# Introduction to Space Vector Modulation Terminology and Illustrations

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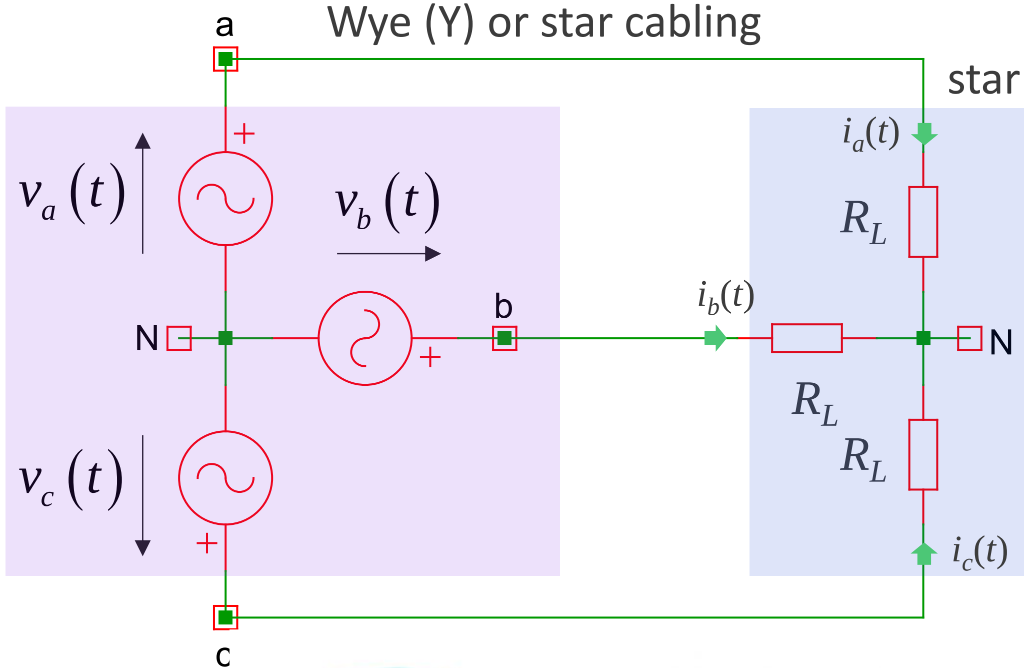
# Agenda

- Symmetrical Components
- Clarke Transform
- Park Transform
- Power Processing
- Power Factor Correction

# Three-Phase Generators

- A power plant delivers 3 voltages out of phase by 120° and referenced to a neutral point
- For a given wire gauge, more power is conveyed in a 3-phase than in a 1-phase network
- ✓ In a balanced system, there is no current flowing in the neutral wire


Wye (Y) or star cabling

Power plant

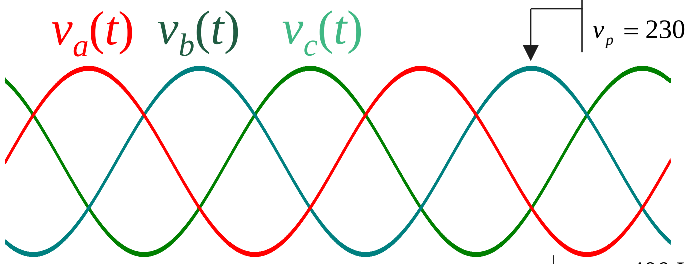
Mathworks video

3-phase load

Phase-neutral voltage

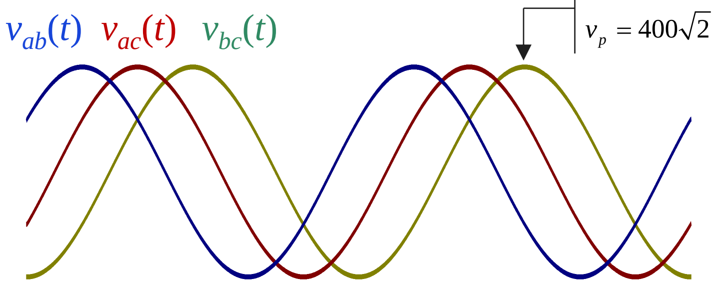
$v_{rms} = \frac{400\text{ V}}{\sqrt{3}} = 230\text{ V}$

$v_p = 230\sqrt{2} = 325\text{ V}$



$v_{rms} = 400\text{ V}$

$v_p = 400\sqrt{2} = 565\text{ V}$



Inter-phase voltage

$v_a(t) = V_{AC}\cdot\sqrt{2}\cdot\sin(\omega\cdot t + \varphi_1)$ $\varphi_1 = 0°$

$v_b(t) = V_{AC}\cdot\sqrt{2}\cdot\sin(\omega\cdot t + \varphi_2)$ $\varphi_2 = -120°$

$v_c(t) = V_{AC}\cdot\sqrt{2}\cdot\sin(\omega\cdot t + \varphi_3)$ $\varphi_3 = -240°$

Balanced system

$$\sum_{k \in \{a,b,c\}} v_k(t) = 0 \text{ and } \sum_{k \in \{a,b,c\}} i_k(t) = 0$$

$R_L := 50\Omega$

$P_L := \frac{V_{AC}^2}{R_L} = 1.058\text{kW}$

Total power is 3 times the power per phase

$$P_{3tot}(t) := \frac{v_a(t)^2}{R_L} + \frac{v_b(t)^2}{R_L} + \frac{v_c(t)^2}{R_L}$$

$$P_{avg} := F_{line}\cdot\int_0^{T_{line}} P_{3tot}(t)\,dt = 3.174\text{kW}$$

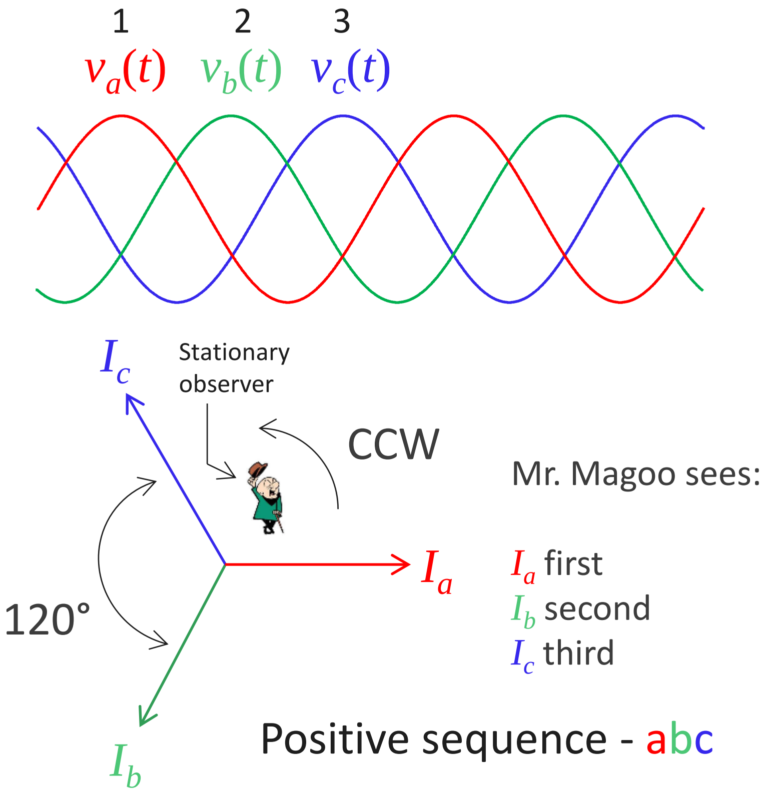
# A Balanced System

- A tri-phase system is balanced when the three conditions below are respected:
- ✓ Angle: there is a constant 120° angle between each of the sinusoidal waveforms
- ✓ All phase currents are of equal magnitude: $\overline{I}_a = \overline{I}_b = \overline{I}_c$
- ✓ The phase sequence is positive with A, B and C or negative with A, C and B

Time-domain representation

Phasor representation

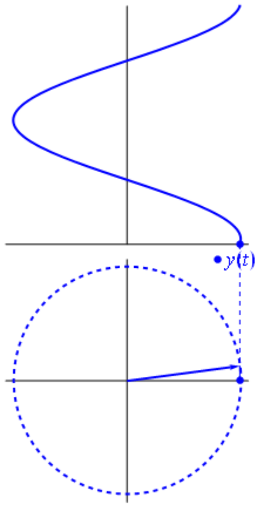
$I \cdot e^{j(\omega t+\theta)}$



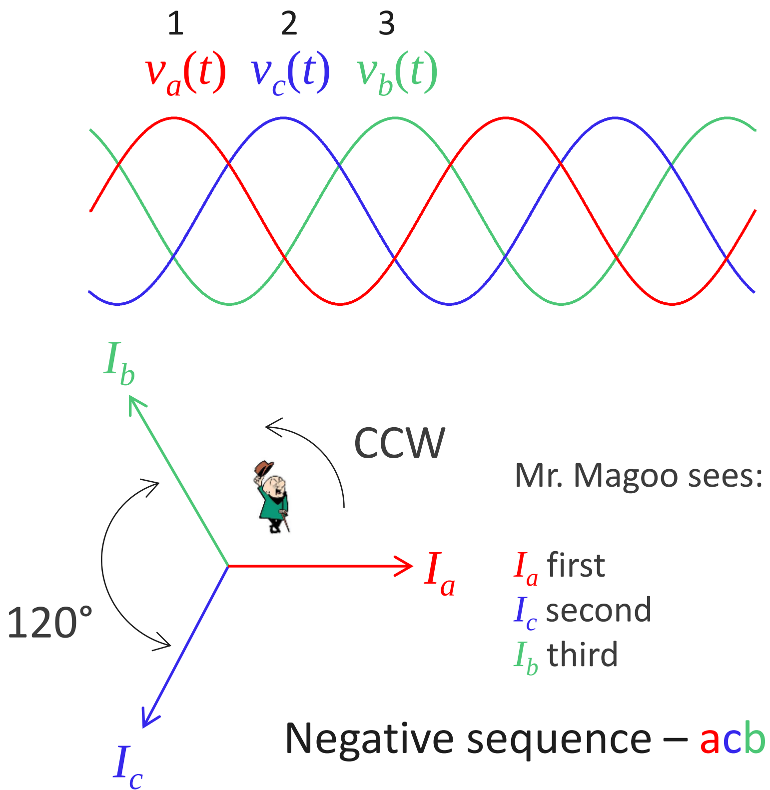
Mr. Magoo sees:

$I_a$ first
$I_b$ second
$I_c$ third

Positive sequence - abc



Wikipedia



Mr. Magoo sees:

