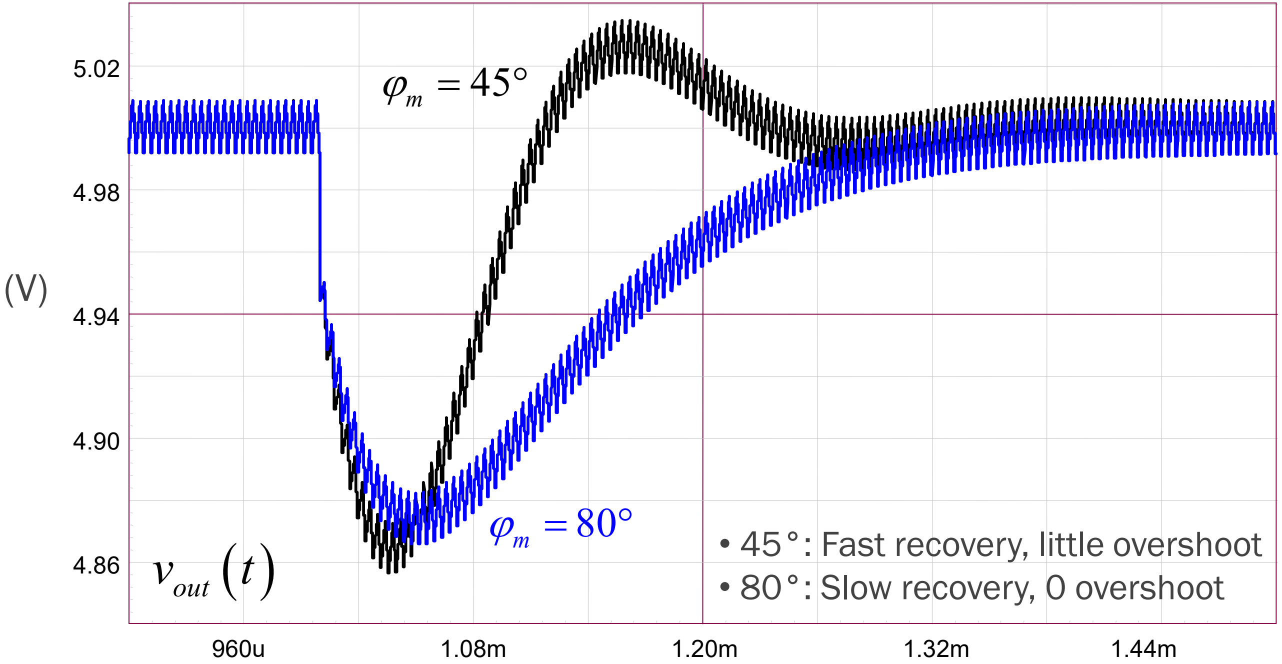
Closed-loop response

✓ The *open-loop* phase margin affects the *closed-loop* quality factor

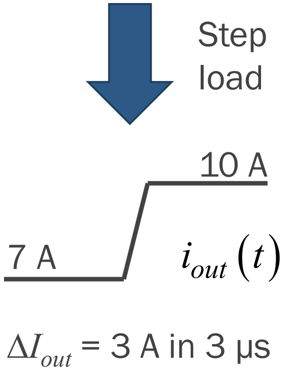
C. Basso, *The Dark Side of Loop Control Theory*, APEC 2012 Professional Seminar

What Transient Response is Needed?

- Choose phase margin based on the transient response you want



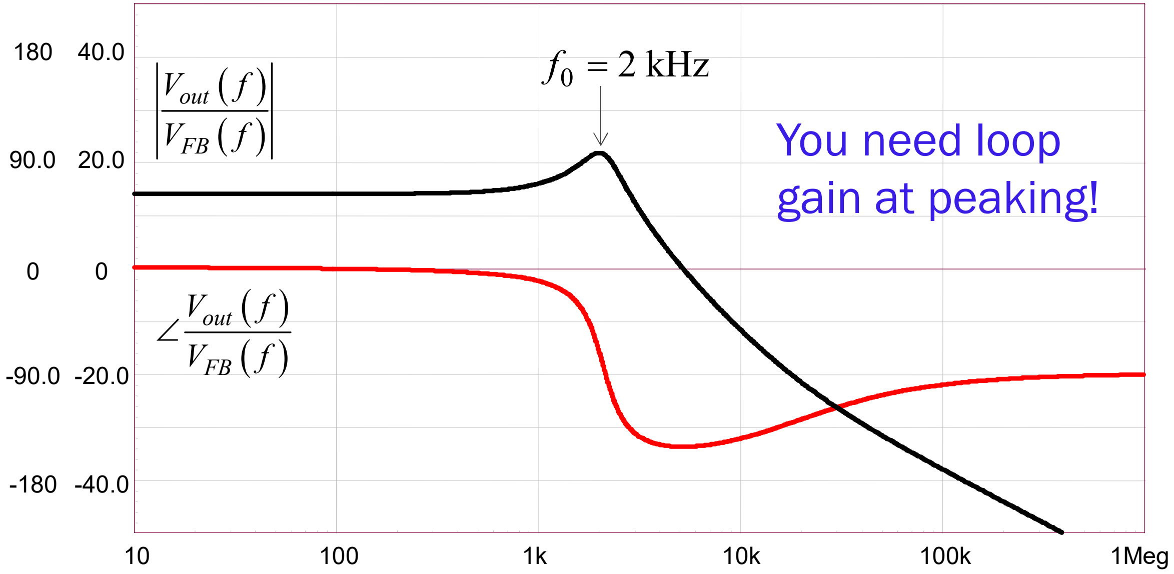
- 45°: Fast recovery, little overshoot
- 80°: Slow recovery, 0 overshoot



ΔI_{out} = 3 A in 3 μs

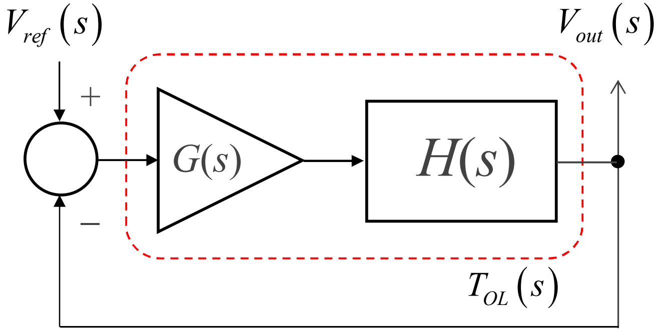
Crossover Selection for the Buck VM Converter

☐ Before selecting a value, there are limits to respect



$f_0 = 2\ \text{kHz}$

You need loop gain at peaking!



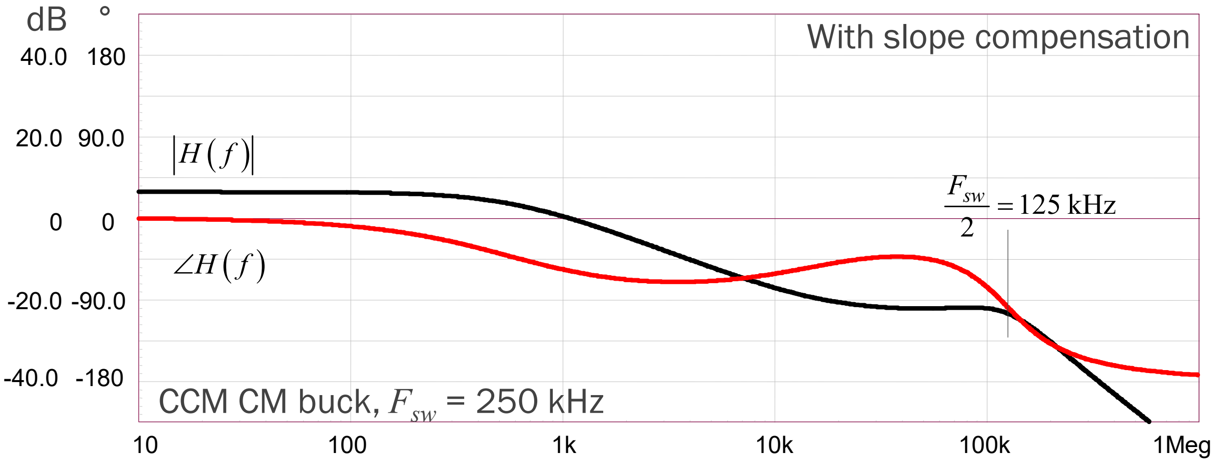
✓ You must have gain in the system at the perturbation frequency

☐ Crossover for a CCM VM buck must be: $3 \cdot f_0 < f_c < \frac{F_{sw}}{2}$

❖ In this example: f_c > 6 kHz

Compensation in Current Mode is Easier

- ☐ Once the poles are damped, the *theoretical* crossover limit is $F_{sw}/2$



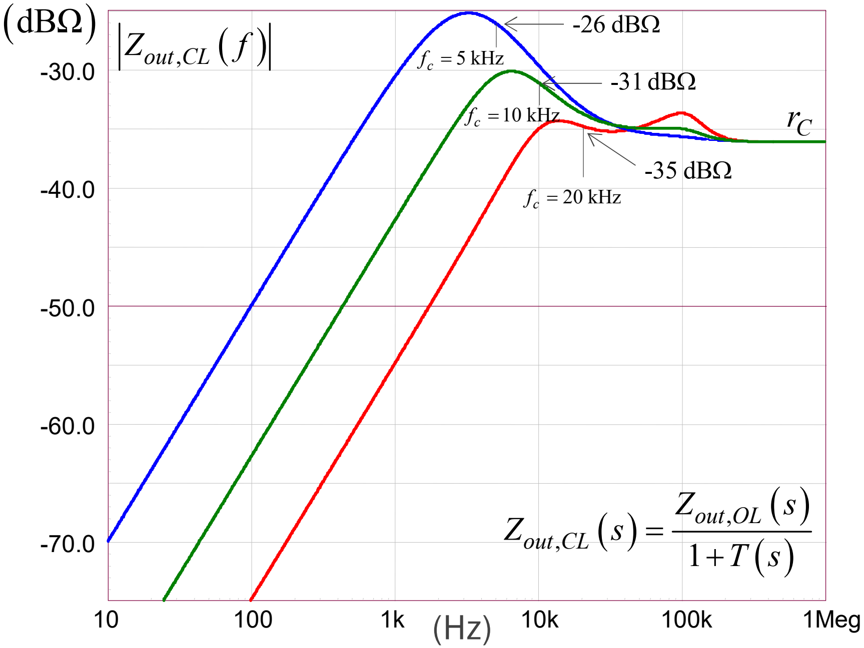
- ❖ Don't push f_c too far as it increases susceptibility to noise

- ❖ As f_c goes up, beware of various delays (conversion time, prop. del.)

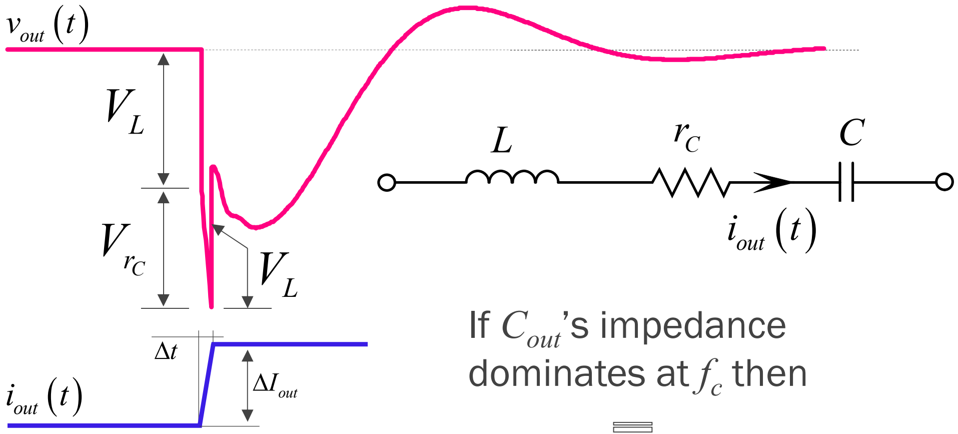
⇨ Choose crossover to meet transient response, not more!

