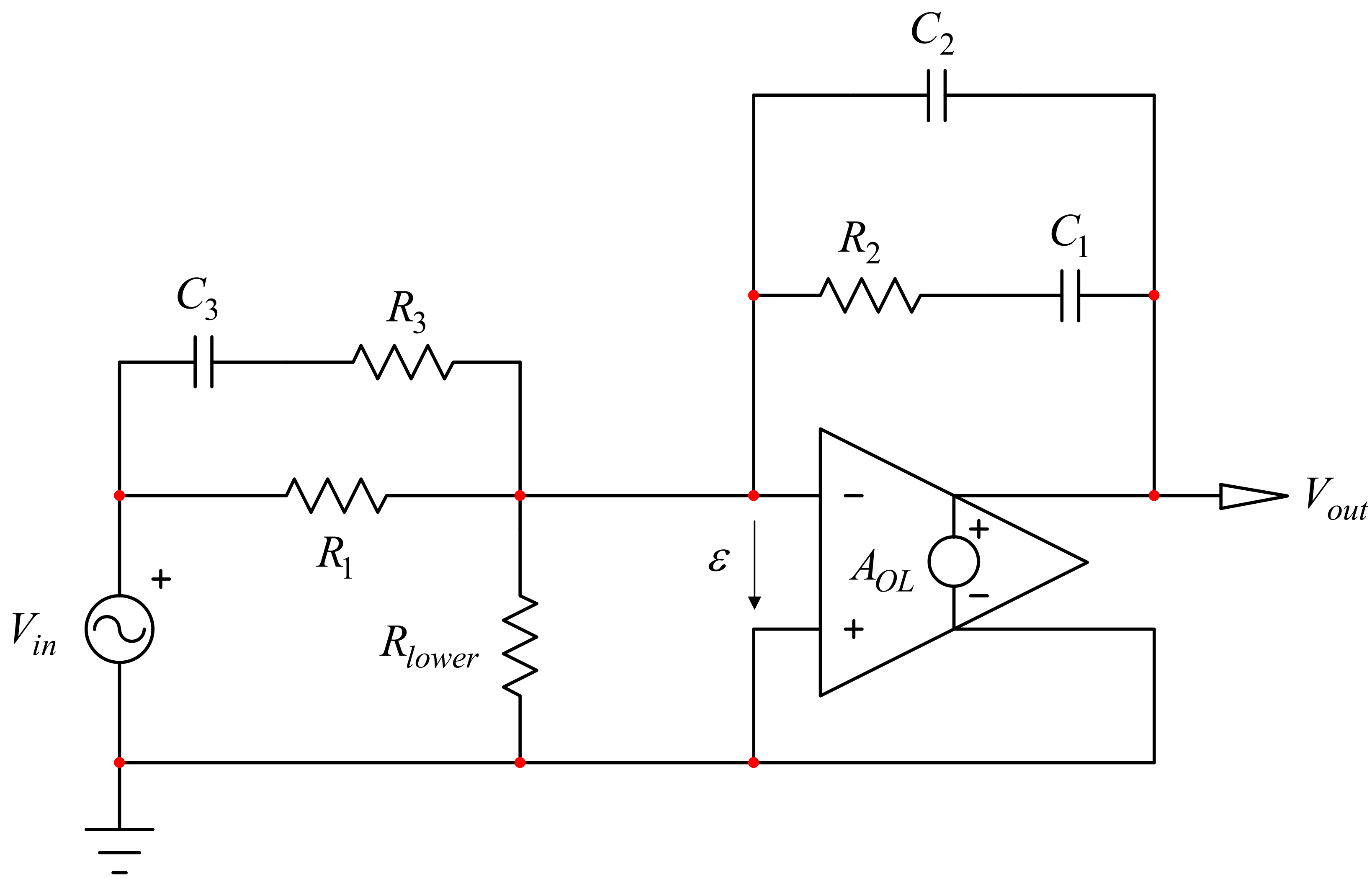
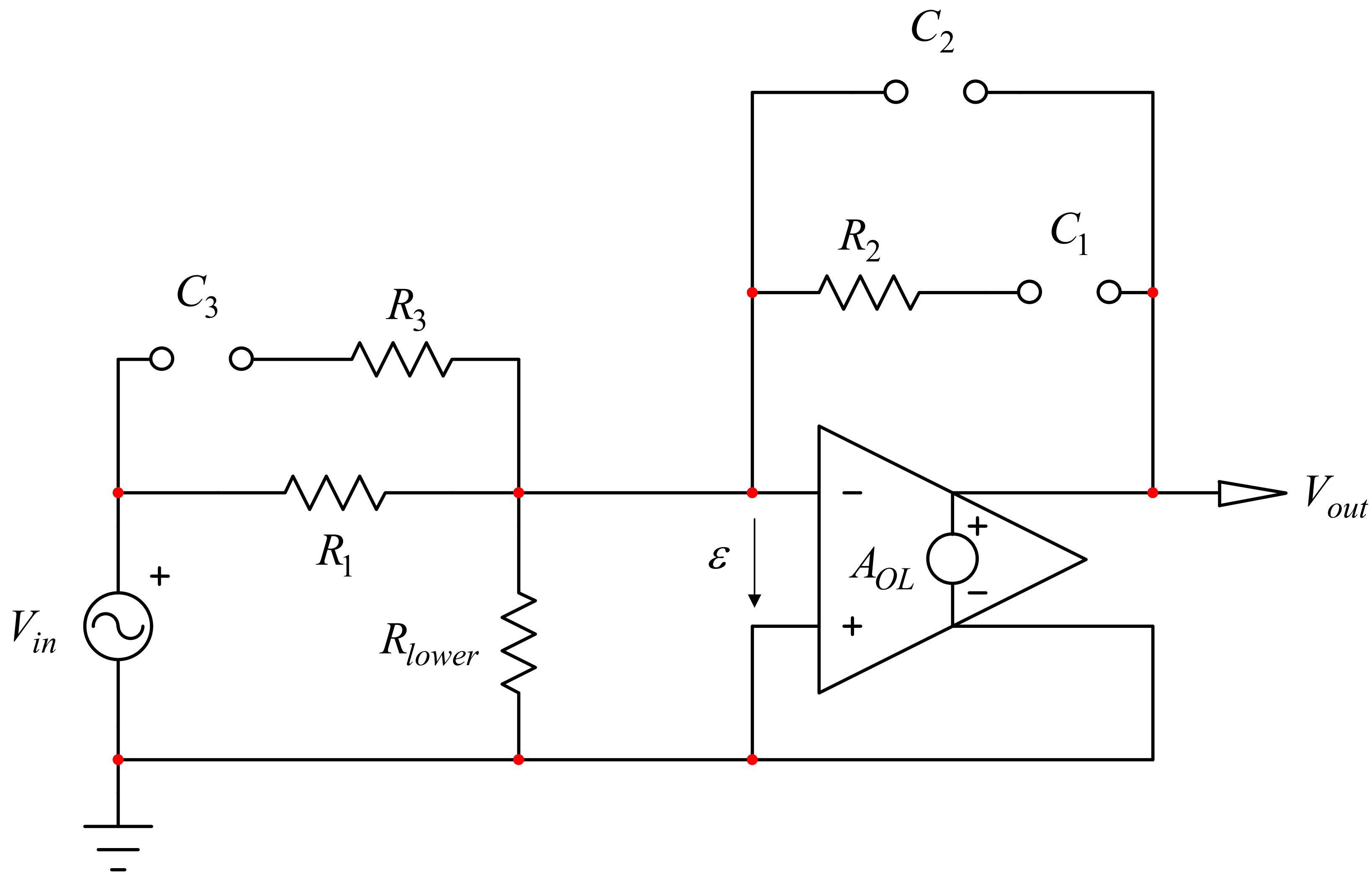
FACTs applied to a type 3 compensator – Christophe Basso – August 2017