$I_a$ first
$I_c$ second
$I_b$ third

Negative sequence – acb

Principle of Symmetrical Components - YouTube

# An Unbalanced System

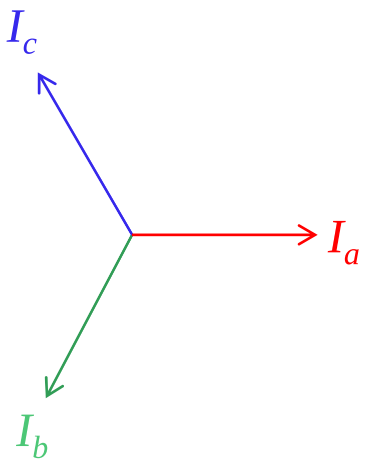
- A system becomes unbalanced or unsymmetrical if:

✓ The output impedances of the distribution networks are unbalanced

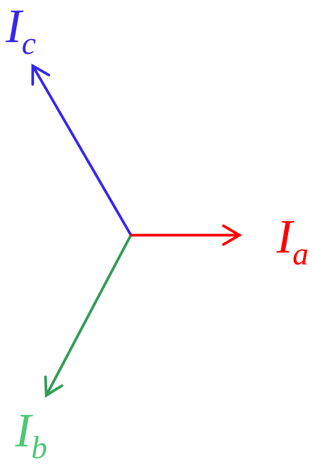
✓ A single- or a two-phase loads exhibit unequal consumptions

✓ Highly nonlinear loads are powered from the network: distorted currents

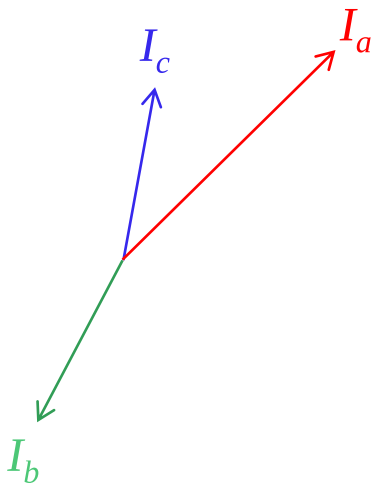
✓ Single phasing occurs in a motor: one of the supply lines opens



Balanced
Symmetrical



Unbalanced
Unsymmetrical

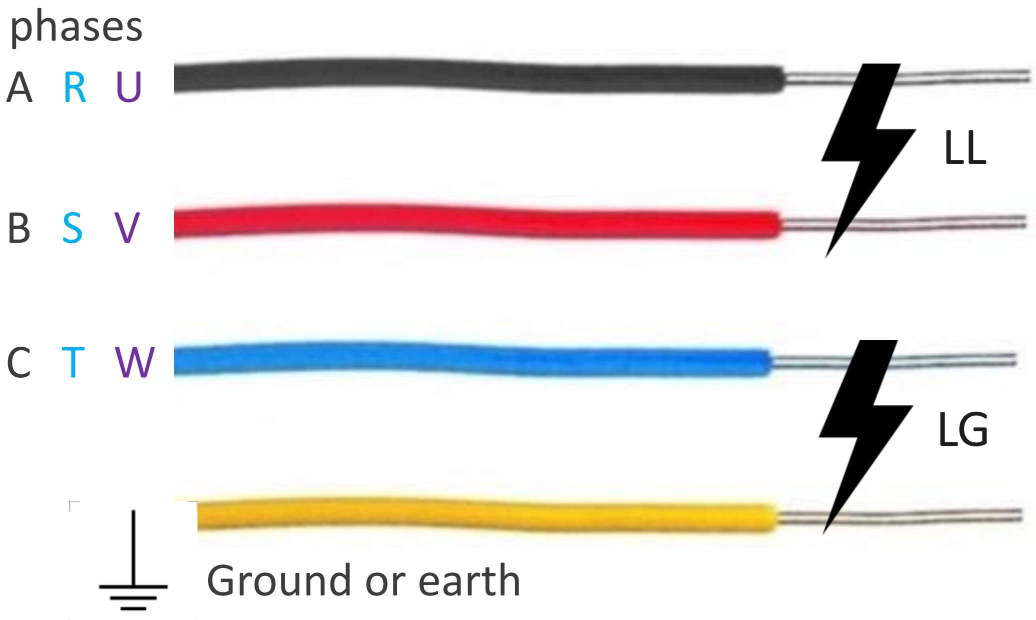


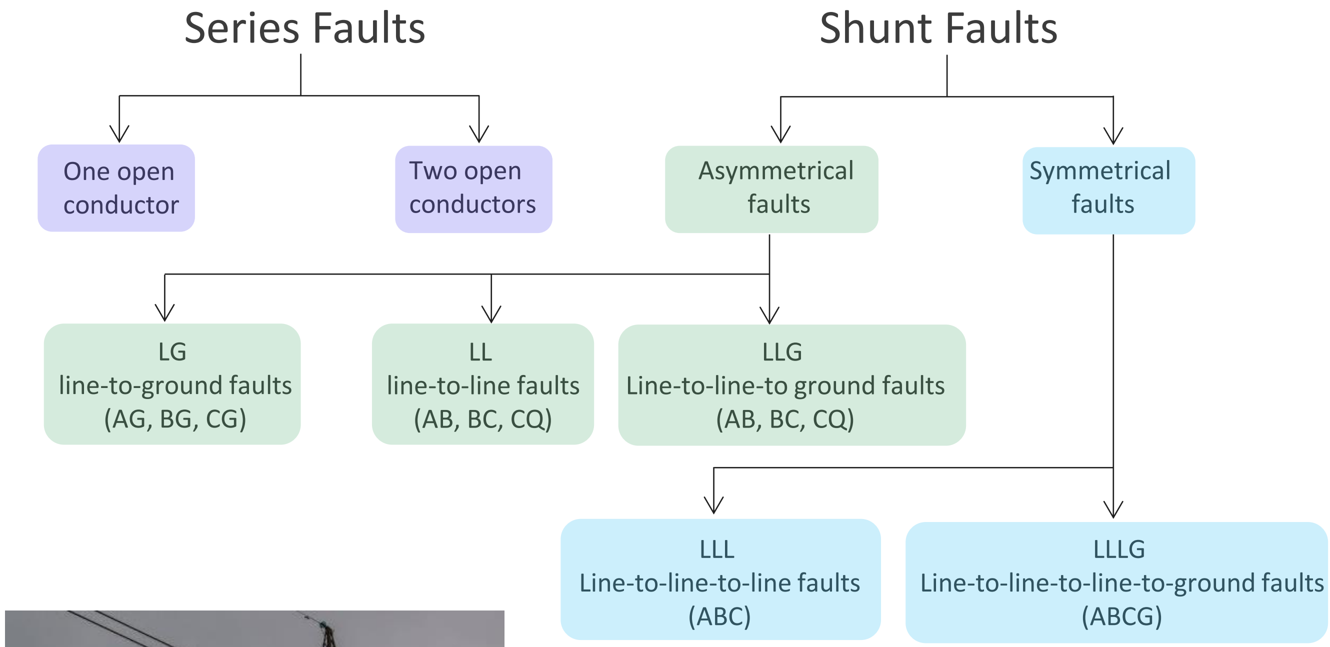
Unbalanced
Unsymmetrical

❖ Unbalanced operation leads to negative sequence in 3-phase motors

# Faults in a Three-Phase System

- During operations, short circuits can occur on distribution lines
- It is important for utility companies to understand what the type of short circuits:
  - ➢ Line-to-line, line-to-ground, line-to-line-to-ground and so on





Series Faults

- One open conductor
- Two open conductors

Shunt Faults

- Asymmetrical faults
  - LG line-to-ground faults (AG, BG, CG)
  - LL line-to-line faults (AB, BC, CQ)
  - LLG Line-to-line-to ground faults (AB, BC, CQ)
- Symmetrical faults
  - LLL Line-to-line-to-line faults (ABC)
  - LLLG Line-to-line-to-line-to-ground faults (ABCG)

How to determine circulating currents in a faulty 3-phase system?

Very Dangerous Electrical Short Circuit Top Five

# Symmetrical Components

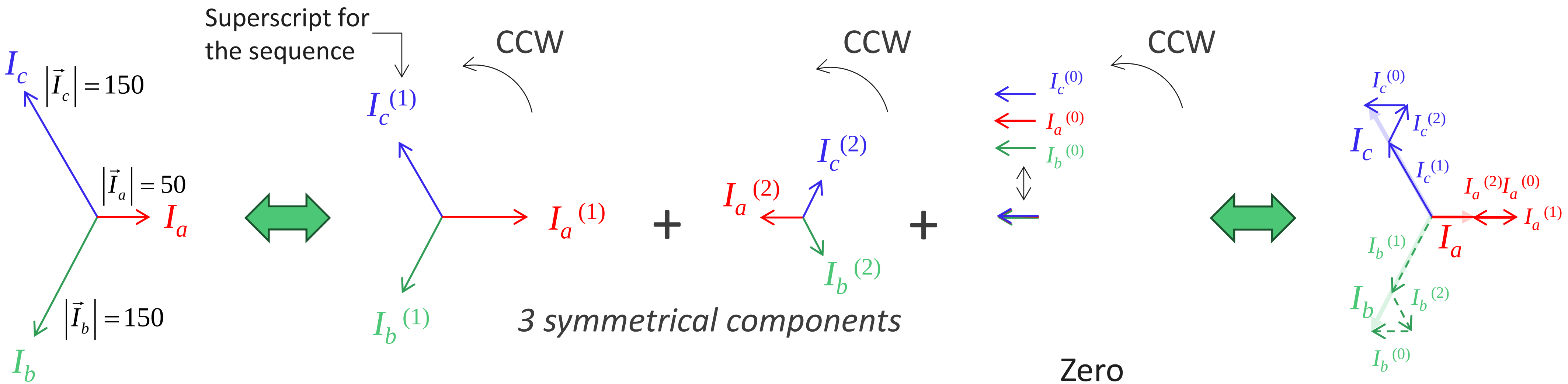
- It is possible to show that a set of 3 unbalanced phasors can be expressed as the sum of:

✓ A set of three *positive-sequence* vectors

✓ A set of three *negative-sequence* vectors

} **Symmetrical** components: same magnitude, constant 120° angle

✓ A set of three *zero-sequence* vectors → Same magnitude, all colinear



Unbalanced

Positive sequence $^{(1)}$

$$\vec{I}_a = 118.33\angle 359.84°$$
$$\vec{I}_b = 118.33\angle 239.84°$$
$$\vec{I}_c = 118.33\angle 119.84°$$

Magnitude ↑ ↑ Angle

Negative sequence $^{(2)}$

$$\vec{I}_a = 31.67\angle 180.6°$$
$$\vec{I}_b = 31.67\angle 300.6°$$
$$\vec{I}_c = 31.67\angle 60.6°$$

Zero Sequence $^{(0)}$

$$\vec{I}_a = 31.67\angle 180.6°$$
$$\vec{I}_b = 31.67\angle 180.6°$$
$$\vec{I}_c = 31.67\angle 180.6°$$