Approximating the Transient Response

- ❑ The capacitor impedance at crossover dominates the output impedance



$$Z_{out,CL}(s) = \frac{Z_{out,OL}(s)}{1+T(s)}$$



If C_{out}'s impedance dominates at f_c then

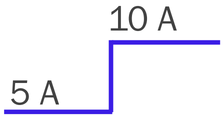
$$\Delta V_{out} \approx \frac{\Delta I_{out}}{2\pi f_c C_{out}}$$

- ❑ As crossover increases, Z_{out} approaches the minimum set by r_C

$$r_C << \frac{1}{2\pi f_c C_{out}}$$

A Simple Guide to Crossover Selection

- ☐ Select the output capacitor based on ripple current, ESR etc.
- ✓ Choose the crossover frequency to meet undershoot specs

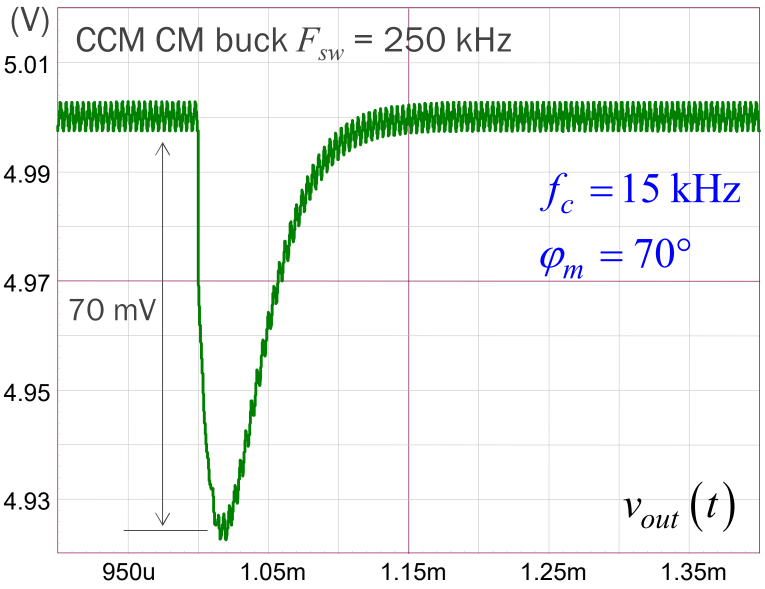


$$\Delta V_{out} < 100\text{ mV}$$

$$C_{out} = 560\ \mu\text{F}$$

$$r_C = 5\text{ m}\Omega$$

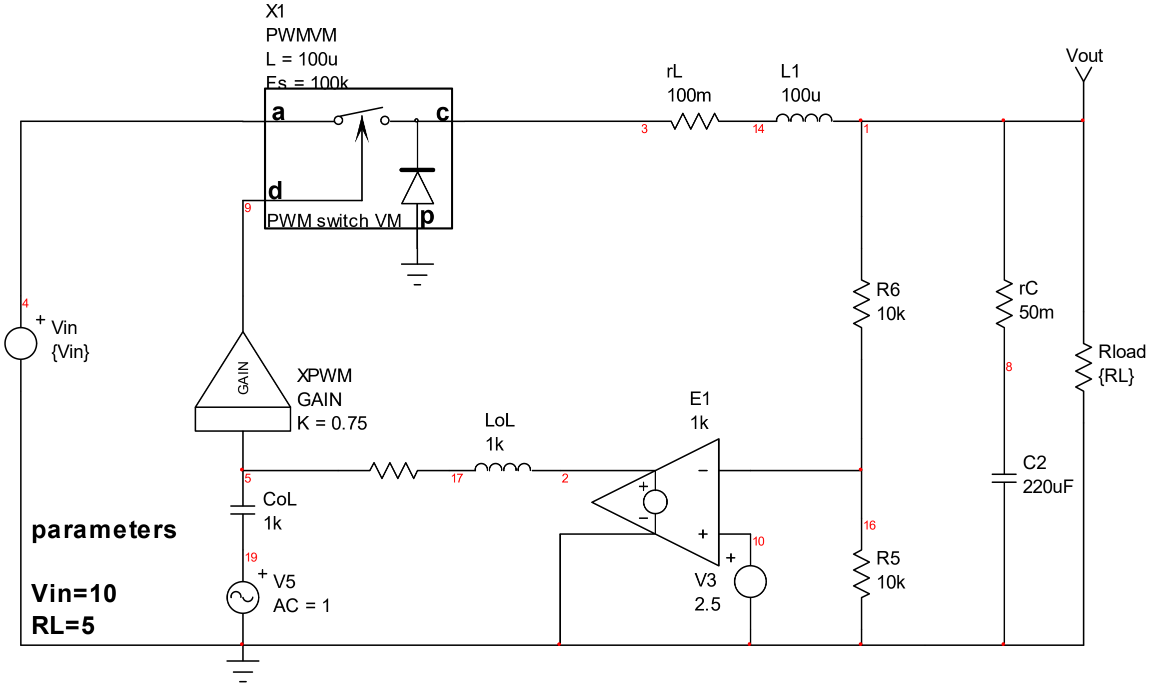
$$f_c \approx \frac{\Delta I_{out}}{2\pi C_{out} \Delta V_{out}} = 15\text{ kHz}$$

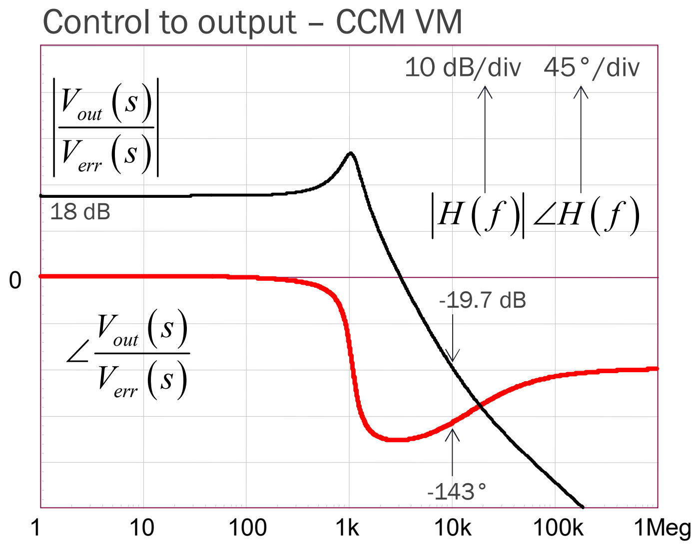


- ☐ It surely is an approximation as the system is nonlinear but it's a guide

Stabilizing the Buck Converter Operated in Voltage Mode

- ☐ Display the control-to-output transfer function with SPICE, SIMPLIS® etc…



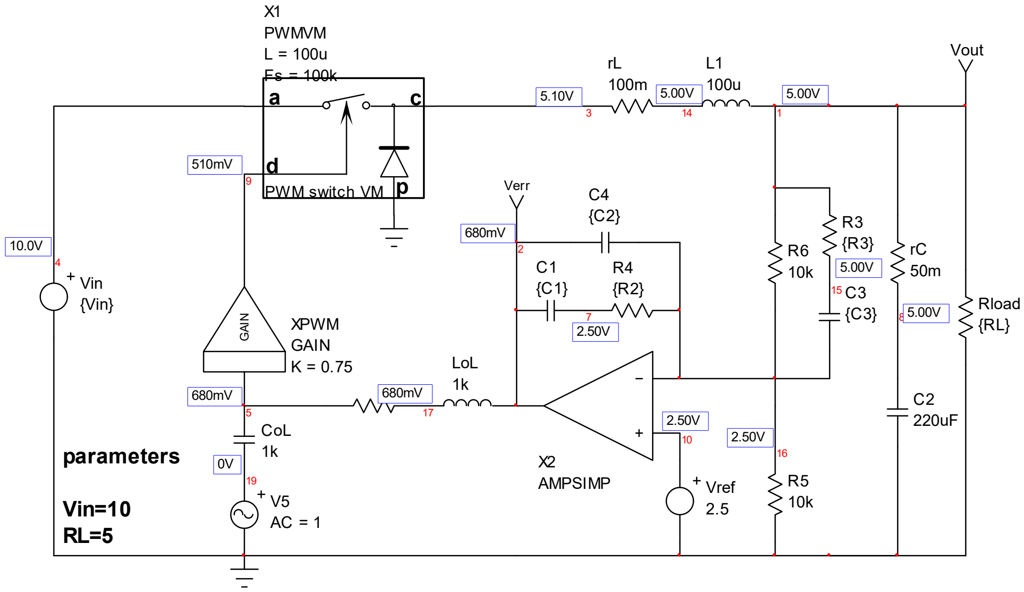


- ➢ Extract magnitude and phase data at 10 kHz: a type 3 is required

Place Poles and Zeros

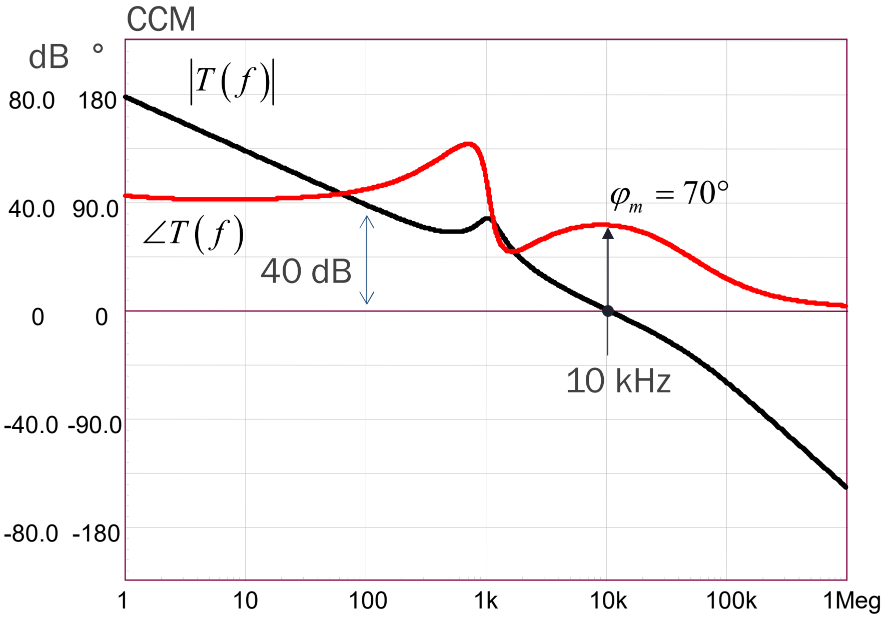
- ✓ Place two coincident zeroes at the resonant frequency (1 kHz)
- ✓ Place the first pole to build phase margin
- ✓ Place the second pole at $F_{sw}/2$ to roll-off the gain at high frequency