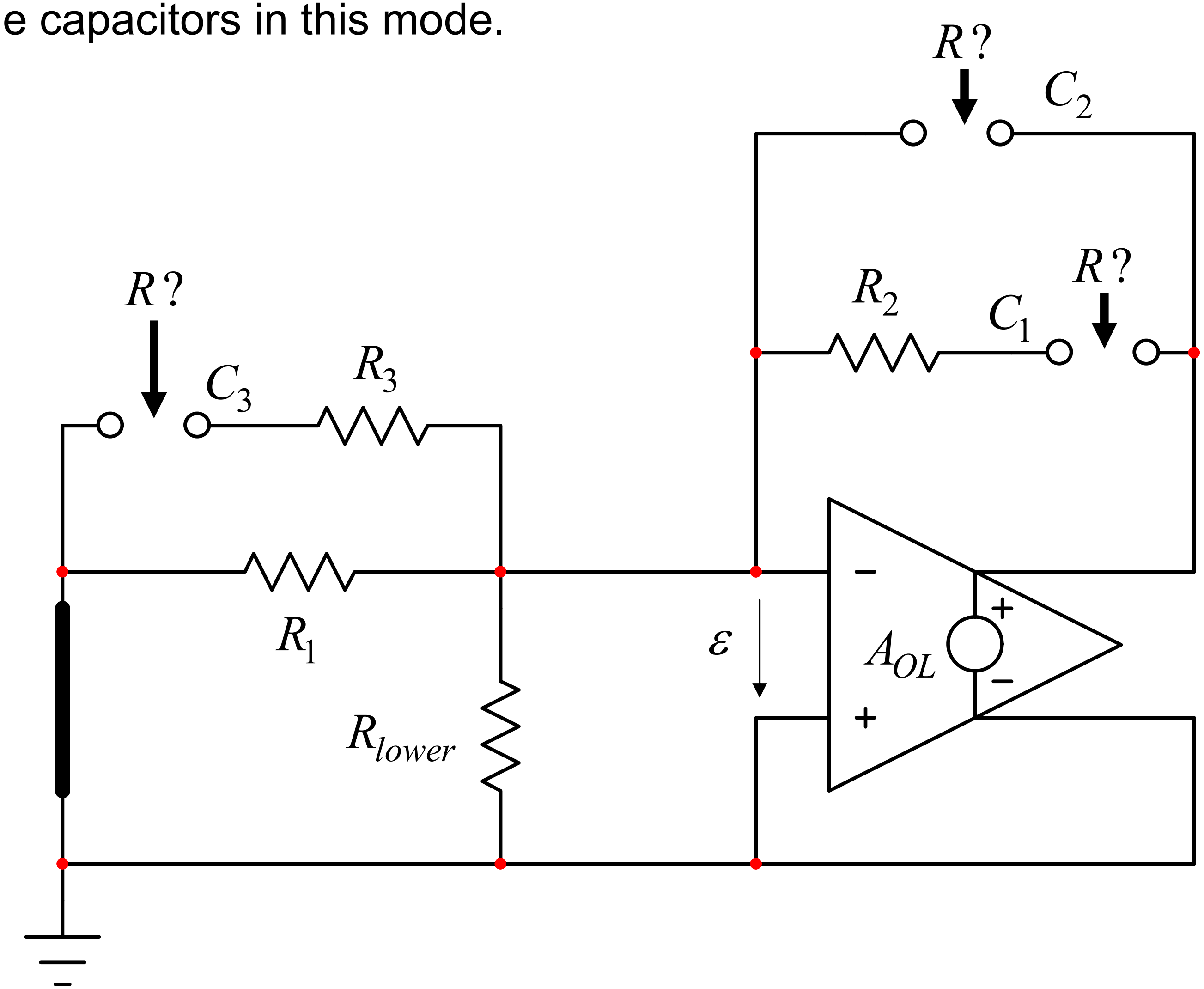


Dc gain, s = 0, open all the caps:

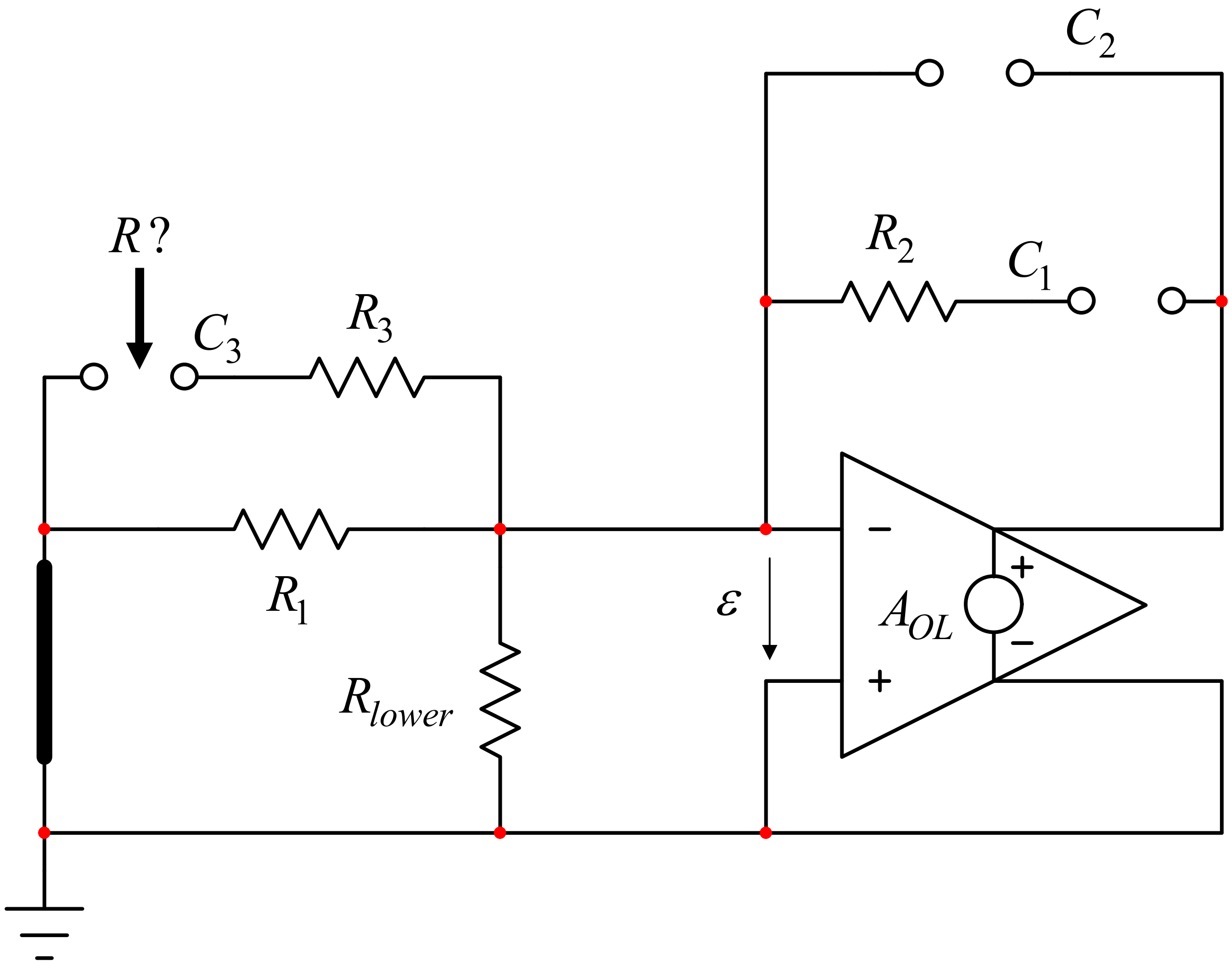


$$H_0 = -\frac{R_{lower}}{R_{lower} + R_1} A_{OL}$$

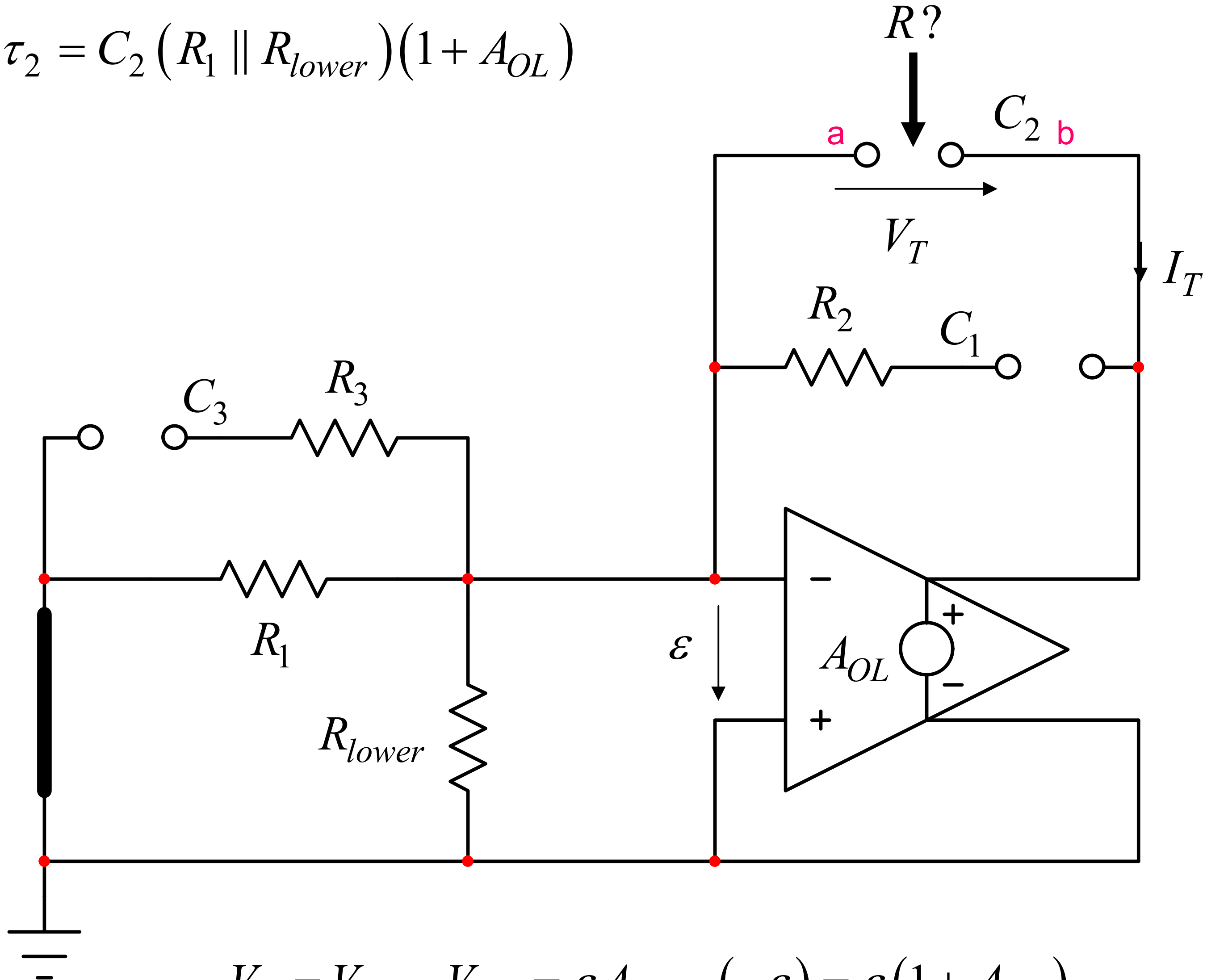
Reduce the excitation (V_{in}) to 0 V and determine the resistances seen from the capacitors in this mode.