The vector-sum of the three sequences leads to the original system

CCW: counter clockwise

US POWER PARTNERS – Symmetrical Components Visualization

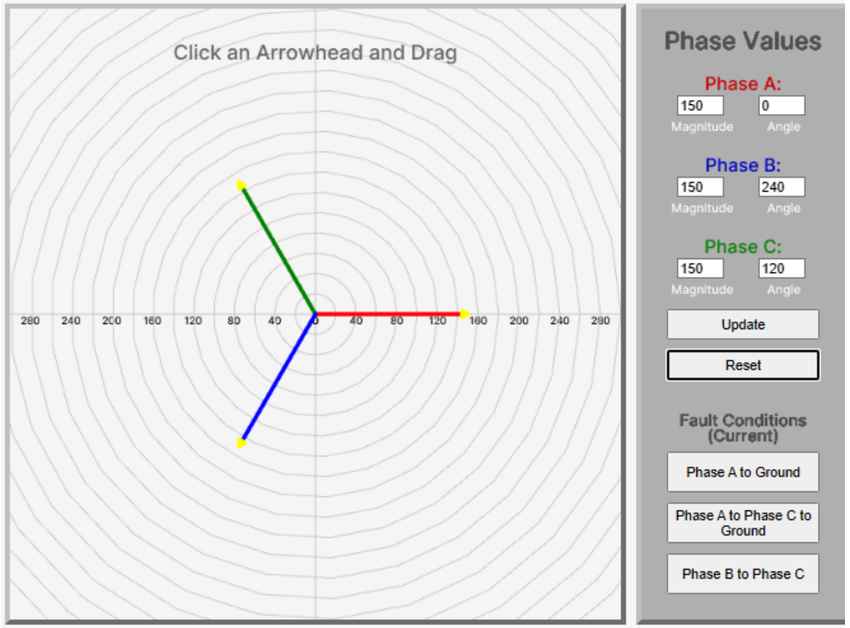
# Symmetrical Components in Absence of Fault

- In normal operation, when the system is balanced:

✓ The positive-sequence is the same as the original set of phasors

✓ The negative-sequence component is zero

✓ The zero-sequence component is also zero: $I^{(0)} = I_a + I_b + I_c = 0$



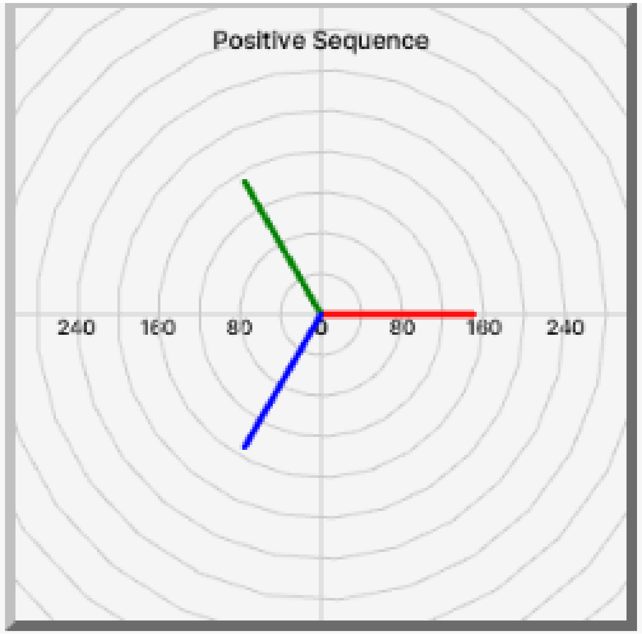
Phase Values

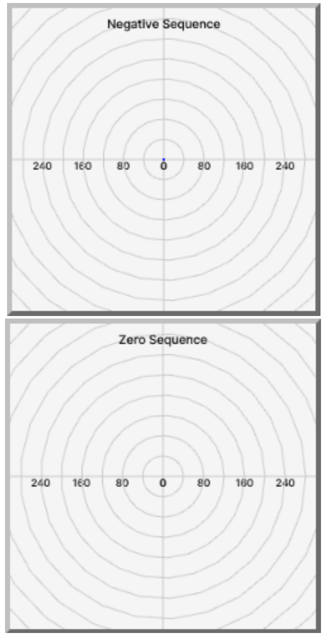
Phase A: Magnitude 150, Angle 0

Phase B: Magnitude 150, Angle 240

Phase C: Magnitude 150, Angle 120

Balanced system

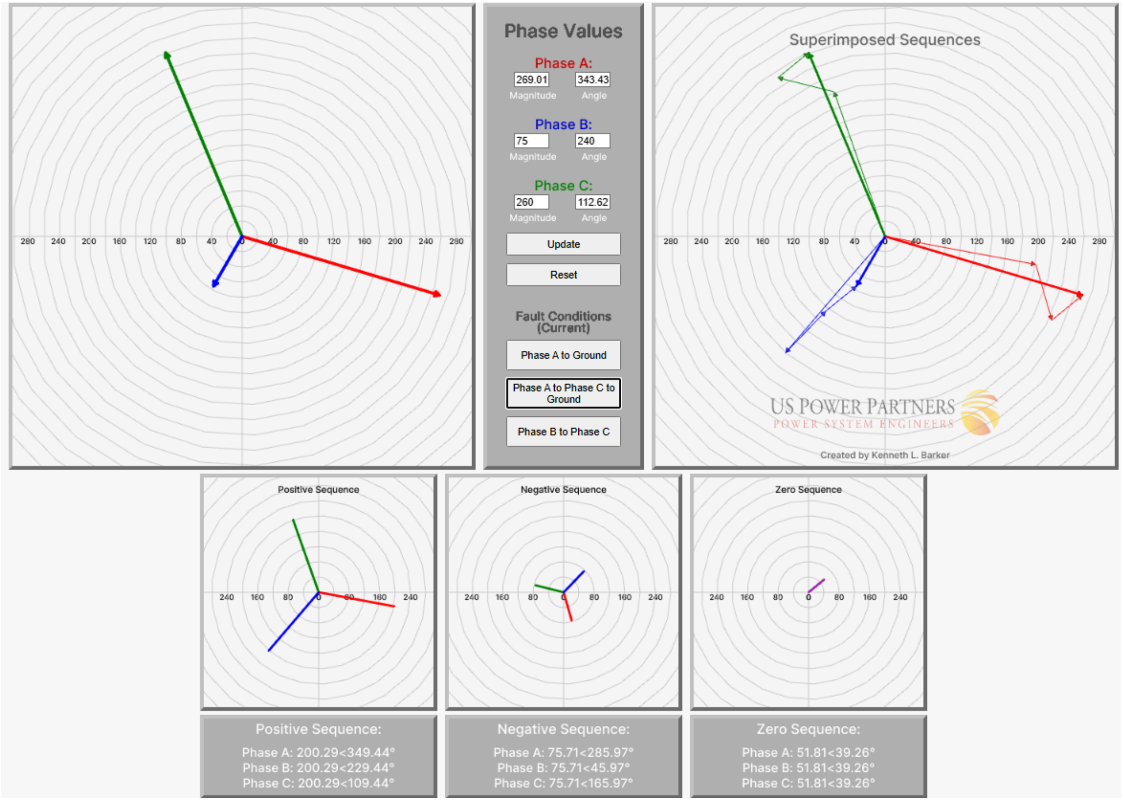




US POWER PARTNERS – Symmetrical Components Visualization

# Analyzing Fault Signatures

- With fault signature analysis, you can identify the issue and react accordingly



Phase Values

| | Magnitude | Angle |
|---|---|---|
| Phase A: | 269.01 | 343.43 |
| Phase B: | 75 | 240 |
| Phase C: | 260 | 112.62 |

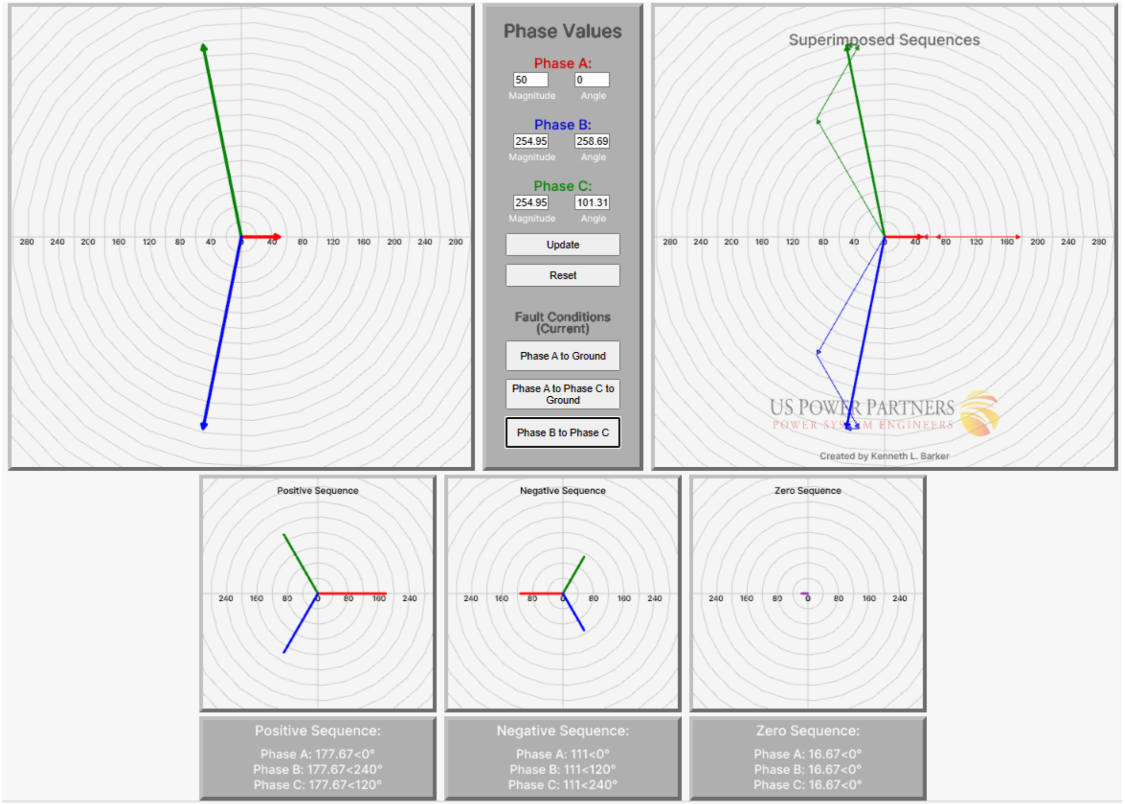
Fault Conditions (Current)

Positive Sequence:
Phase A: 200.29<349.44°
Phase B: 200.29<229.44°
Phase C: 200.29<109.44°

Negative Sequence:
Phase A: 75.71<285.97°
Phase B: 75.71<45.97°
Phase C: 75.71<165.97°