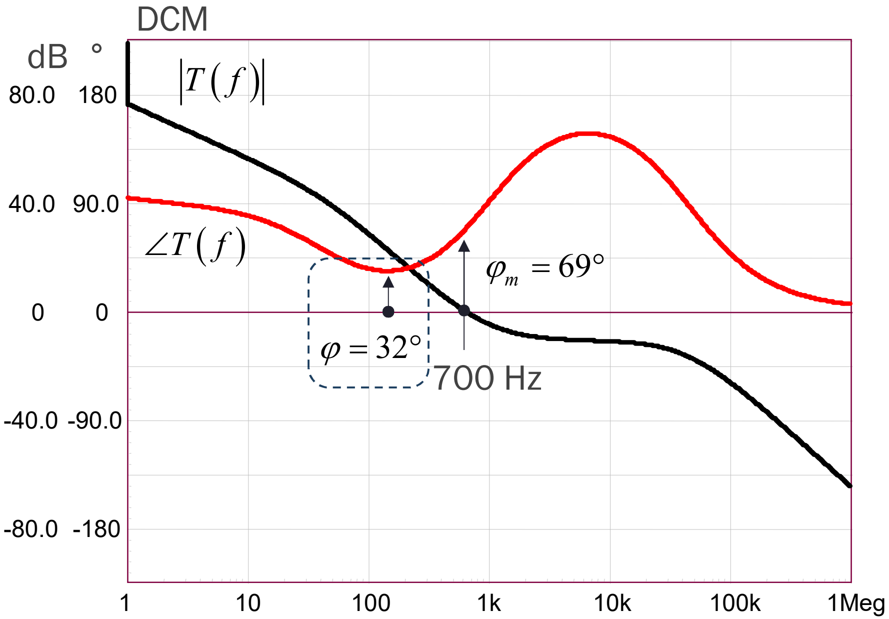
```
parameters

Rupper=10k              a=sqrt((fc^2/fp1^2)+1)
fc=10k                  b=sqrt((fc^2/fp2^2)+1)
Gfc=-19.6               c=sqrt((fz1^2/fc^2)+1)
pfc=-143                d=sqrt((fc^2/fz2^2)+1)
pm=70
boost=pm-(pfc)-90       R2=((a*b/(c*d))/(fp1-fz1))*Rupper*G*fp1
G=10^(-Gfc/20)
pi=3.14159

fz1=1k
fz2=1k
fp1=fc/tan((2*atan(fc/fz1)-tan(fc/fp2))-boost*pi/180)
fp2=50k

C1=1/(2*pi*fz1*R2)
C2=C1/(C1*R2*2*pi*fp1-1)
C3=(fp2-fz2)/(2*pi*Rupper*fp2*fz2)
R3=Rupper*fz2/(fp2-fz2)
```



Check the Loop Gain in CCM and DCM

- ❑ Reducing the load to 50 Ω forces discontinuous operation

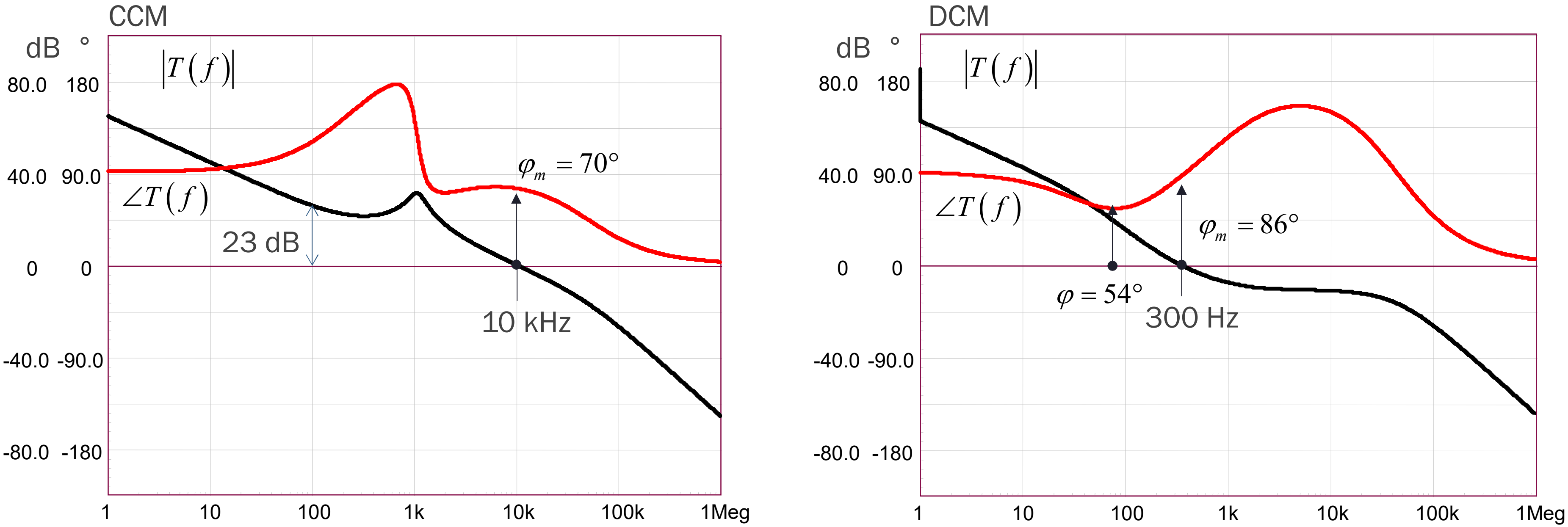




➢ Loop gain changes in DCM: watch weak phase margin zone

Split the Zeroes to Improve the Phase Response

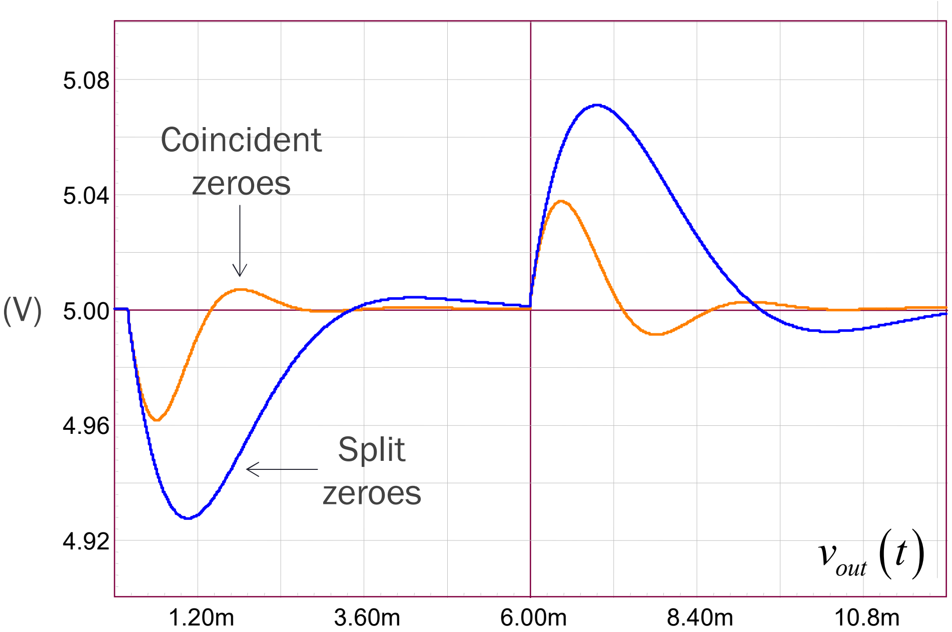
- ❑ Rather than two coincident zeroes, place one at f_0 and another at 200 Hz



- ➢ Lowering the zero with a constant f_c reduces the gain at low frequency

A Better Phase Margin in DCM Slows the Response

- More pronounced undershoot with split zeroes and longer recovery

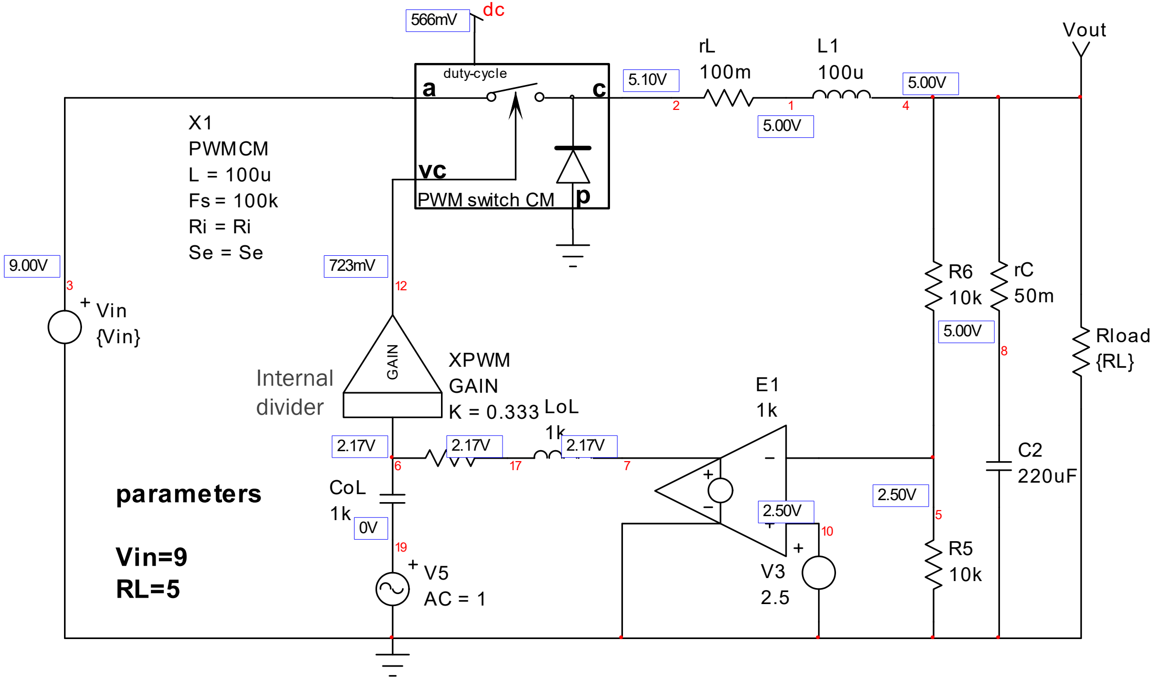


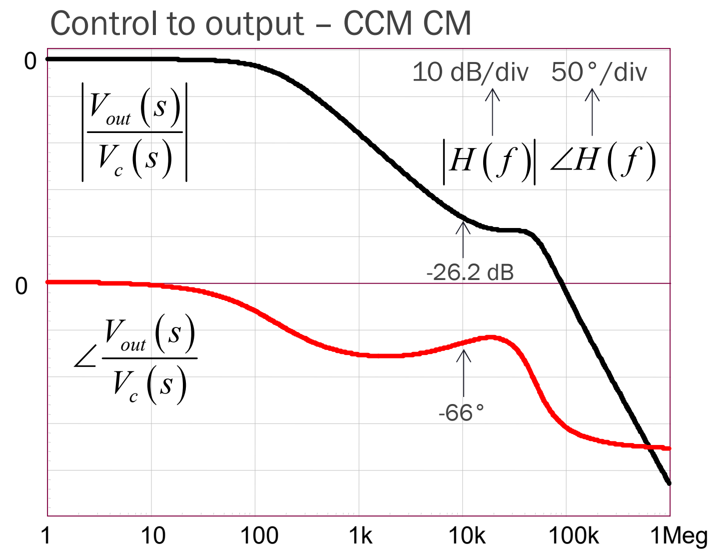
	Frequency	Overshoot	Settling time	Phase margin
f_{z_1}	↗	↗	faster	↘
	↘	↘	slower	↗
f_{z_2}	↗	↗	faster	↘