$$\tau_3 = C_3\left(R_3 + R_1 \,||\, R_{lower}\right)$$

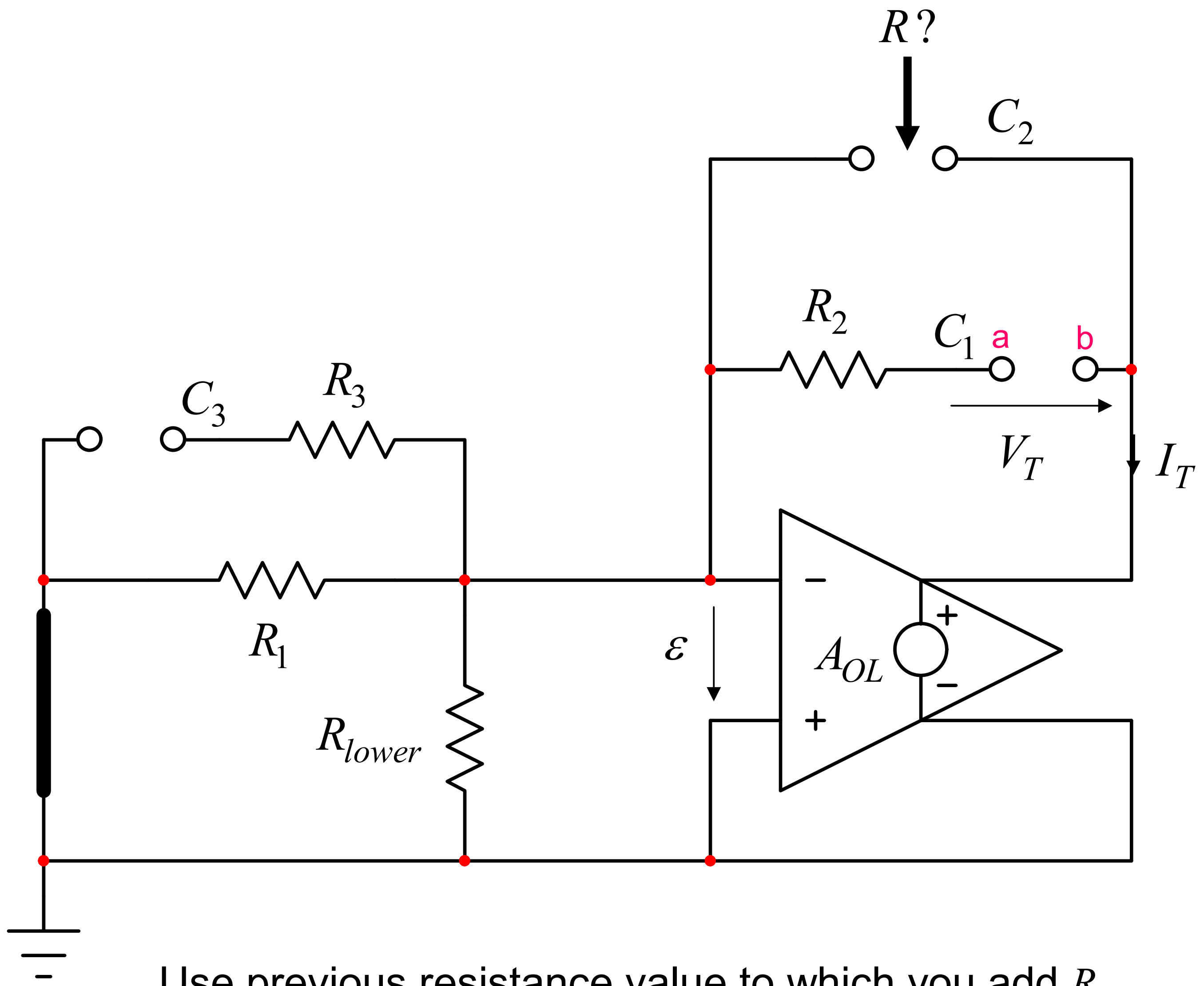


$$\tau_2 = C_2 \left(R_1 \parallel R_{lower}\right)\left(1 + A_{OL}\right)$$



$$V_T = V_{(b)} - V_{(a)} = \varepsilon A_{OL} - (-\varepsilon) = \varepsilon\left(1 + A_{OL}\right)$$

$$\varepsilon = I_T \left(R_1 \parallel R_{lower}\right)$$



Use previous resistance value to which you add R_2

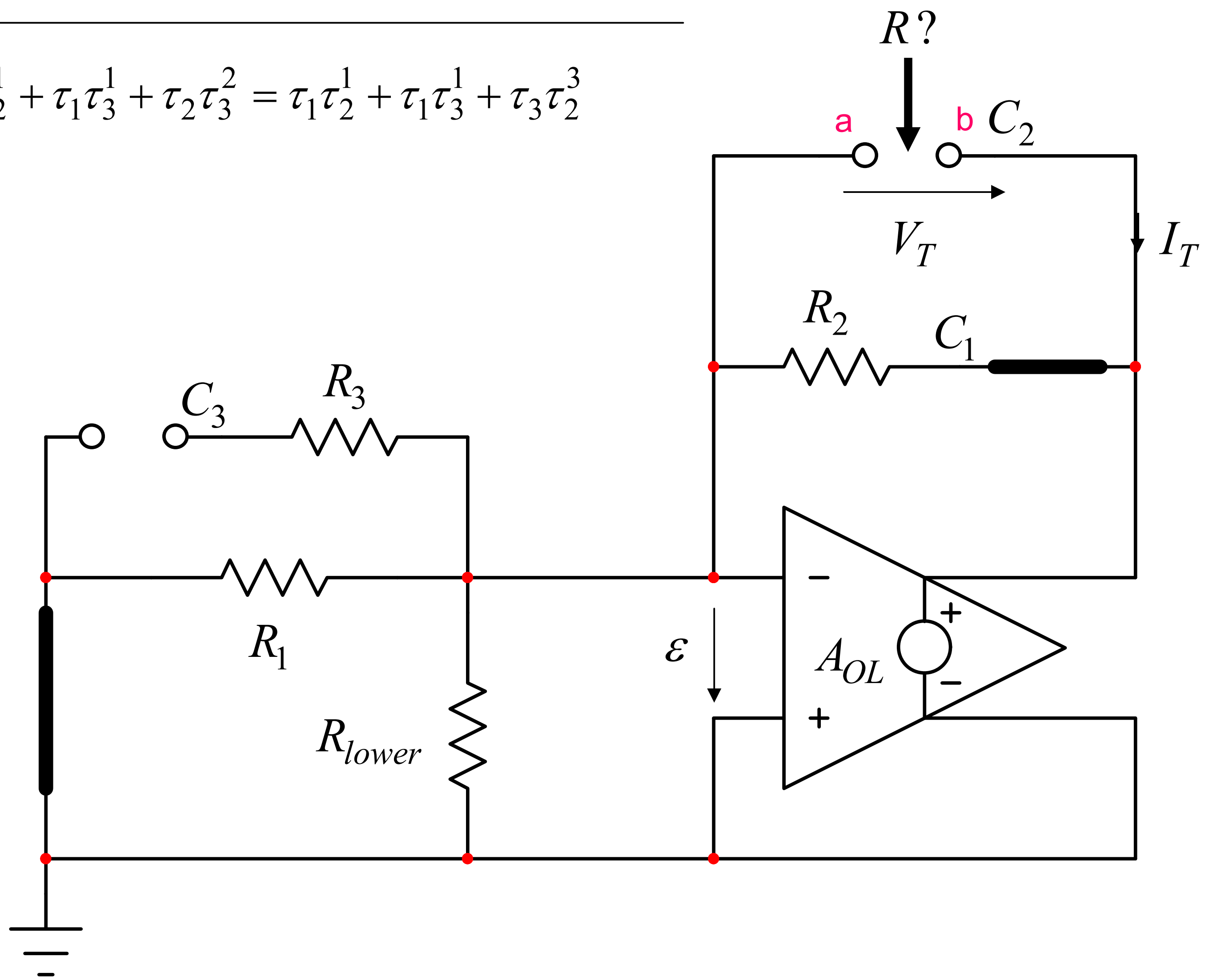
$$\tau_1 = C_1\left[\left(R_1 \parallel R_{lower}\right)\left(1 + A_{OL}\right) + R_2\right]$$