Zero Sequence:
Phase A: 51.81<39.26°
Phase B: 51.81<39.26°
Phase C: 51.81<39.26°

Phase A to phase C shorted to ground



Phase Values

| | Magnitude | Angle |
|---|---|---|
| Phase A: | 50 | 0 |
| Phase B: | 254.95 | 258.69 |
| Phase C: | 254.95 | 101.31 |

Fault Conditions (Current)

Positive Sequence:
Phase A: 177.67<0°
Phase B: 177.67<240°
Phase C: 177.67<120°

Negative Sequence:
Phase A: 111<0°
Phase B: 111<120°
Phase C: 111<240°

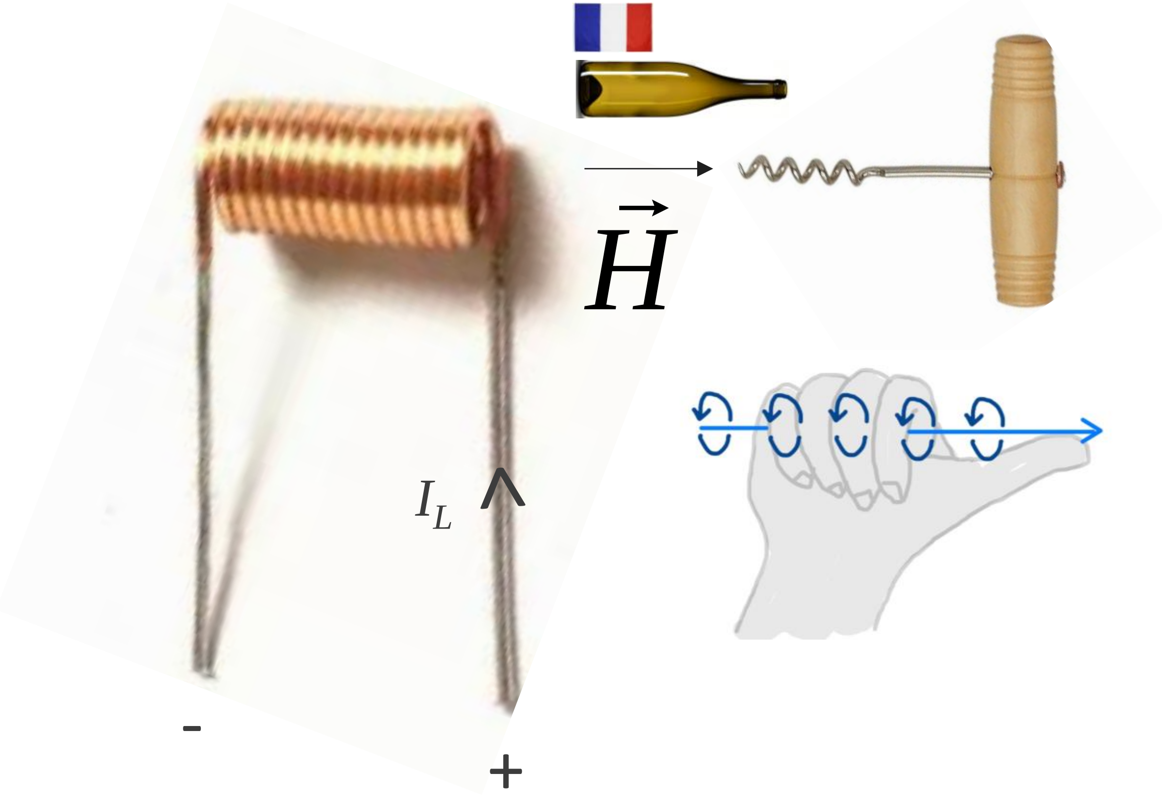
Zero Sequence:
Phase A: 16.67<0°
Phase B: 16.67<0°
Phase C: 16.67<0°

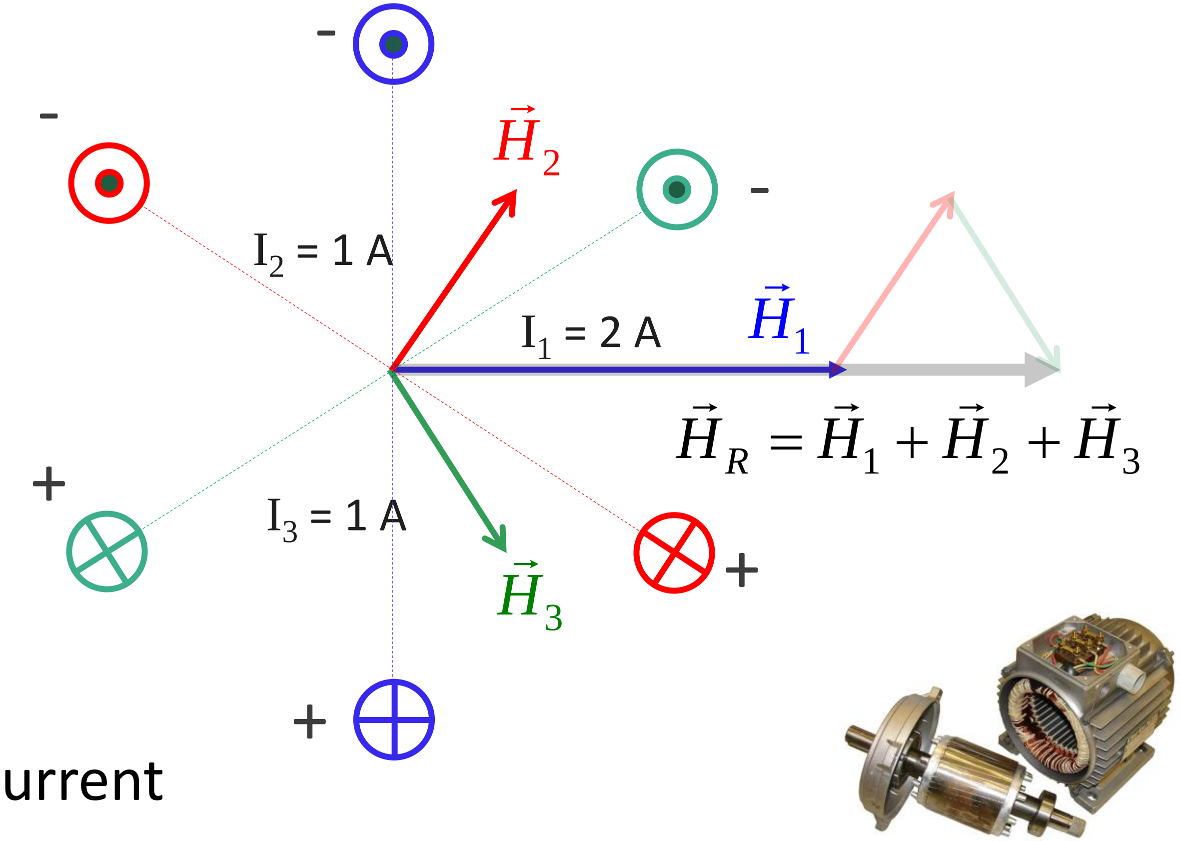
Phase B to phase C short circuit

US POWER PARTNERS – Symmetrical Components Visualization

# Magnetic Field

- When a coil is subject to a current, it produces a magnetic field
- The direction of the field is determined by the right-hand rule





- Consider three individual coils biased by a current

✓ The total magnetic field is the vector sum of the fields

Rotating Magnetic Fields – Prof. Essam Hamdi - YouTube

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- Park Transform
- Power Processing
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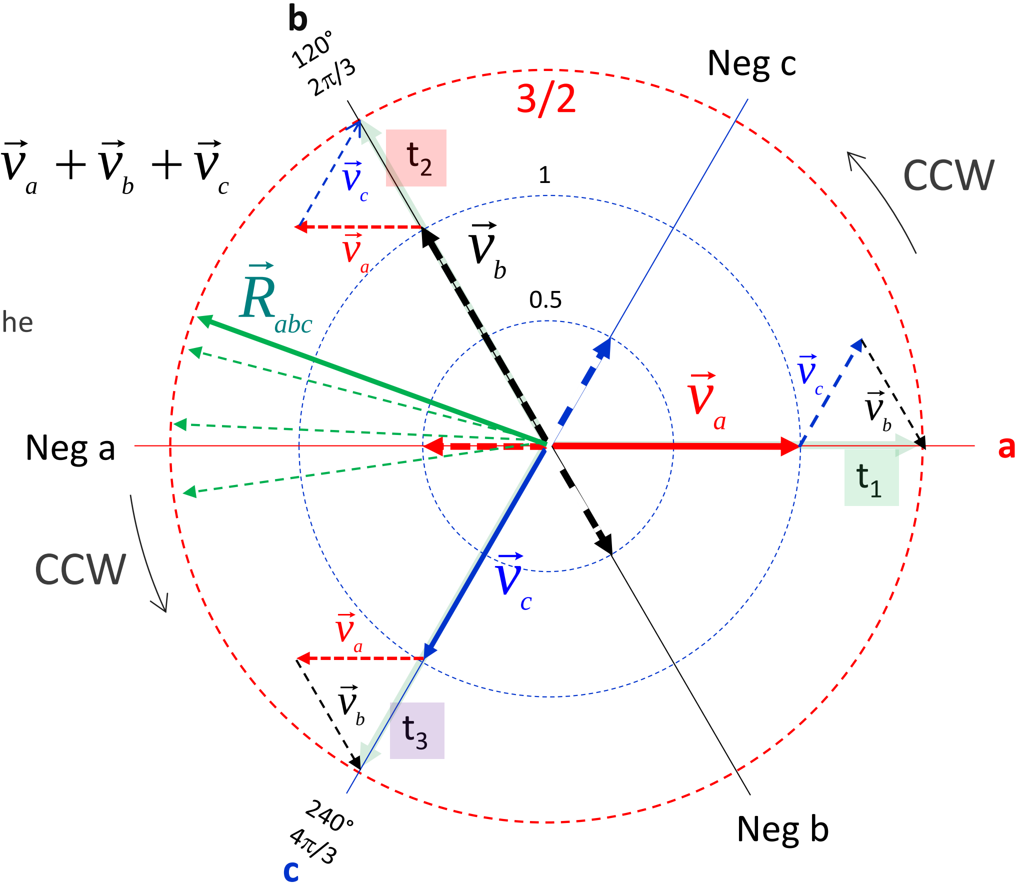
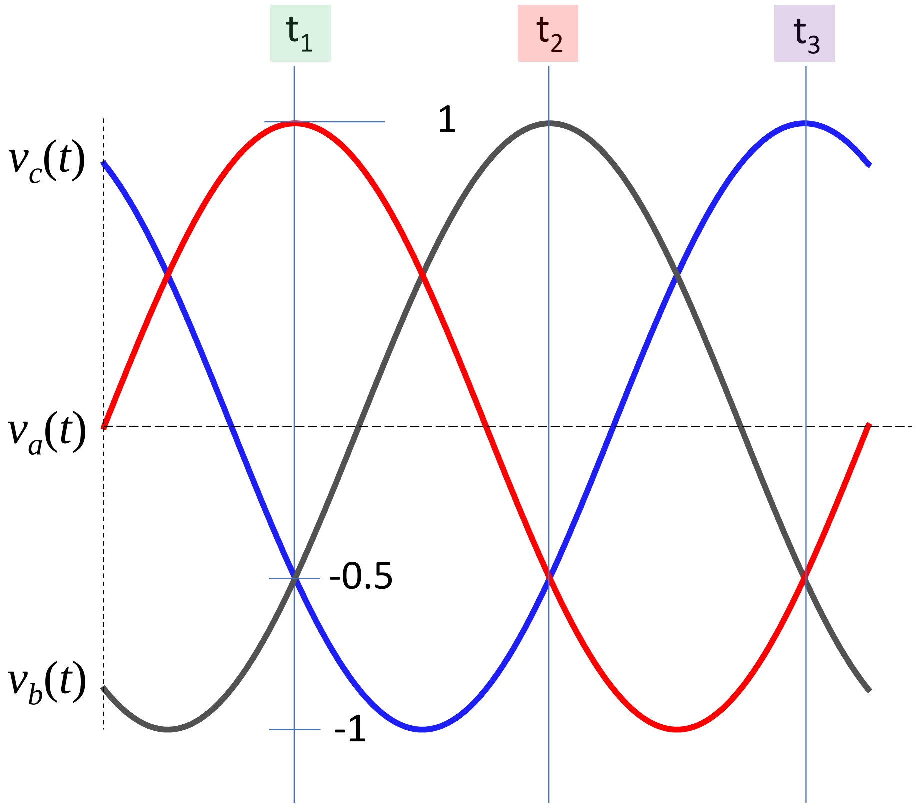
# A Rotating Field Vector