- Adjusting the second zero position changes the transient response

Stabilizing the Buck Converter Operated in Current Mode

- ❑ Use the PWM switch to plot the control-to-output transfer function





- ➢ Extract magnitude and phase data at 10 kHz: a type 2 is required

Pole and Zero to Compensate the CM Buck

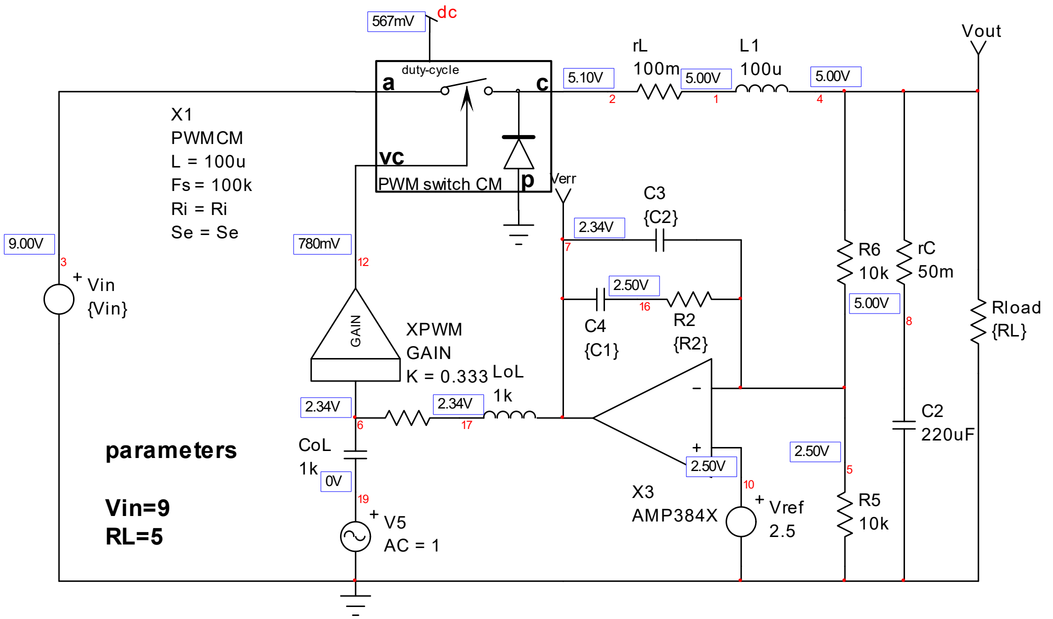
- ☐ You can use the k-factor method in CM designs
- ✓ Zero and pole are spread to boost the phase at crossover
- ✓ The pole at the origin provides dc gain to minimize the static error

parameters

Rupper=10k
fc=10k
Gfc=-26.2
pfc=-66
pm=70
Se=20k
Ri=600m

boost=pm-(pfc)-90

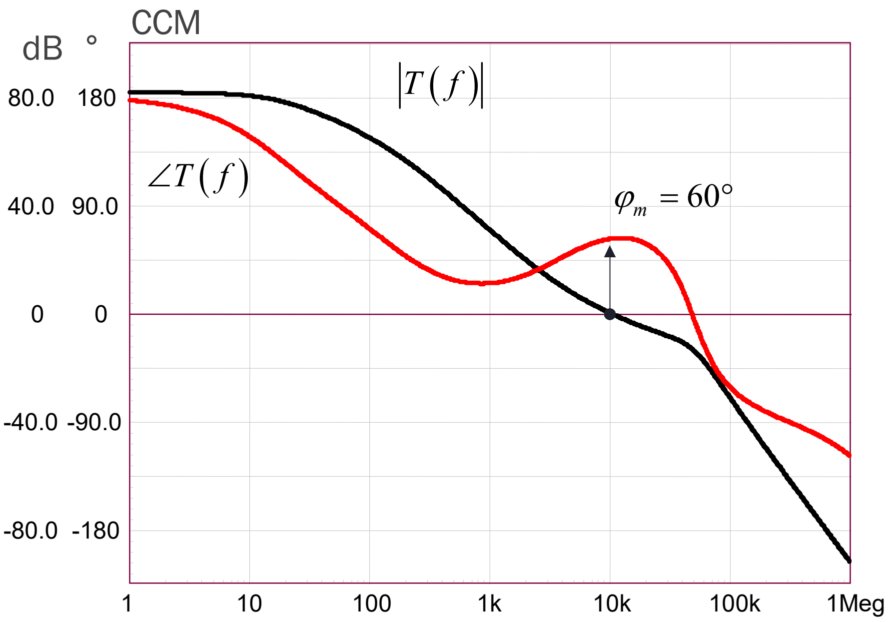
G=10^(-Gfc/20)
boost=pm-(pfc)-90
pi=3.14159
K=tan((boost/2+45)*pi/180)
C2=1/(2*pi*fc*G*k*Rupper)
C1=C2*(K^2-1)
R2=k/(2*pi*fc*C1)

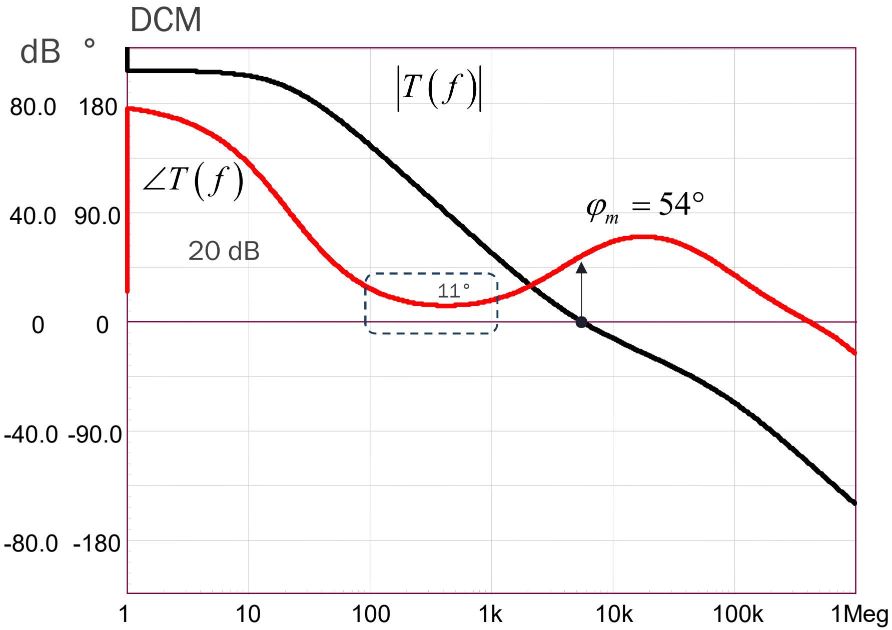


D. Venable, *The k factor: A New Mathematical Tool for Stability Analysis and Synthesis*, Proceedings of Powercon 10, 1983, pp. 1-12

Check the Loop Gain in CCM and DCM

☐ Crossover frequency does not significantly change in DCM





➢ Less loop gain impact in DCM: 20-dB margin in magnitude curve

Course Agenda

- ☐ The Buck Converter
- ☐ Control Schemes
- ☐ Introduction to Modeling
- ☐ The PWM Switch in Current-Mode Control
- ☐ Introduction to the Fast Analytical Techniques
- ☐ The Four Transfer Functions in CCM Current-Mode Control
- ☐ Compensation Strategy
- ☐ Prototype Measurements

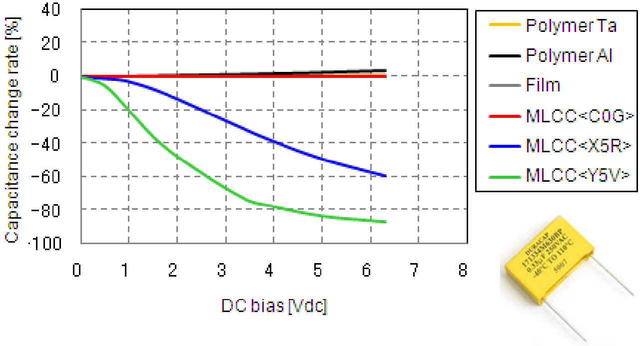
Characterize your Components on the Bench

☐ Bench experiments are a mandatory step

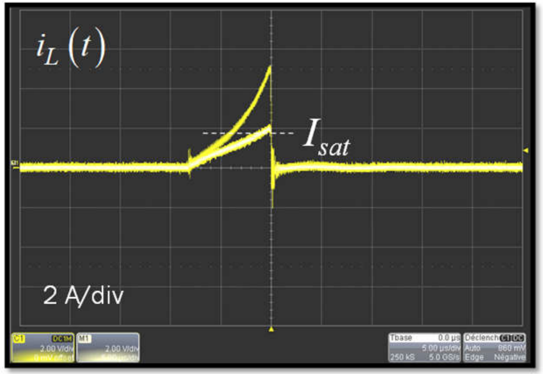
- ➢ Are the assumptions adopted for the analytical model confirmed?
- ➢ Feed the model back with real measurements, refine simulations
- ➢ Check the behavior in temperature, particularly stability

❖ Capacitance change with bias voltage?

❖ Margin before inductor saturation?