You can form b_1 for the denominator

$$b_1 = \tau_1 + \tau_2 + \tau_3$$

$$= C_1\left[\left(R_1 \parallel R_{lower}\right)\left(1 + A_{OL}\right) + R_2\right] + C_2\left(R_1 \parallel R_{lower}\right)\left(1 + A_{OL}\right) + C_3\left(R_3 + R_1 \parallel R_{lower}\right)$$

$$b_2 = \tau_1\tau_2^1 + \tau_1\tau_3^1 + \tau_2\tau_3^2 = \tau_1\tau_2^1 + \tau_1\tau_3^1 + \tau_3\tau_2^3$$



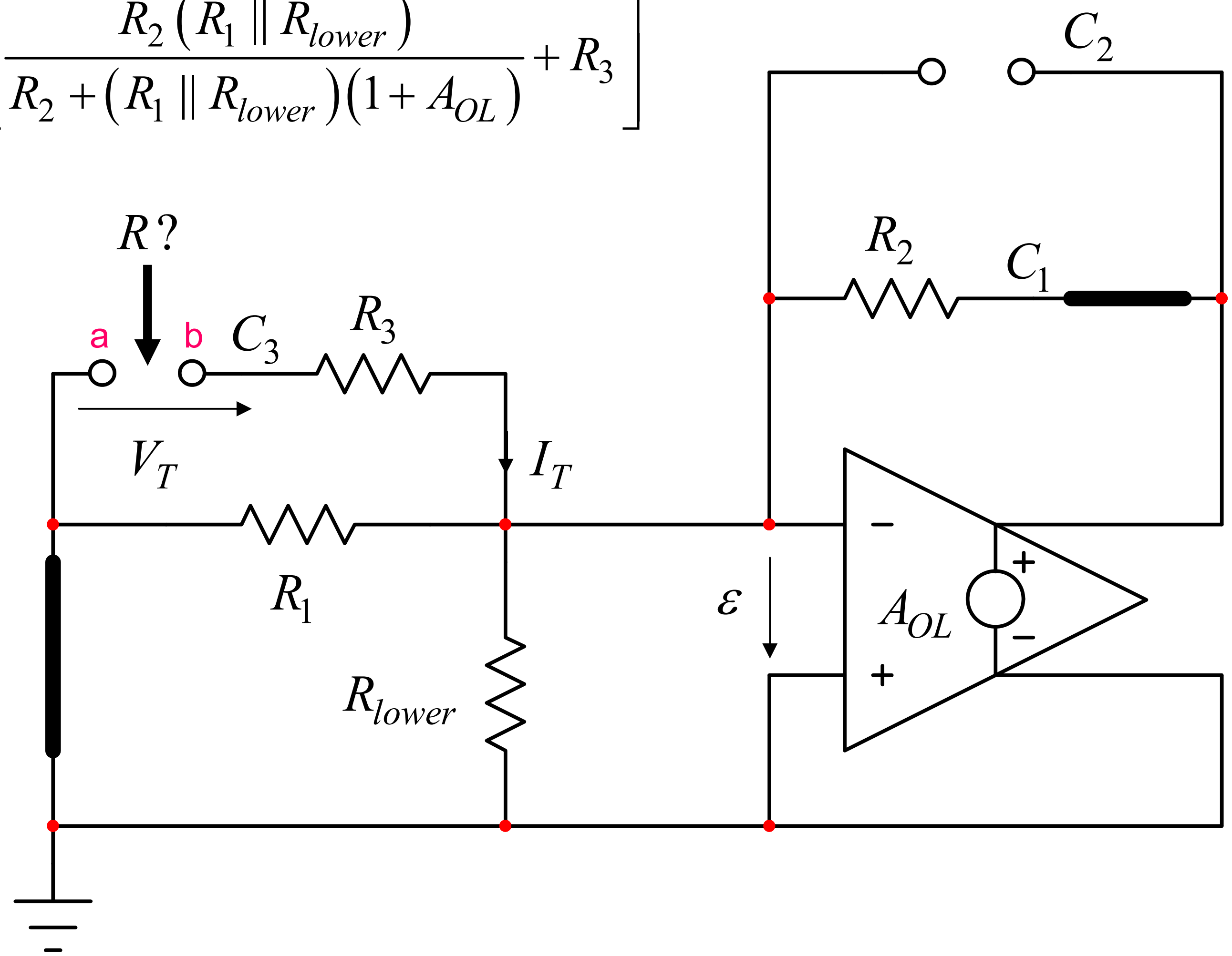
τ_2^1 ← short ← R?

$$V_T = V_{(b)} - V_{(a)} = \varepsilon A_{OL} - (-\varepsilon) = \varepsilon(1 + A_{OL})$$

$$\varepsilon = \left(I_T - \frac{V_T}{R_2}\right)(R_1 \parallel R_{lower})$$

$$\tau_2^1 = C_2\left[\frac{R_2(R_1 \parallel R_{lower})(1 + A_{OL})}{R_2 + (R_1 \parallel R_{lower})(1 + A_{OL})}\right]$$

$$\tau_3^1 = C_3\left[\frac{R_2(R_1 \parallel R_{lower})}{R_2 + (R_1 \parallel R_{lower})(1 + A_{OL})} + R_3\right]$$