- As each phase current changes amplitude and angle, a rotating ***space*** vector appears
- The magnitude of this vector is 1.5 time the peak amplitude of the phases

$$\vec{R}_{abc} = \vec{v}_a + \vec{v}_b + \vec{v}_c$$

Rotating resulting vector in the *abc* frame





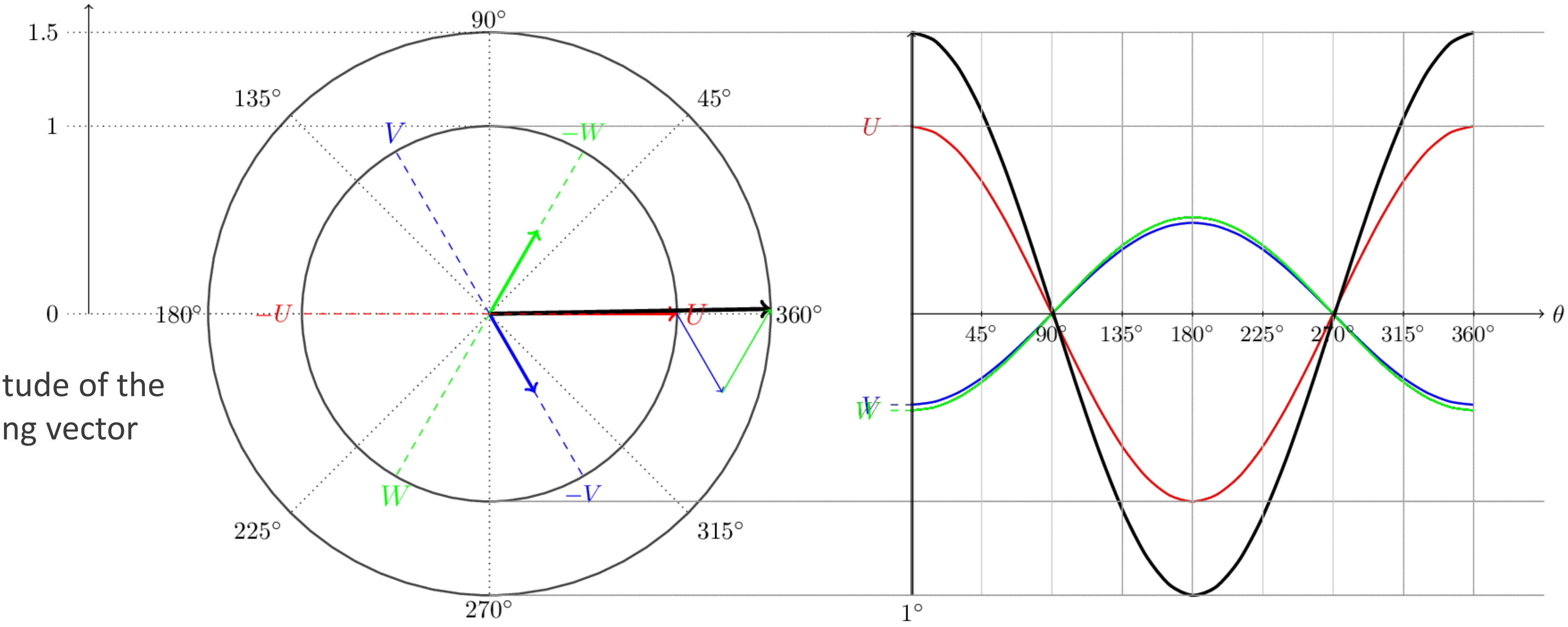
The amplitude of the vector varies with time.

Phase currents

- The resulting vector is made of three components, represented in the *abc* frame

# Animated Field

- The equivalent *space* vector rotates along the sinusoidal waveforms



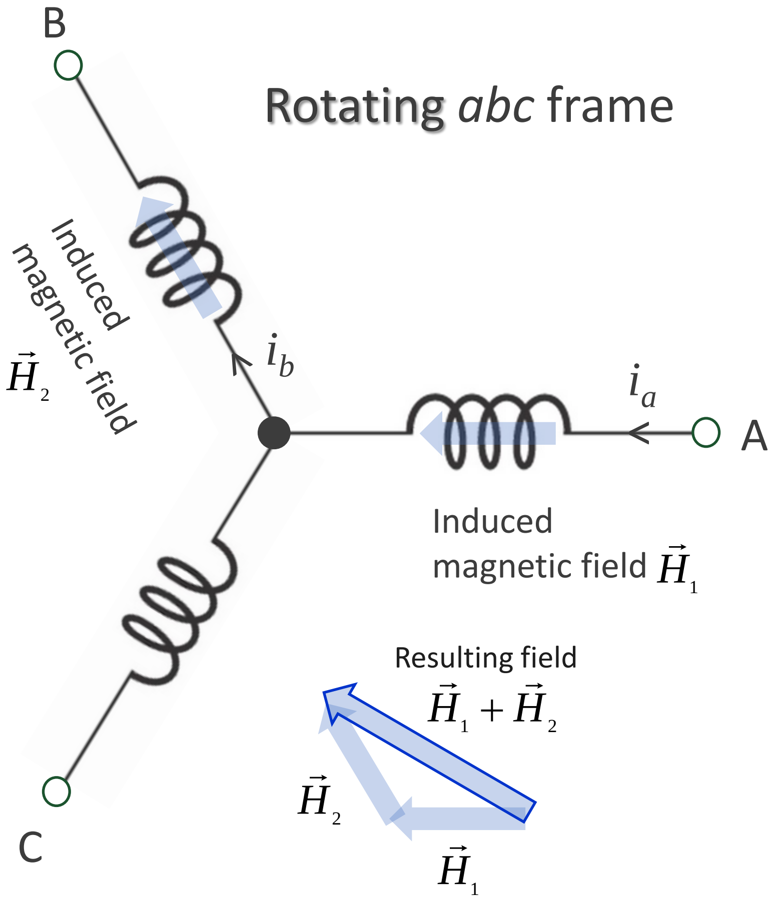
Magnitude of the resulting vector

Phase currents

https://www.switchcraft.org/learning/2017/3/15/space-vector-pwm-intro

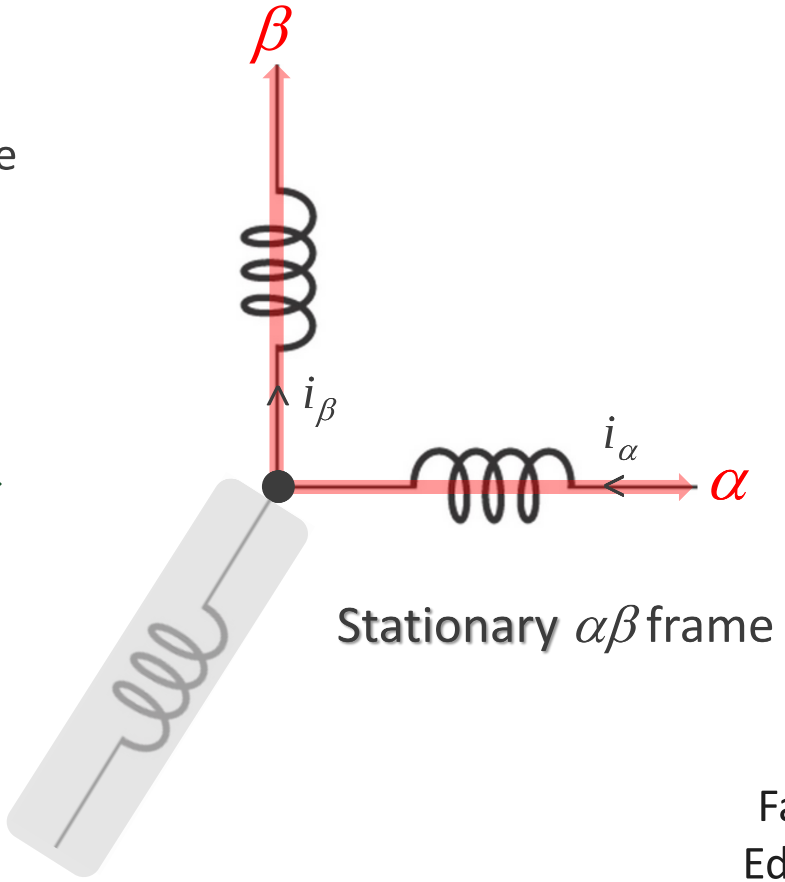
# Changing the Frame – *abc* to $\alpha\beta$

- The magnetic field is synthetized by three varying components: 3-dimension frame
- The idea is to go from a **three**-dimension (abc) to a **two**-dimension system (αβ)