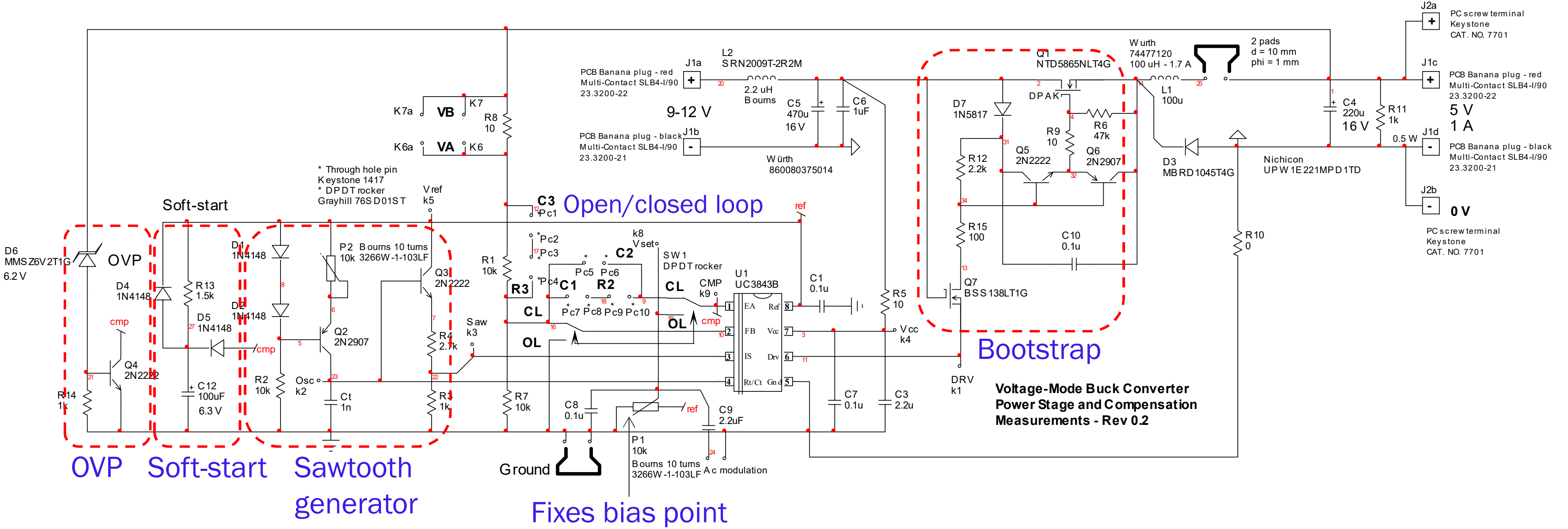


Characterization



A Buck Converter in Voltage-Mode Control

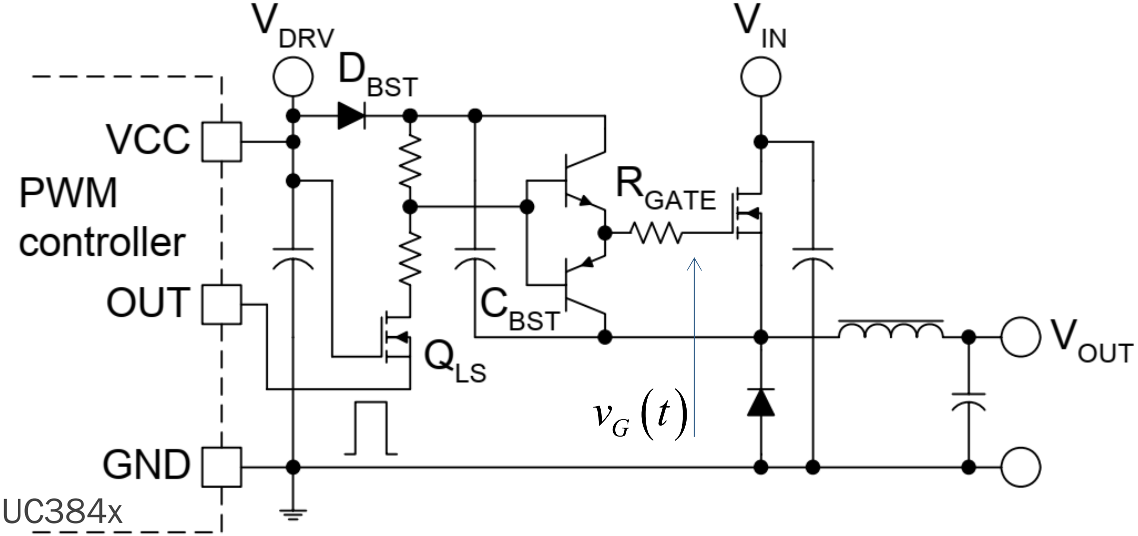
- ❑ Turn the UC3843 into a voltage-mode controller

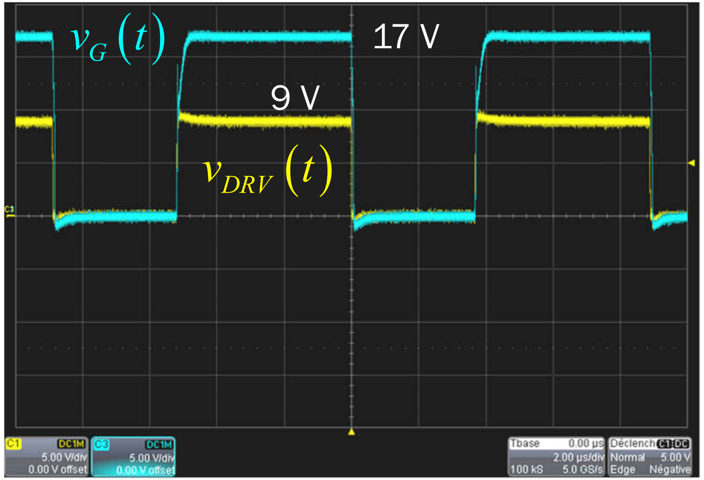


- ❖ The soft-start provides a smooth start-up sequence
- ❖ The OVP circuit protects the converter in open-loop operations

The Need for a Bootstrapped Driver

- ❑ The UC384x delivers a ground-referenced drive signal



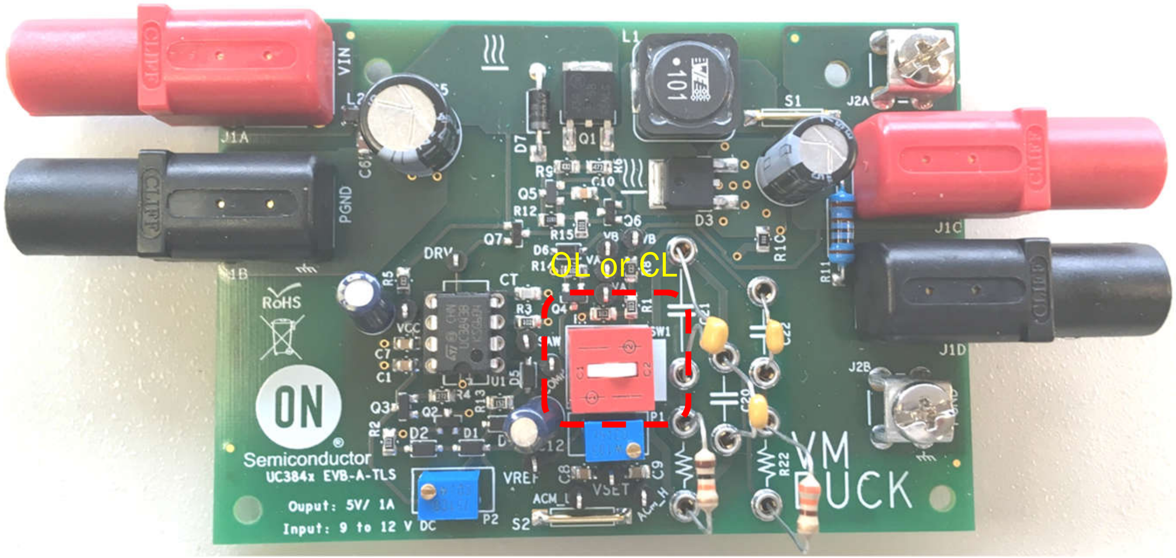


- ➢ The circuit brings the gate 8 V above V_{cc} and enhances the MOSFET

L. Balogh, *Design and Application Guide for High-Speed MOSFET Gate Drive Circuit*, TI SLUA 618

The Prototype in Voltage-Mode Control

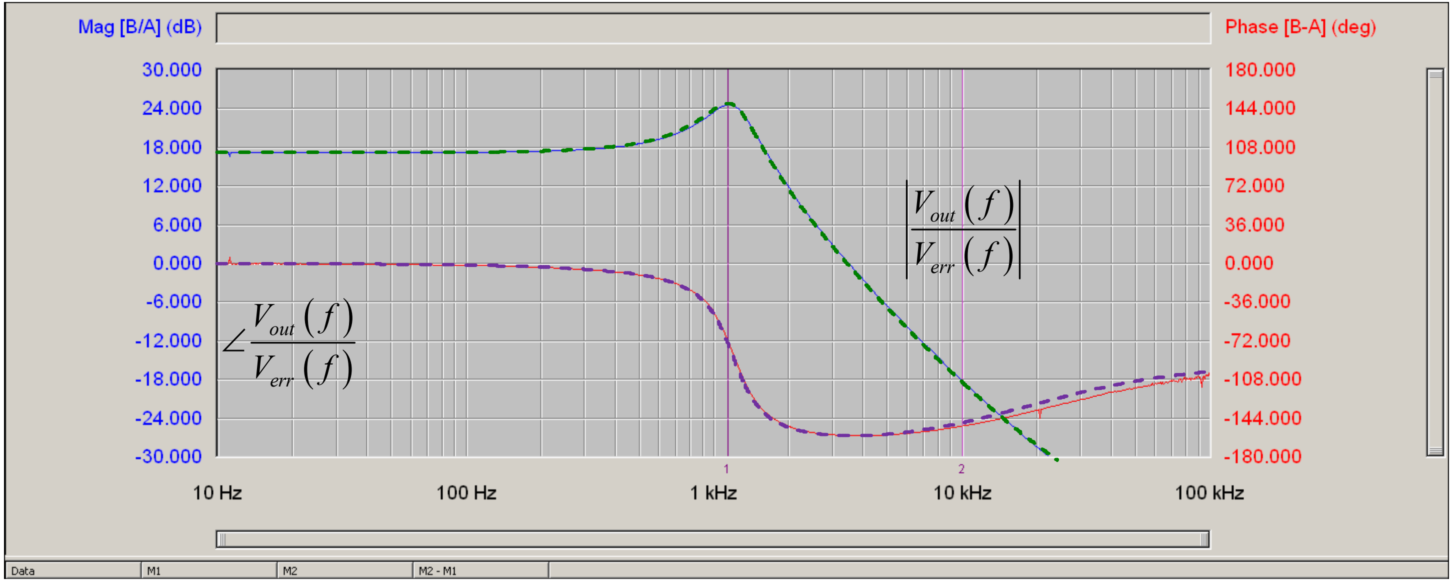
- ☐ The board allows the easy testing of various compensation strategies



- ☐ You can toggle between open- and closed-loop configurations

Extract the Control-to-Output Transfer Function

- ☐ Simulation and bench experiments agree well with each other

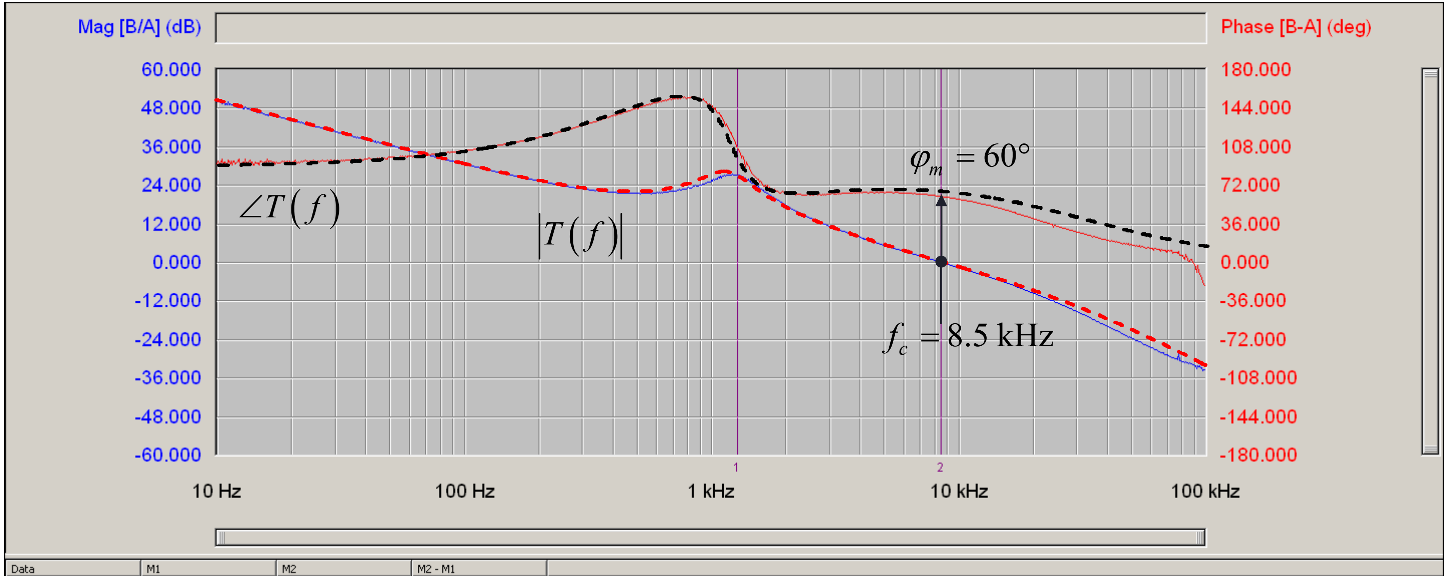


Mr Natural

- ➢ Good parasitics extraction is key to validate the model

Loop Gain after Compensation Strategy

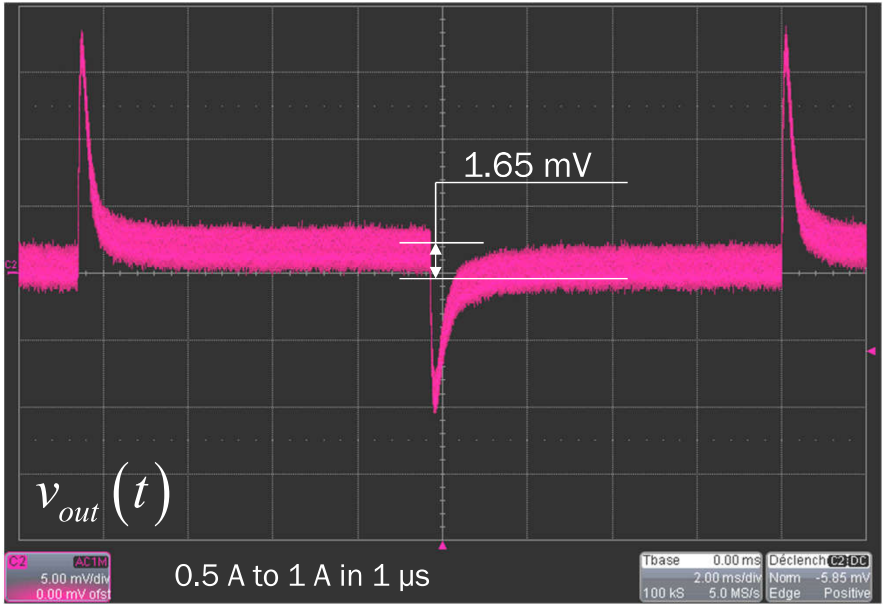
- ☐ The crossover frequency is slightly below 10 kHz, good phase margin