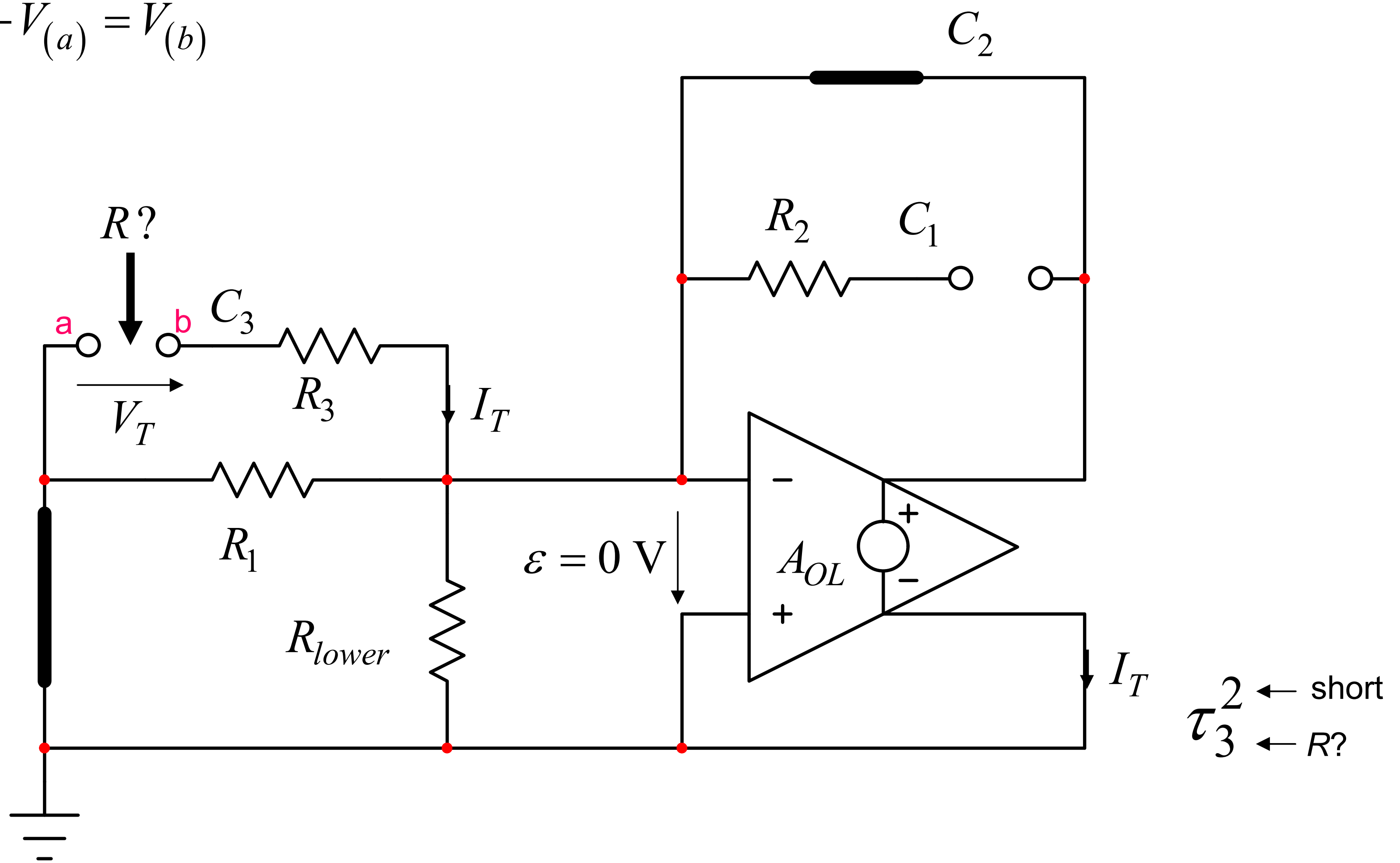


τ_3^1 ← short
← R?

In this configuration, IT crosses the op amp but not R_1 and R_{lower}.