Rotating *abc* frame

Edith Clarke
1884-1959



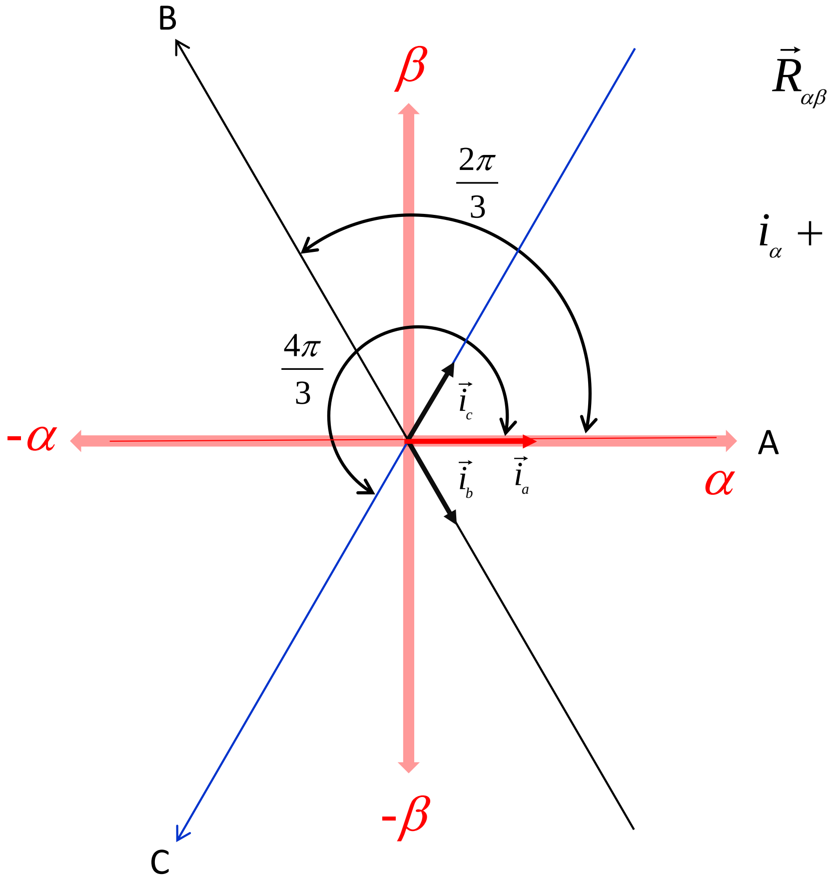
Stationary $\alpha\beta$ frame

Fast
Eddie
Clark
♠

C. O'Rourke, M. Qasim, M. Overlin, J. Kirtley, *A Geometric Interpretation of Reference Frames and Transformations*, IEEE, 2019

# Understanding the Transform

- You are projecting three vectors onto two axes, α and β
- Use Euler’s notation to write the sum in phasor notation and rearrange



$$\vec{R} = \vec{i}_a + \vec{i}_b + \vec{i}_c = i_a e^{j0} + i_b e^{j\frac{2\pi}{3}} + i_c e^{j\frac{4\pi}{3}} \longrightarrow Ae^{j\theta} = A(\cos\theta + j\sin\theta)$$

$$\vec{R}_{\alpha\beta} = \left[ i_a \cdot 1 + i_b \cos\left(\frac{2\pi}{3}\right) + i_c \cos\left(\frac{4\pi}{3}\right)\right] + j\left[ i_a \cdot \sin(0) + i_b \sin\left(\frac{2\pi}{3}\right) + i_c \sin\left(\frac{4\pi}{3}\right)\right]$$

$$i_\alpha + j \cdot i_\beta = \left[ i_a - 0.5 i_b - 0.5 i_c \right] + j\left[ 0 + \frac{\sqrt{3}}{2} i_b - \frac{\sqrt{3}}{2} i_c \right]$$

Real

Imaginary

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

direct

$$i_\alpha(t) = \frac{2}{3} i_a(t) - \frac{1}{3}\left[ i_b(t) + i_c(t) \right]$$

$$i_\beta = \frac{1}{\sqrt{3}} \left[ i_b(t) - i_c(t) \right]$$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

inverse

$$i_a(t) = i_\alpha(t)$$

$$i_b(t) = -\frac{1}{2} i_\alpha(t) + \frac{\sqrt{3}}{2} i_\beta(t)$$

$$i_c(t) = -\frac{1}{2} i_\alpha(t) - \frac{\sqrt{3}}{2} i_\beta(t)$$

Simplified expressions for a balanced system in which the 0-sequence is null.

# Power-Invariant Transform

- Clarke's transform preserves current and voltage values but not power
- Concordia's transform is similar to Clarke's but is now power-invariant

Direct

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

Inverse

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

Assume the following instantaneous values at *t* = 0

$$v_a = V_p \qquad i_a = I_p$$

$$v_b = -0.5V_p \quad i_b = -0.5I_p$$

$$v_c = -0.5V_p \quad i_c = -0.5I_p$$

$$\Downarrow$$

$$P = v_a i_a + v_b i_b + v_c i_c = \frac{3}{2} V_p I_p$$

Using Clarke's transform:

$$v_\alpha = V_p \qquad i_\alpha = I_p$$

$$v_\beta = 0 \quad i_\beta = 0$$

$$\Downarrow$$

wrong

$$P = v_\alpha i_\alpha + v_\beta i_\beta = V_p I_p$$

Using Concordia's transform:

$$v_\alpha = \sqrt{\frac{3}{2}} V_p \qquad i_\alpha = \sqrt{\frac{3}{2}} I_p$$

$$v_\beta = 0 \qquad i_\beta = 0$$

$$\Downarrow$$

$$P = v_\alpha i_\alpha + v_\beta i_\beta = \frac{3}{2} V_p I_p$$

Simplified expressions for a balanced system in which the 0-sequence is null.

General Theory of Electric Machines – Dr. A. Rahideh

# Simulation Waveforms

$V_{in} := 100\text{V} \qquad F_{line} := 50\text{Hz}$

$t := 10\mu\text{s}, 20\mu\text{s} .. 100\text{ms} \qquad T_{line} := \frac{1}{F_{line}}$

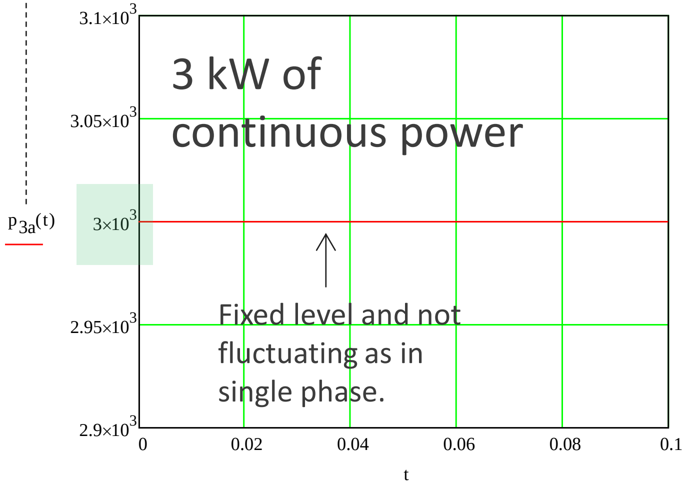
$\omega := 2\cdot\pi\cdot F_{line} \qquad R_L := 10\Omega$

$v_a(t) := V_{in}\cdot\sqrt{2}\cdot\sin(\omega\cdot t)$

$v_b(t) := V_{in}\cdot\sqrt{2}\cdot\sin(\omega\cdot t - 120°)$

$v_c(t) := V_{in}\cdot\sqrt{2}\cdot\sin(\omega\cdot t - 240°)$

$p_{3a}(t) := \frac{v_a(t)^2}{R_L} + \frac{v_b(t)^2}{R_L} + \frac{v_c(t)^2}{R_L}$ Instantaneous power



3 kW of continuous power

Fixed level and not fluctuating as in single phase.

$$V_\alpha(t) := \sqrt{\frac{2}{3}}\cdot\left(v_a(t) - \frac{1}{2}\cdot v_b(t) - \frac{1}{2}\cdot v_c(t)\right)$$

$$V_\beta(t) := \sqrt{\frac{2}{3}}\cdot\left(0\cdot v_a(t) + \frac{\sqrt{3}}{2}\cdot v_b(t) - \frac{\sqrt{3}}{2}\cdot v_c(t)\right)$$

$$V_0(t) := \sqrt{\frac{2}{3}}\cdot\left(\sqrt{\frac{1}{2}}\cdot v_a(t) + \sqrt{\frac{1}{2}}\cdot v_b(t) + \sqrt{\frac{1}{2}}\cdot v_c(t)\right)$$

$$MagV := \sqrt{\left(V_\alpha(5\text{ms})\right)^2 + \left(V_\beta(5\text{ms})\right)^2} = 173.205\,\text{V}$$

$$I_\alpha(t) := \sqrt{\frac{2}{3}}\cdot\left(i_a(t) - \frac{1}{2}\cdot i_b(t) - \frac{1}{2}\cdot i_c(t)\right)$$

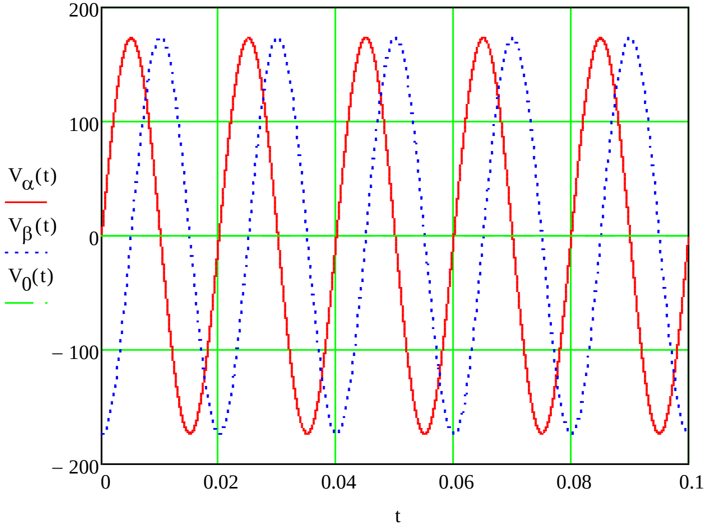
$$I_\beta(t) := \sqrt{\frac{2}{3}}\cdot\left(0\cdot i_a(t) + \frac{\sqrt{3}}{2}\cdot i_b(t) - \frac{\sqrt{3}}{2}\cdot i_c(t)\right)$$

$$I_0(t) := \sqrt{\frac{2}{3}}\cdot\left(\sqrt{\frac{1}{2}}\cdot i_a(t) + \sqrt{\frac{1}{2}}\cdot i_b(t) + \sqrt{\frac{1}{2}}\cdot i_c(t)\right)$$