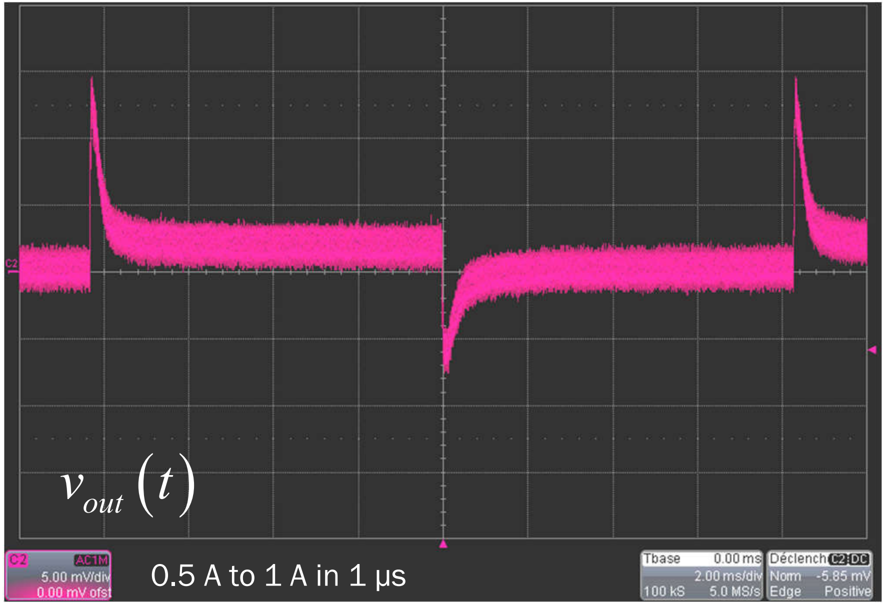
- ➢ This plot confirms the model is representative of the hardware

Transient Tests in Closed-Loop Conditions

☐ The transient response is excellent and shows a low dc deviation



8.5-kHz crossover with 60° PM

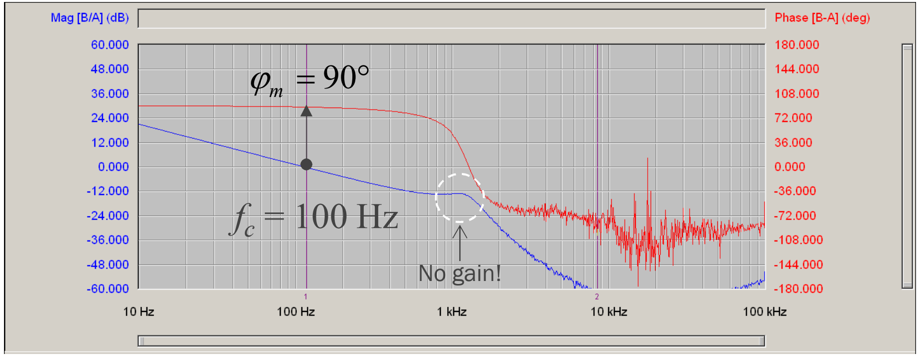


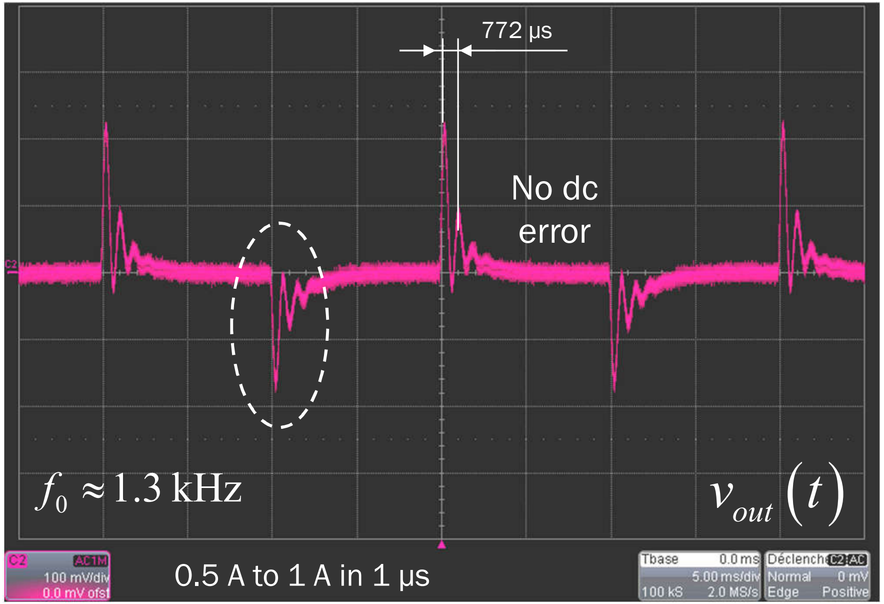
9.5-kHz crossover with 45° PM

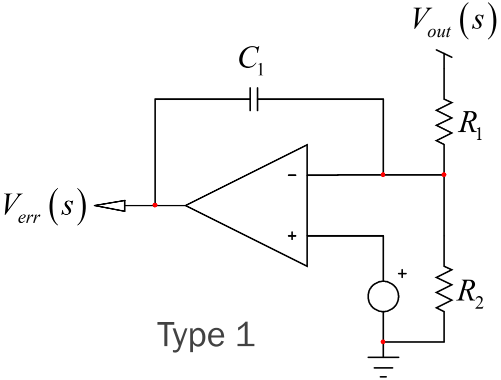
➢ It's impossible to infer margins from transient tests: always go for Bode plot

No Gain, No Feedback!

- ☐ Wrongly selecting the crossover shows oscillations in the response





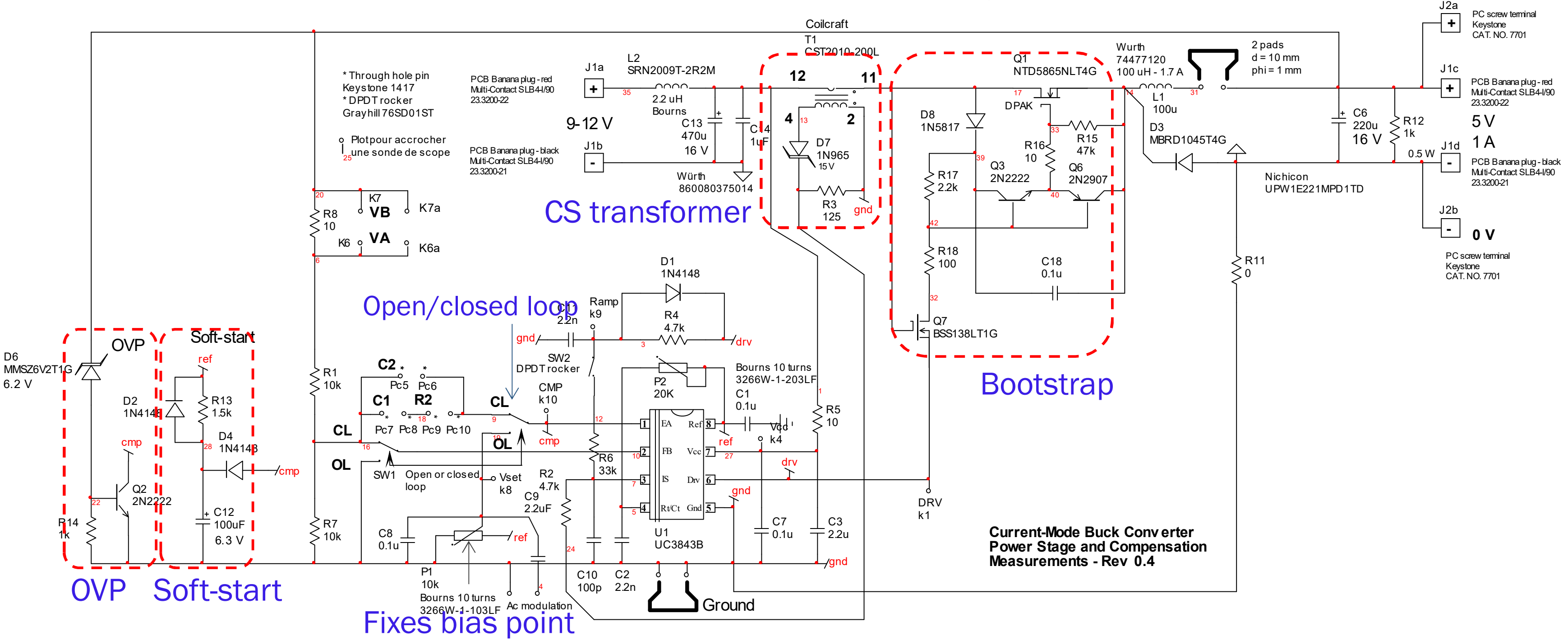


The system regulates well in dc but cannot fight the oscillations brought by the LC filter: increase crossover!