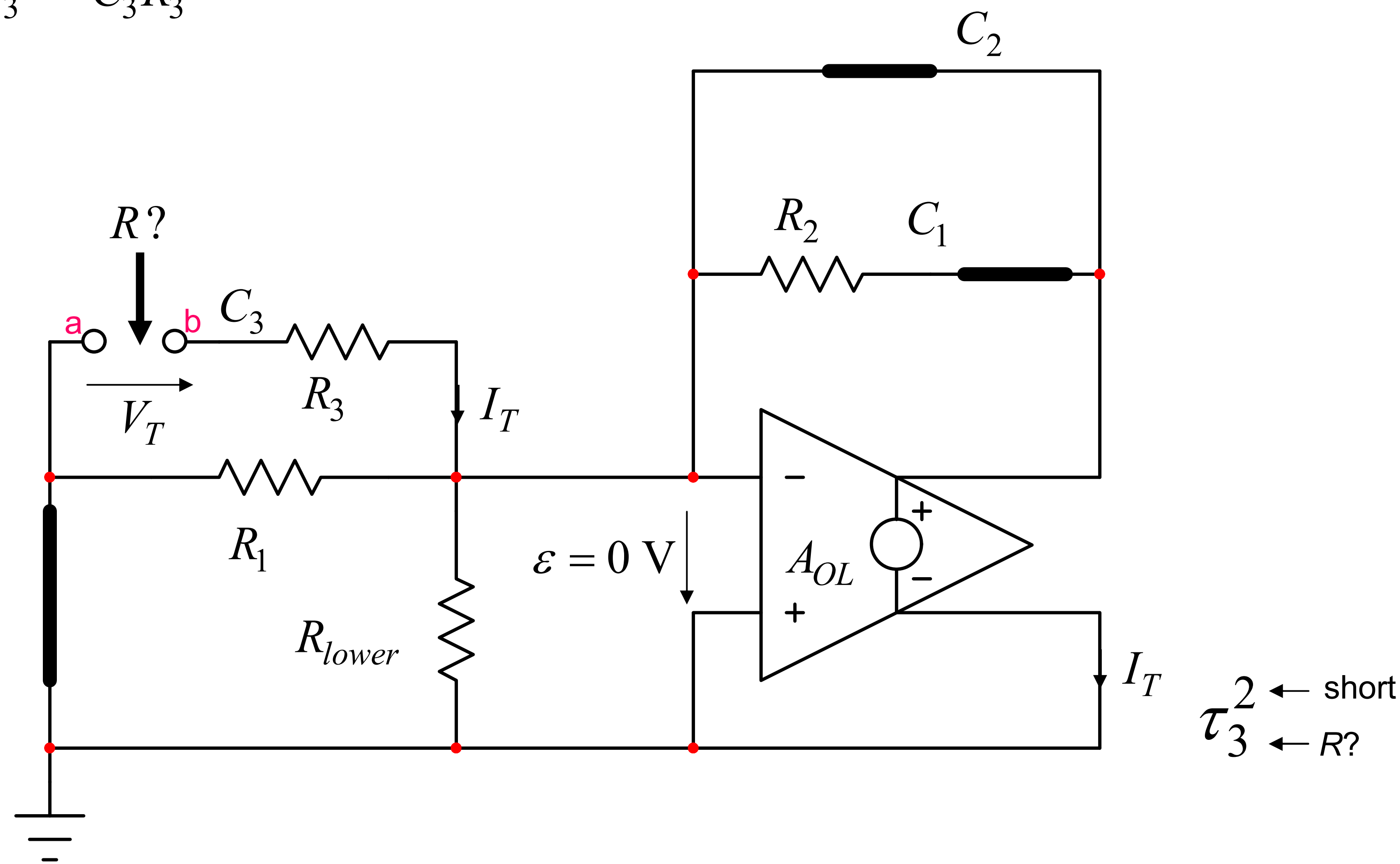
$$V_T = V_{(b)} - V_{(a)} = V_{(b)}$$

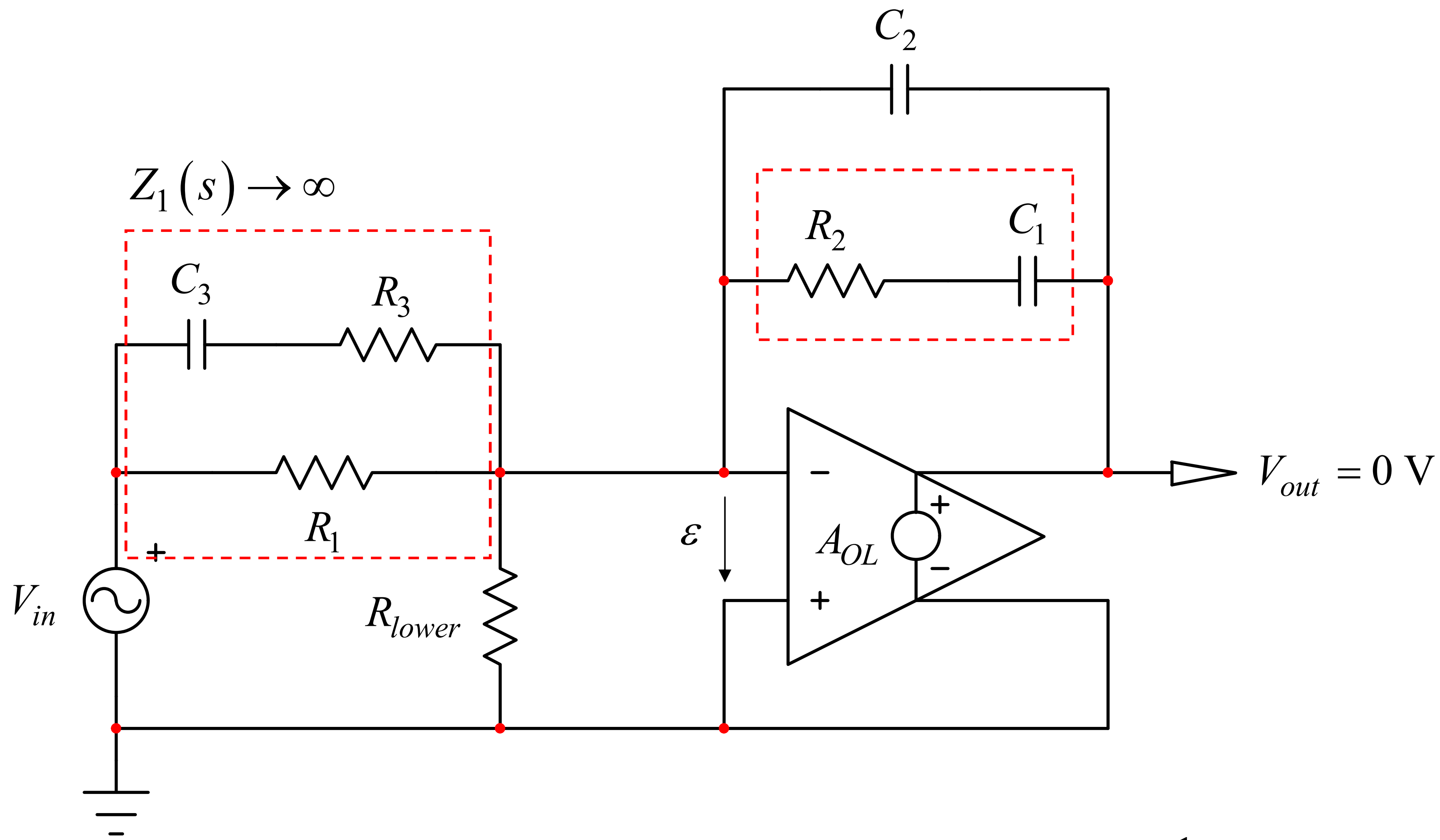
$$\tau_3^2 = C_3 R_3$$



$$b_3 = \tau_1 \tau_2^1 \tau_3^{12}$$

$$\tau_3^{12} = C_3 R_3$$





$$\left.\begin{array}{l} Z_1(s) \to \infty \\ \dfrac{N(s)}{D(s)} \to \infty \\ D(s) = 0 \end{array}\right\}$$

$$D(s) = 1 + sC_3(R_3 + R_1)$$

$$\Downarrow$$

$$\omega_{z_1} = \frac{1}{C_3(R_3 + R_1)}$$

$$Z_2(s) = 0 \to R_2 + \frac{1}{sC_1} = 0$$

$$\Downarrow$$

$$\omega_{z_2} = \frac{1}{C_1 R_2}$$

The final expressions is thus:

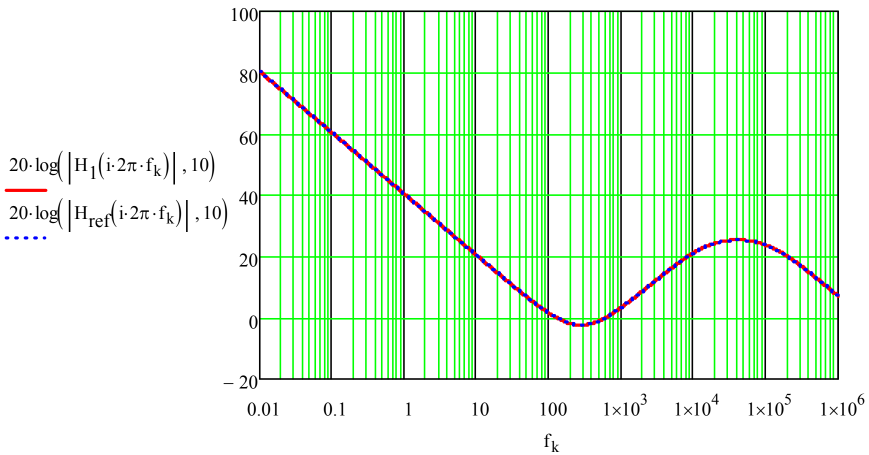
$$H(s) = H_0 \frac{\left(1+\frac{s}{\omega_{z_1}}\right)\left(1+\frac{s}{\omega_{z_2}}\right)}{1+b_1 s + b_2 s^2 + b_3 s^3}$$

$$H_0 = -\frac{R_{lower}}{R_{lower}+R_1} A_{OL}$$

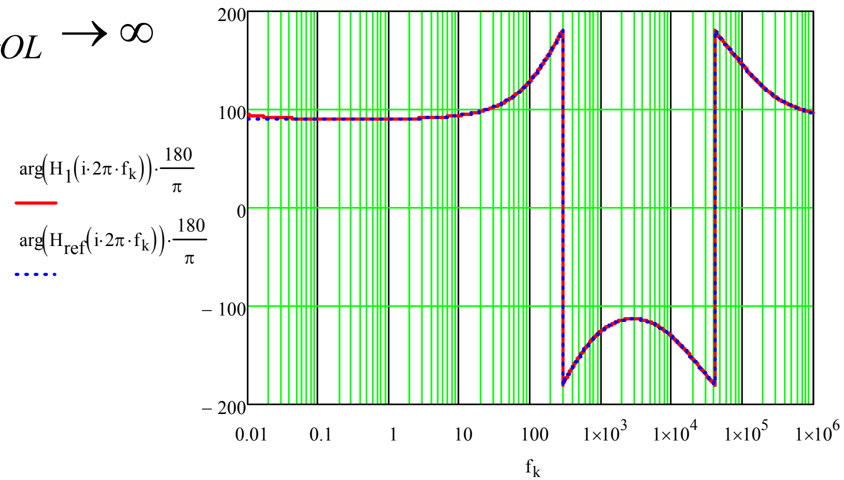
$$\omega_{z_2} = \frac{1}{C_1 R_2} \qquad \omega_{z_1} = \frac{1}{C_3\left(R_3+R_1\right)}$$

The simplified expressions is:

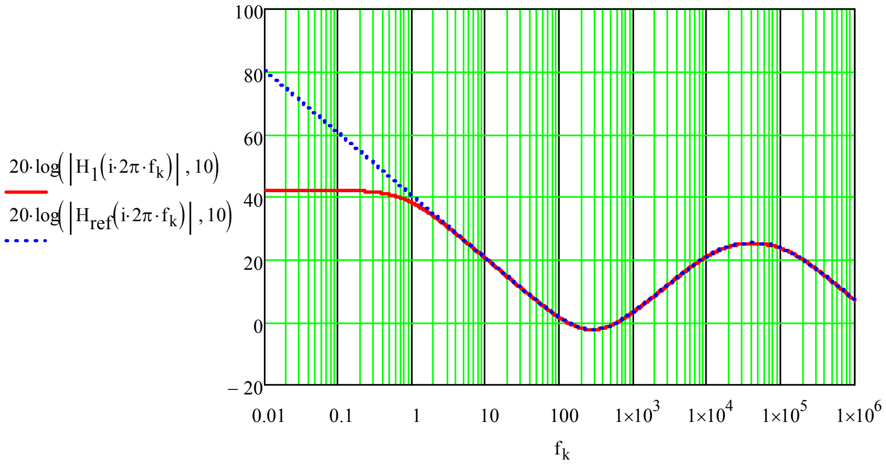
$$H(s) = -\frac{R_2 C_1}{R_1\left(C_1+C_2\right)} \frac{1+\frac{1}{sR_2C_1}}{1+sR_2\frac{C_1C_2}{C_1+C_2}} \frac{1+sC_3\left(R_1+R_3\right)}{1+sR_3C_3}$$

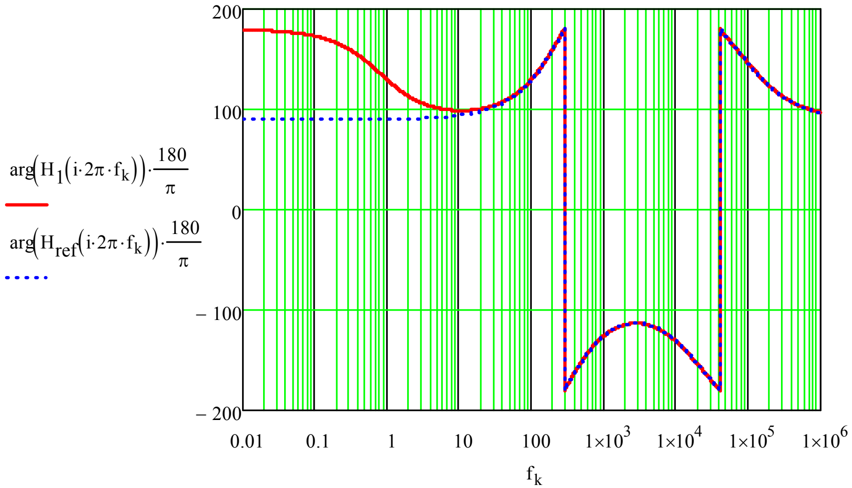


$$A_{OL} \to \infty$$



$$A_{OL} = 60\ \text{dB}$$





Considering the open-loop gain helps figuring out the role of the resistive network which, in the simplified analysis, does not play a role.

$$H(s) = H_0 \frac{\left(1+\frac{s}{\omega_{z_1}}\right)\left(1+\frac{s}{\omega_{z_2}}\right)}{1+b_1 s+b_2 s^2+b_3 s^3} \approx -\frac{\frac{R_{lower}}{R_{lower}+R_1} A_{OL}}{b_1 \omega_{z_1}} \frac{\left(1+\frac{\omega_{z_1}}{s}\right)\left(1+\frac{s}{\omega_{z_2}}\right)}{\left(1+\frac{s}{Q\omega_0}\right)\left(1+\frac{sQ}{\omega_o}\right)}$$

$$Q = \frac{\sqrt{b_1 b_3}}{b_2}$$

$$\omega_o = \sqrt{\frac{b_1}{b_3}}$$