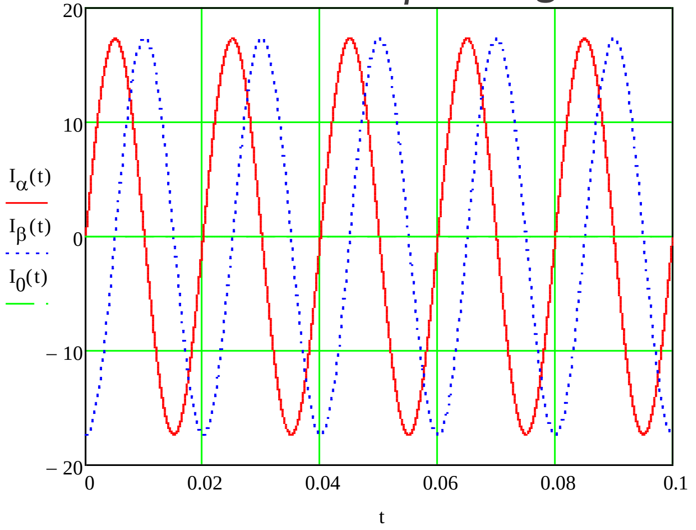
$$MagI := \sqrt{\left(I_\alpha(5\text{ms})\right)^2 + \left(I_\beta(5\text{ms})\right)^2} = 17.321\,\text{A}$$

$P_{INV} := MagI\cdot MagV = 3\,\text{kW}$ The computed averaged power is correct

Instantaneous αβ currents



Instantaneous αβ voltages



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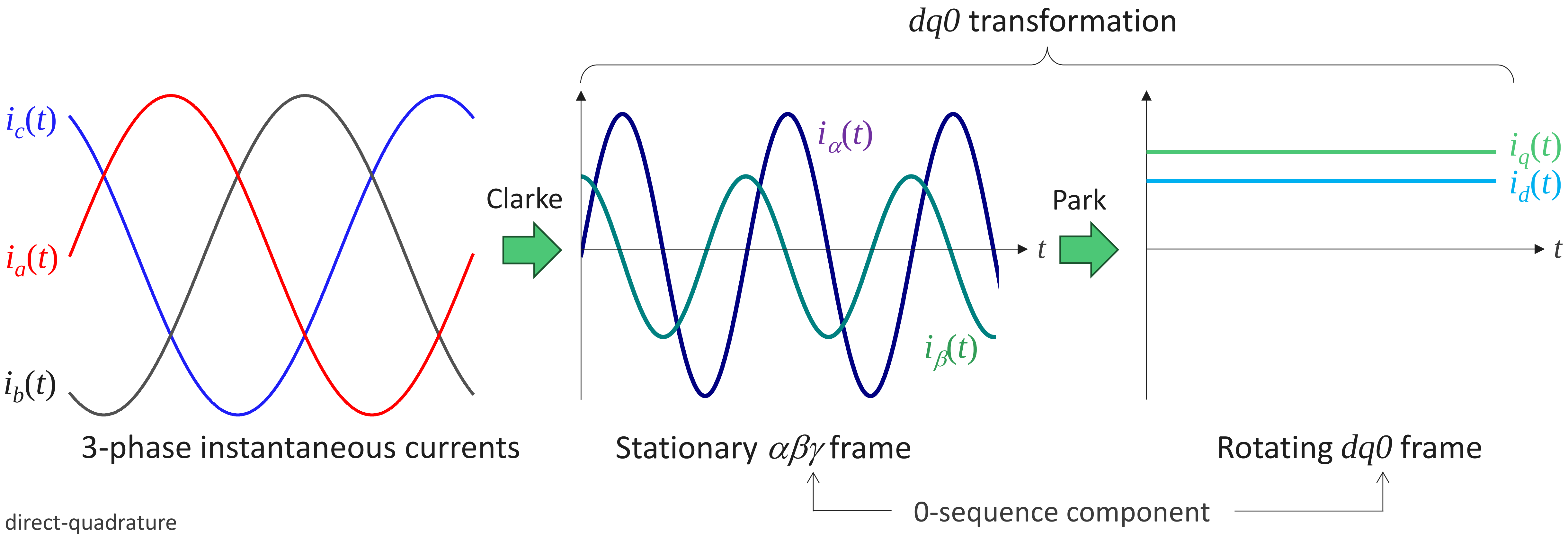
# The Park Transformation

- ■ Clarke’s transform turns a three-phase system into a two-phase reference frame
- ❖ The variables $\alpha$ and $\beta$ are still fluctuating: building a control system is difficult
- ✓ Park’s transform turns these two waveforms into two dc components, $d$ and $q$
- ➢ Implementing a control system with dc references now becomes easier

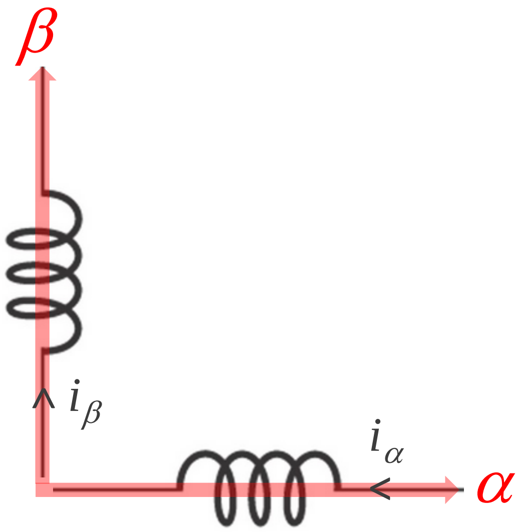


dq: direct-quadrature

# Changing the Frame – $\alpha\beta$ to $dq$

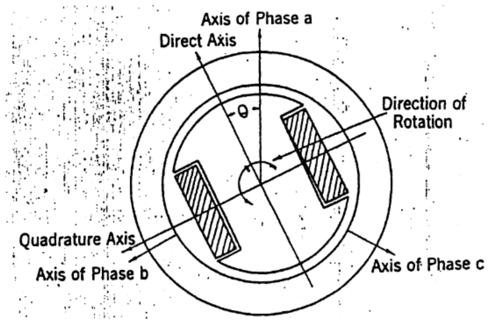
- The $\alpha\beta$ frame is *stationary* and impractical for a control system
- Park's transform converts this *stationary* frame into a *rotating* frame

✓ The resulting $dq$ values are dc signals representing active and reactive components

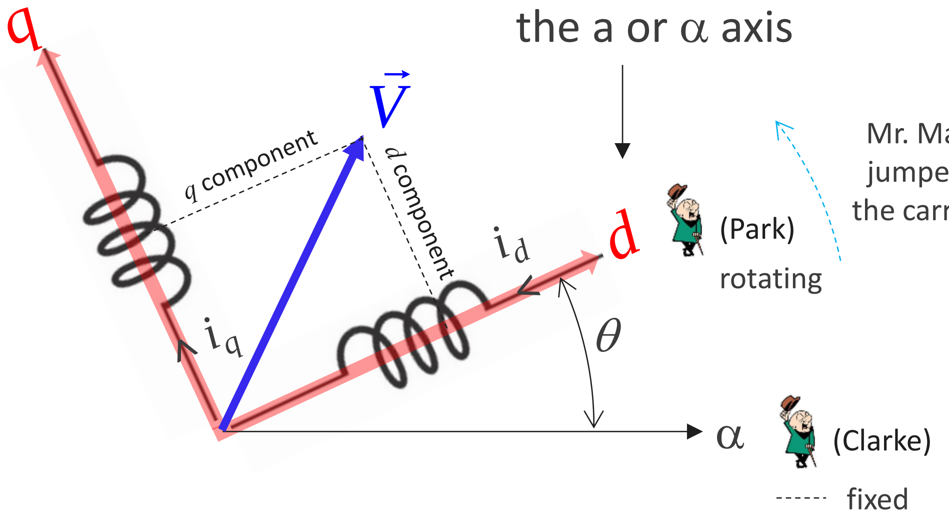


Stationary $\alpha\beta$ frame

Robert H. Park
1902-1994



$d$ is aligned with the a or $\alpha$ axis



Mr. Magoo jumped on the carrousel

Mr. Magoo on solid ground

Rotating $dq$ frame

R. H. Park, *Two-Reaction Theory of Synchronous Machines*, N.Y., Jan. 28-Feb. 1, 1929

# Park's Transformation – a Simple Explanation

- Stationary reference frame phasor projected into a rotating reference frame
- Vector **V** has the same magnitude but two different angles in $\alpha\beta$ and $dq$ frames

$$\vec{V}_{\alpha\beta} = \left|\vec{V}\right|\angle\varphi$$

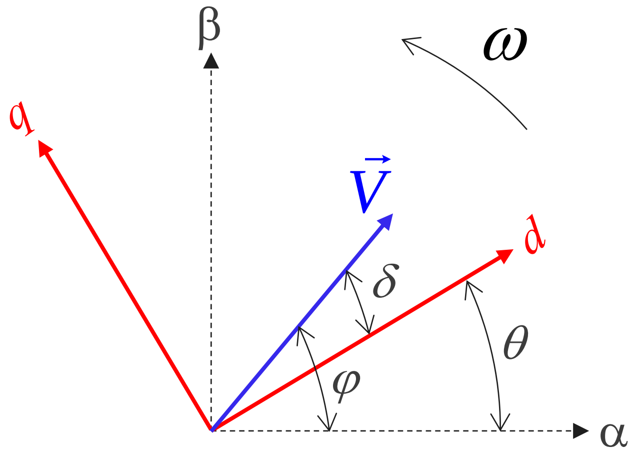
$$\vec{V}_{dq} = \left|\vec{V}\right|\angle\delta$$

$$\Rightarrow$$

$$\vec{V}_{dq} = \left|\vec{V}\right|\angle(\varphi-\theta) = \left|\vec{V}\right|\angle\varphi\cdot 1\angle-\theta$$

$$\vec{V}_{dq} = \vec{V}_{\alpha\beta}\cdot 1\angle-\theta \rightarrow \vec{V}_{dq} = \vec{V}_{\alpha\beta}\cdot e^{-j\theta}$$

$$e^{-j\theta} = \cos(-\theta) + j\sin(-\theta) = \cos(\theta) - j\sin(\theta)$$



$$\theta = \varphi - \delta$$

$$\delta = \varphi - \theta$$

- $d$ and $q$ axes are linked by a 90° angle:

$$\cos\left(\theta+\frac{\pi}{2}\right) = -\sin\theta$$

$$\sin\left(\theta+\frac{\pi}{2}\right) = \cos\theta$$

$$\Rightarrow$$

$$i_d = i_\alpha\cos\theta + i_\beta\sin\theta$$

$$i_q = -i_\alpha\sin\theta + i_\beta\cos\theta$$

Park's transform

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

$$\Leftrightarrow$$

Inverse Park's transform

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

$$\theta = \omega t = 2\pi\cdot F_{line}\cdot t$$

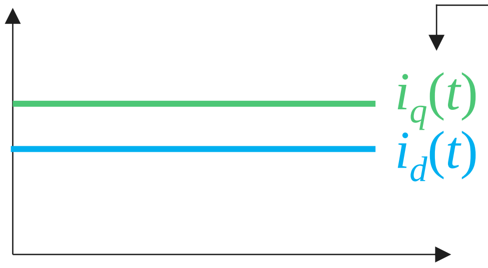
↑

You need this instantaneous angle

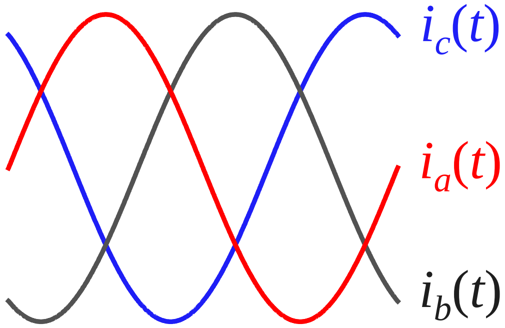
Park's Transformation in 5 mn – PEEMDConcepts - YouTube