An output impedance magnitude plot would show the peaking

A Buck Converter in Current-Mode Control

- ☐ The current mode architecture requires a high-side sense



CS transformer

Bootstrap

Open/closed loop

OVP

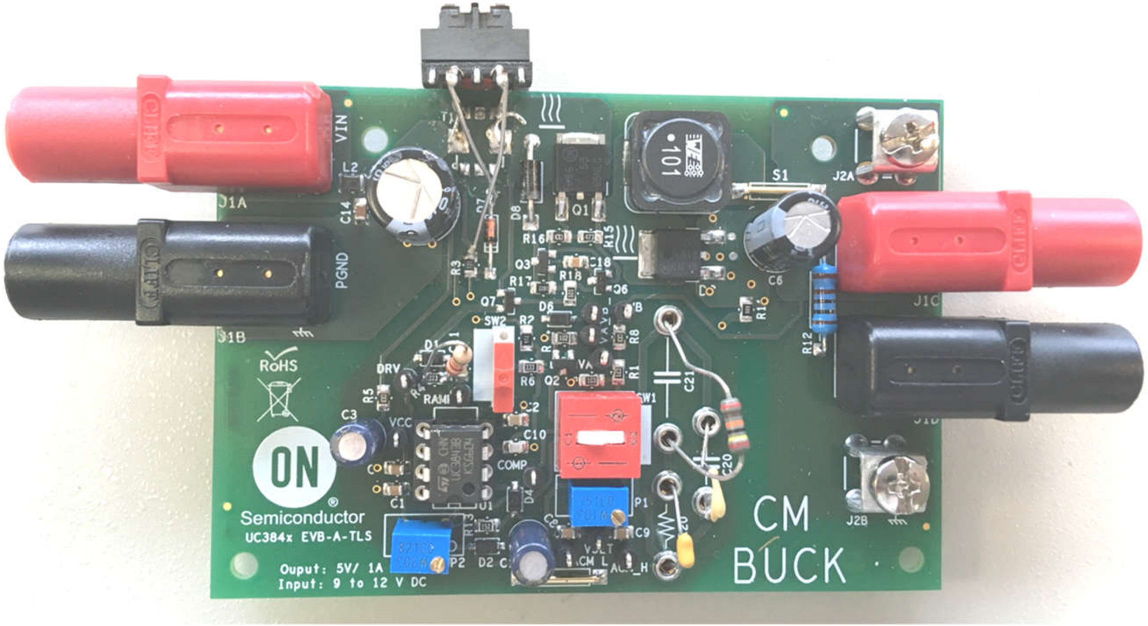
Soft-start

Fixes bias point

- ➢ A current transformer references the inductor current to ground

The Prototype in Current-Mode Control

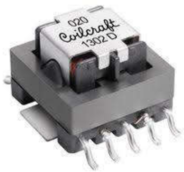
- ☐ A current sense transformer is now needed to sense $i_L(t)$

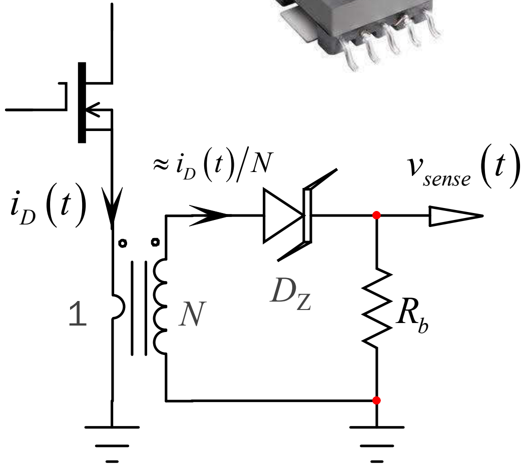


- ☐ You can toggle between open- and closed-loop configurations

Current Sense Transformer Operations

☐ The current is scaled by the turns ratio and generate a sense voltage



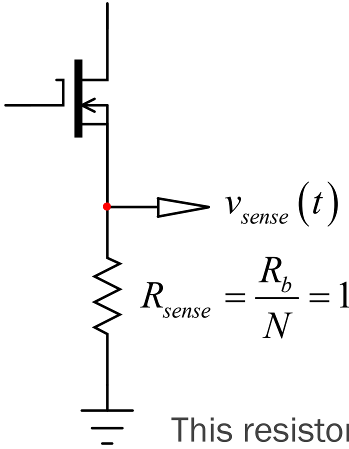


$$V_{sense,\max} = 1\text{ V}$$

$$N = 100$$

$$I_{D,\max} = 8\text{ A}$$

$$R_b = \frac{V_{sense,\max}}{I_{D,\max}} N = \frac{1}{8} 100 = 12.5\ \Omega$$



$$R_{sense} = \frac{R_b}{N} = 125\text{ m}\Omega$$

This resistor value is passed to the model

❖ The equivalent resistance value is used for small-signal analysis

Current Sense Reset Scheme

- ☐ The transformer features a magnetization inductance
  - ➢ Reset is mandatory

Part No.	Turns Ratio	E-T Product (V-µs)	Ls[1] (mH, min.)	Rs[2] (Ω, max.)	Current Rating[5] (A)	Hi-Pot (V_{AC})
CT02-050	1:50	4.0	0.44	1.25	15.0	1500
CT02-100	1:100	8.0	1.8	4.8	18.0	1500
CT02-150	1:150	12.0	4.0	15.0	18.0	1500
CT02-200	1:200	16.0	7.1	25.0	18.0	1500
CT02-250	1:250	20.0	11.1	37.2	18.0	1500
CT02-300	1:300	24.0	15.0	55.0	18.0	1500

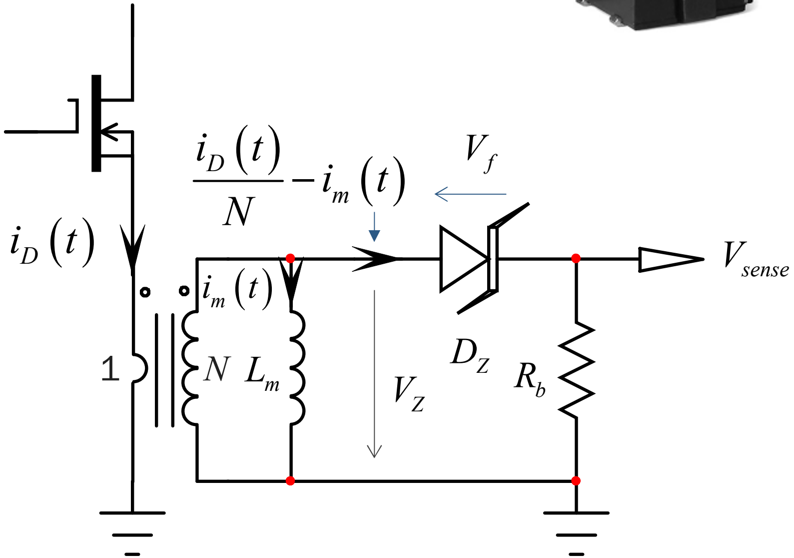
$$V_{mag} D_{\max} T_{sw} = V_{reset} \left(1 - D_{\max}\right) T_{sw}$$

max V.s

$$V_{mag} D_{\max} T_{sw} = \left(\underline{V_{sense,\max}} + V_f\right) D_{\max} T_{sw} \left[\mathrm{V} \cdot \mu\mathrm{s}\right]$$

$$V_{reset} = \frac{V_{mag} D_{\max}}{1 - D_{\max}}$$

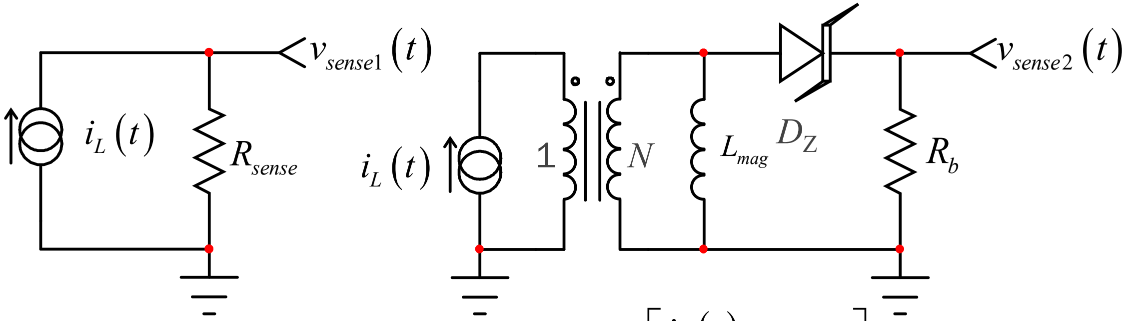
The Zener diode provides the reset



$$i_m(t) \approx \frac{v_{sense}(t) + V_f}{L_{mag}}$$

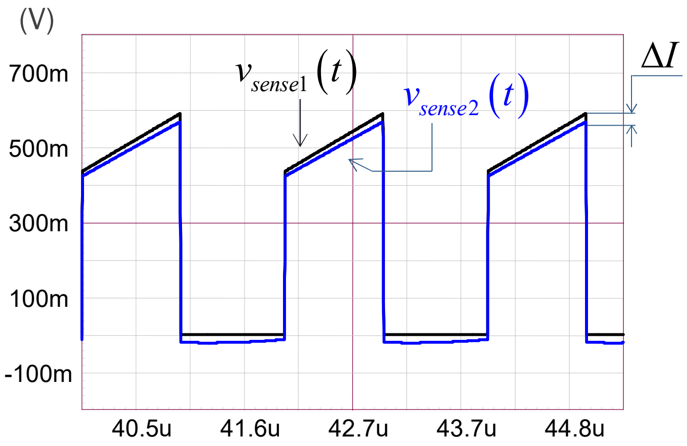
Watch for the Magnetizing Current

- The magnetizing current can affect the precision



$$v_{sense1}(t) = i_L(t) R_{sense}$$

$$v_{sense2}(t) \approx \left[\frac{i_L(t)}{N} - i_m(t)\right] R_b$$



$$\Delta I(\%) = \frac{\frac{V_{CS}+V_f}{L_{mag}} DT_{sw}}{\frac{V_{CS}}{R_b}} \times 100$$

Check the magnetizing inductance value to minimize this error

V_{CS} is the maximum voltage sense limit

The Magnetizing Current Affects Compensation

- ☐ The magnetizing current subtracts from the monitored current
  - ➢ A *wrongly-selected* current transformer can bring instabilities

On-time slope $$S_n = \frac{V_{in} - V_{out}}{L}\frac{R_b}{N}$$

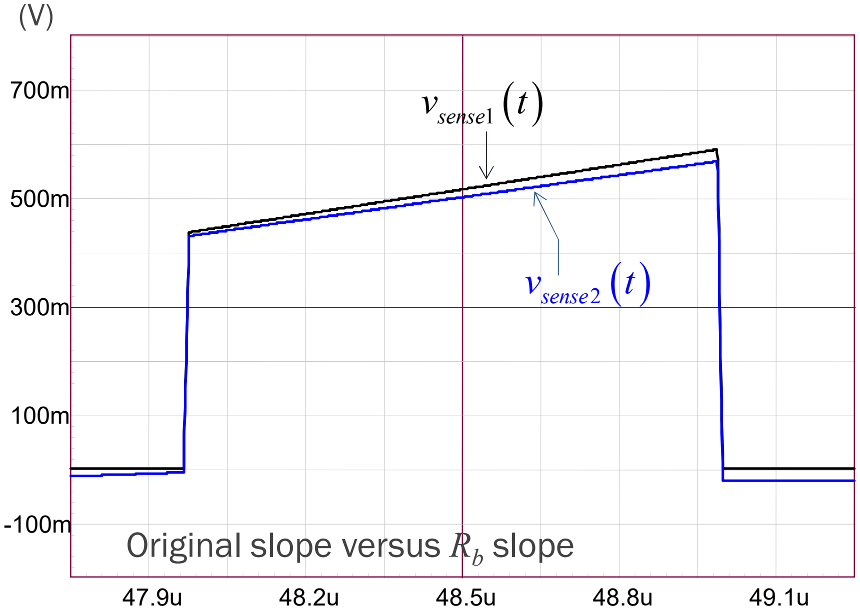
Compensation slope $$S_a = [\mathrm{V/s}]$$

Magnetizing slope $$S_m = \frac{V_f + V_{CS}}{\boxed{L_m}}R_b$$

$v_{CS}(t)$

$$S_{tot} = S_n - S_m + S_a$$

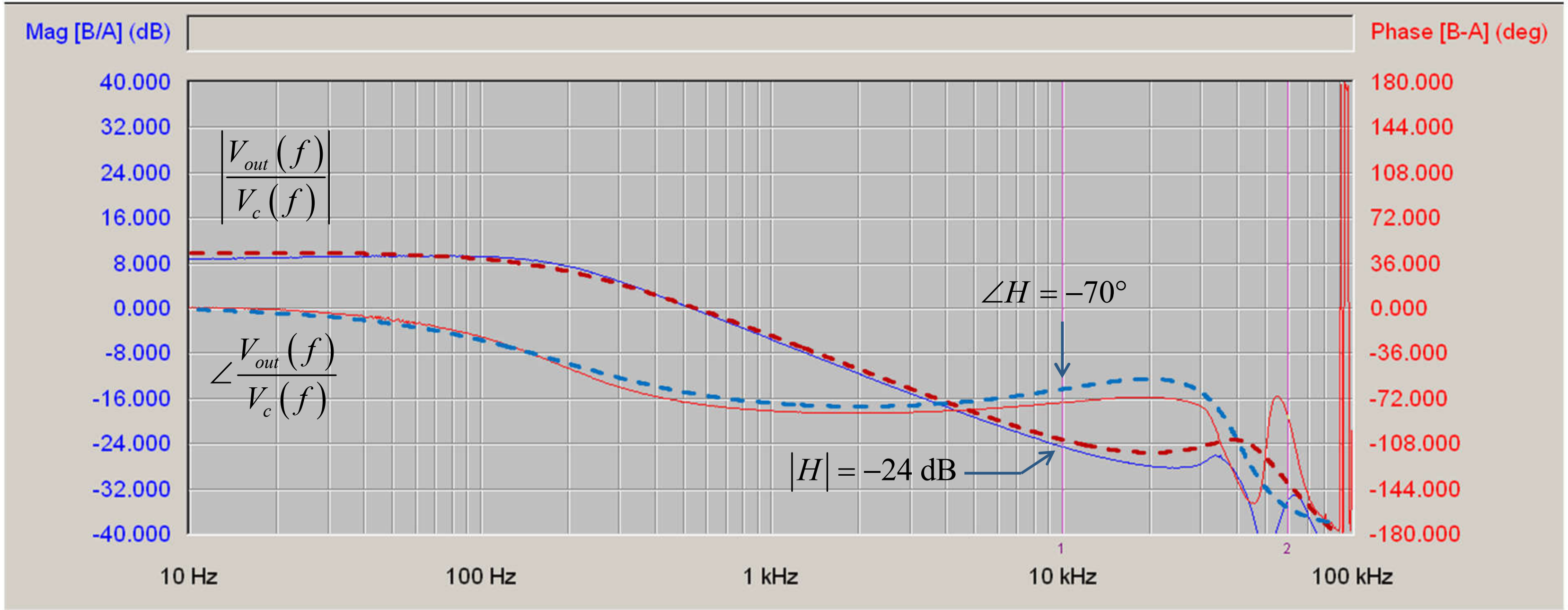
Can affect slope compensation



Original slope versus R_b slope

Extract the Plant Control-to-Output Transfer Function

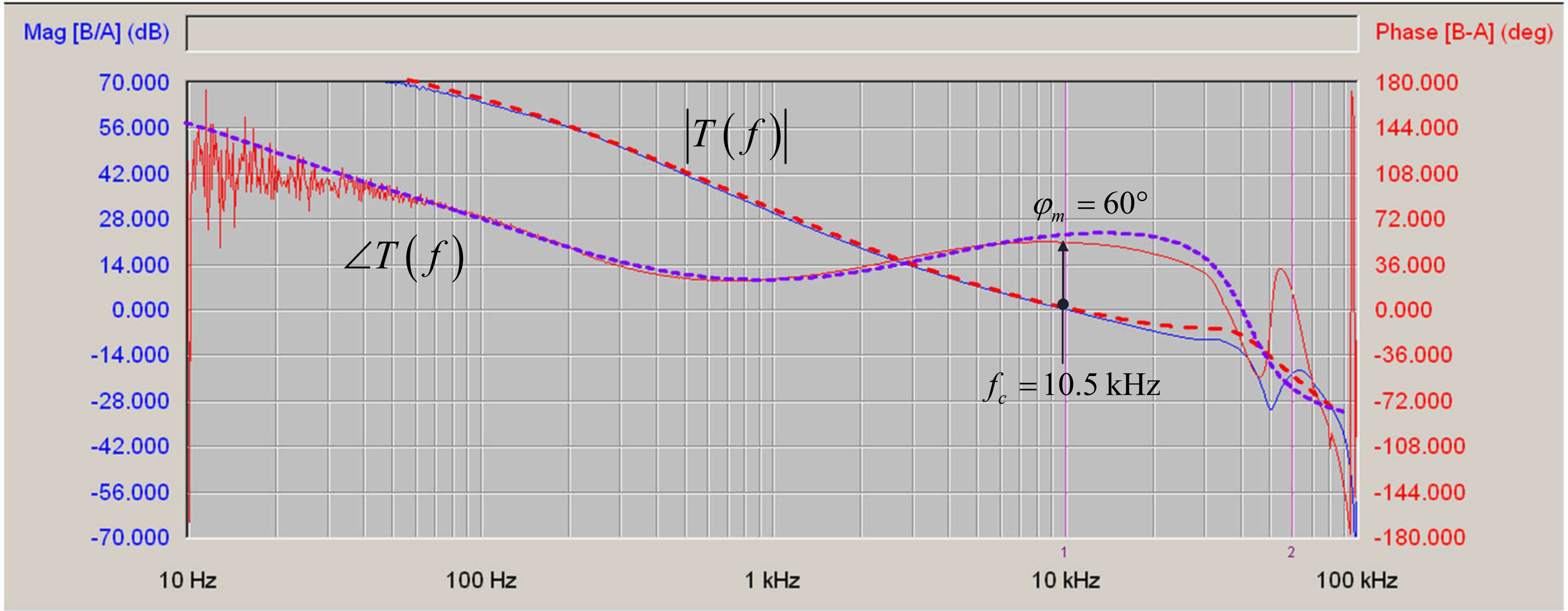
- ☐ Simulation and bench experiments show some discrepancies



➢ Probably a better parasitic extraction is needed from the components

Loop Gain of the Current-Mode Buck Converter

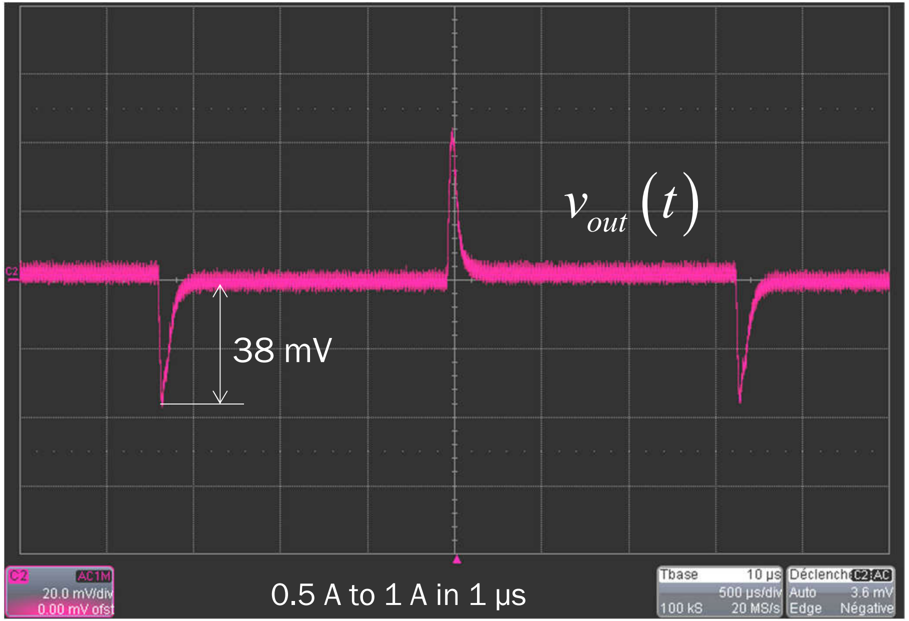
- ☐ The crossover frequency is slightly above 10 kHz, good phase margin



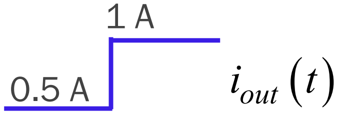
- ➢ A large dc gain induces a low output static error

Predicting the Transient Response Dropout

- ☐ The transient tests confirm a stable converter



V_{in} = 9 V, f_c = 10.5 kHz, PM = 60 °



$$C_{out} = 220\ \mu\text{F}$$

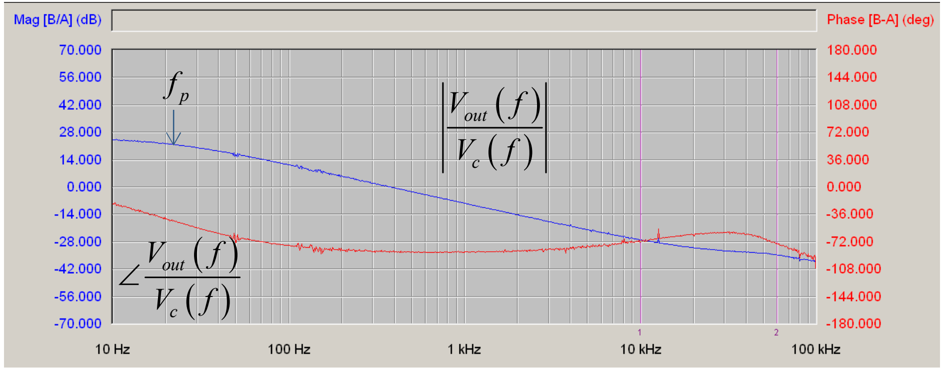
$$r_C = 50\ \text{m}\Omega$$

Approximate expression

$$\Delta V_{out} \approx \Delta I_{out} \frac{1}{2\pi f_c C_{out}} = 34\ \text{mV}$$

Instability in Discontinuous Conduction Mode

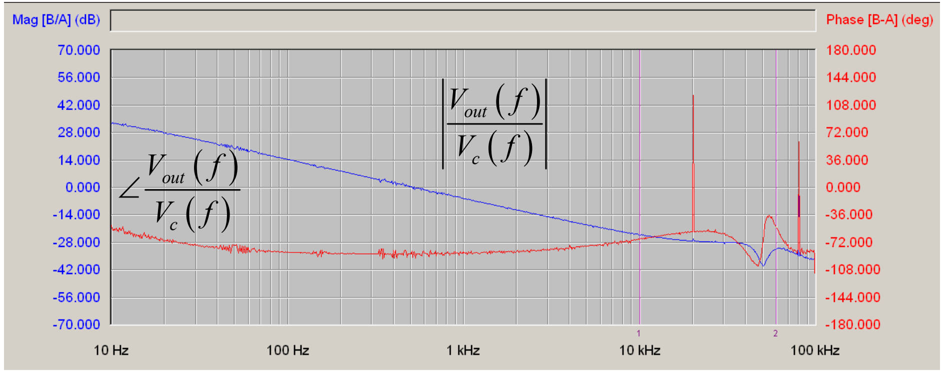
- ☐ As M approaches 0.666, the pole goes to lower frequency



Current mode:
V_{in} = 14 V
V_{out} = 5.8 V
R_{load} = 80 Ω
DCM operation

Inject slope compensation to stabilize the converter

$$S_a > 0.086 \cdot S_{off}$$



Current mode:
V_{in} = 9.55 V
V_{out} = 5.6 V
R_{load} = 80 Ω
DCM operation

B. Erickson, D. Maksimović, *Fundamentals of Power Electronics*, 2nd edition, Springer

Conclusion

- The buck converter can be operated with different schemes
  - Current-mode control is the most popular technique
- Modeling still sees new emerging techniques
  - The PWM switch model is truly the simplest approach
  - You can improve models by accounting for harmonics
- The FACTs are an efficient tool to determine transfer functions
  - Analytical analysis gets you the insight on who does what?
- Slope compensation is necessary in current-mode CCM
  - but also in DCM for the buck converter to keep stability
- As usual, analytical analysis + simulation + bench prototype = success!