# *dq0* Transformation

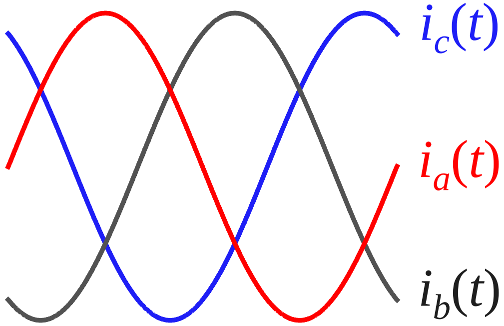
- This *power invariant* transformation combines Clarke and Park transforms
- It directly converts the 3-phase signals into dc $d$ and $q$ variables



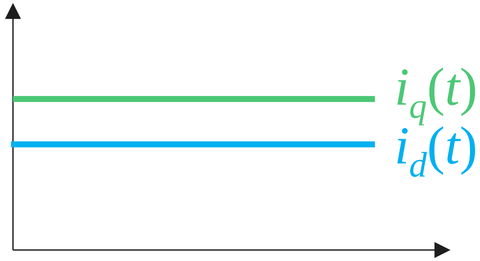
$$\begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & \cos\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) \\ -\sin(\theta) & -\sin\left(\theta-\frac{2\pi}{3}\right) & -\sin\left(\theta+\frac{2\pi}{3}\right) \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$



✓The inverse *dq0* reconstructs the corrected waveforms after corrected $d$ and $q$ variables



$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \frac{\sqrt{2}}{2} \\ \cos(\theta-\frac{2\pi}{3}) & -\sin(\theta-\frac{2\pi}{3}) & \frac{\sqrt{2}}{2} \\ \cos(\theta+\frac{2\pi}{3}) & -\sin(\theta+\frac{2\pi}{3}) & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix}$$

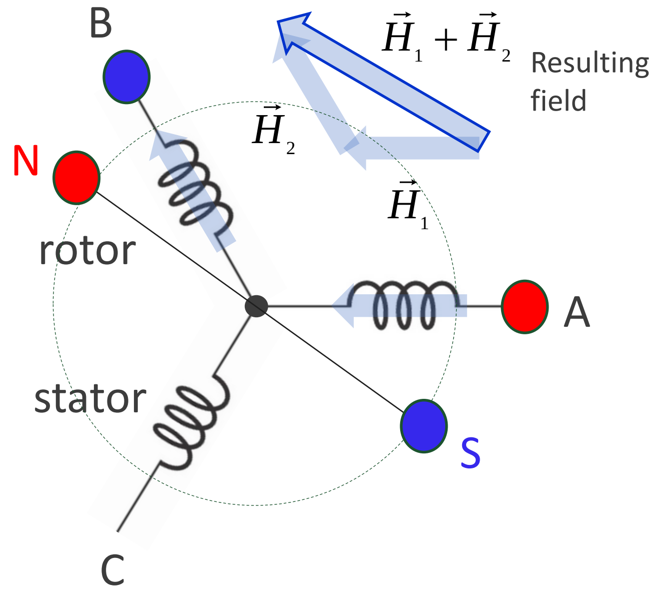


See video 51 here

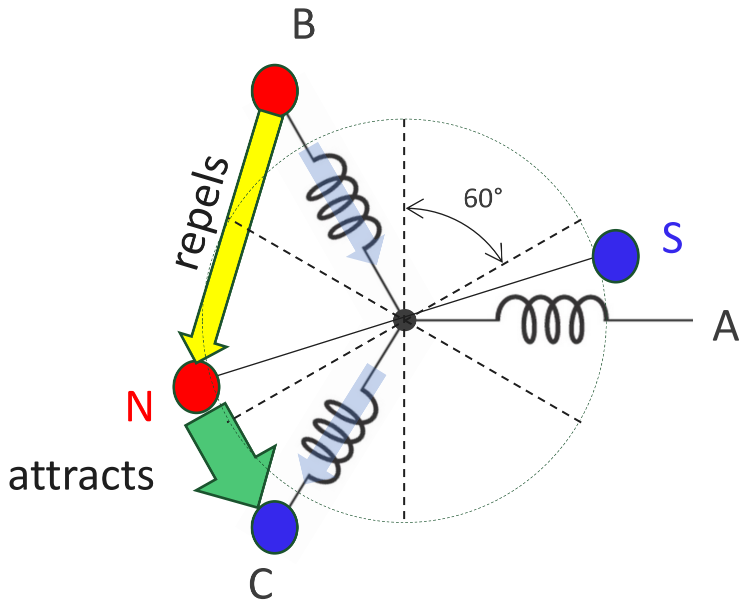
Transform for an alignment of the d-axis with the alpha axis

# Stator and Rotor Fields

- In a brushless dc motor (BLDC), you have stator and rotor fields
- The rotor field is imposed by magnets in a one pole-pair configuration
- The stator field will be controlled by biasing the three windings



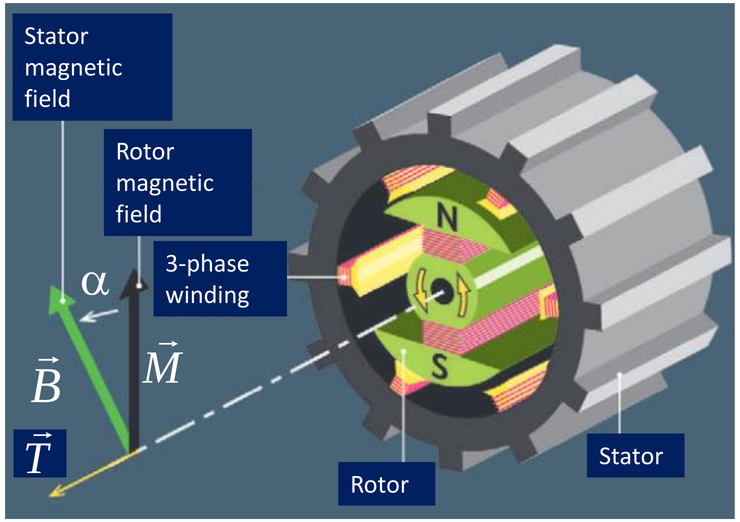
The rotor aligns with the stator field
The torque in this mode is <u>equal to 0</u>



You can energize a stator coil pair in 6 ways
6 possible rotor alignments – 60° apart

6-step commutation
or ***trapezoidal control***

Picture from this article



Torque is maximum when $\alpha = 90°$

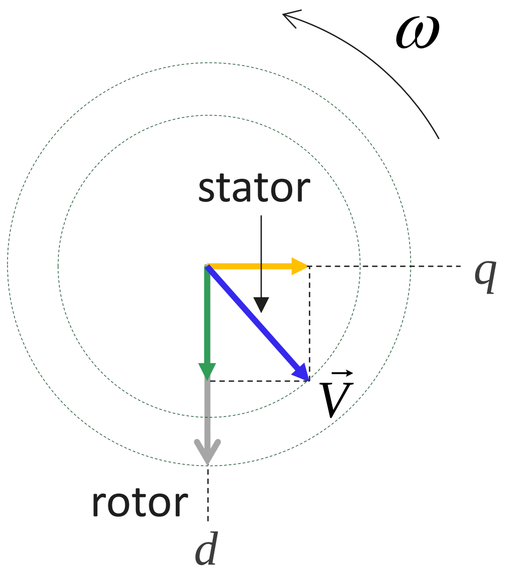
$$\vec{T} = \vec{M} \wedge \vec{B}$$

$$\left\|\vec{T}\right\| = \left\|\vec{M}\right\| \cdot \left\|\vec{B}\right\| \cdot \sin(\alpha)$$

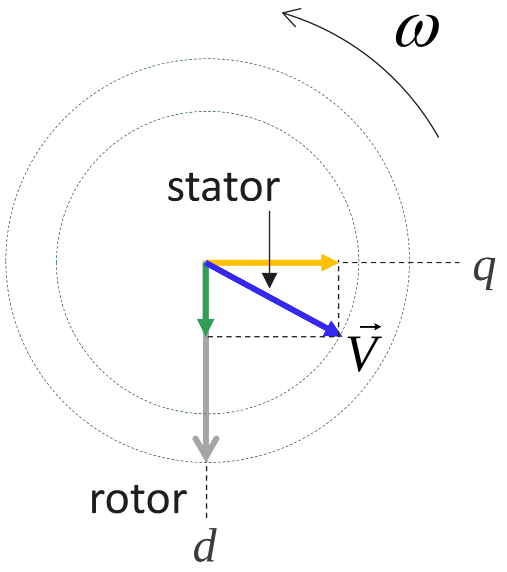
An Introduction to BLDC Motor – 6-part series on YouTube from Melda Ulusoy

# The Need for Orthogonal Quantities

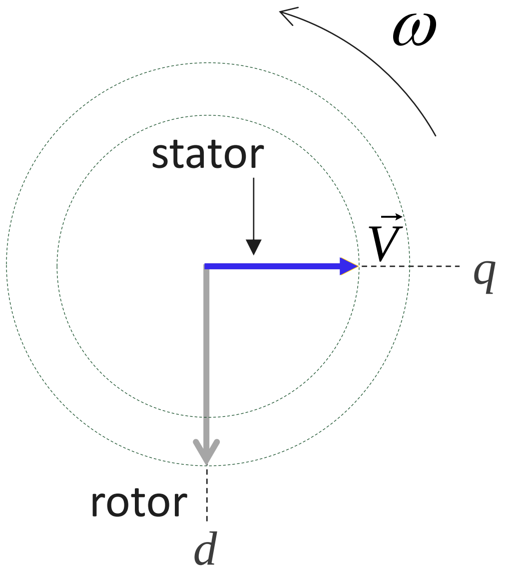
- We want to impose an orthogonal field between the stator and rotor fields
- The stator field is expressed in $d$ and $q$ components via Clarke's and Park's transforms
- ✓ Set $d$ to zero and maximize the $q$ component for orthogonal fields



The rotor field and the stator field are not orthogonal



By reducing the $d$ component, the $q$ component grows

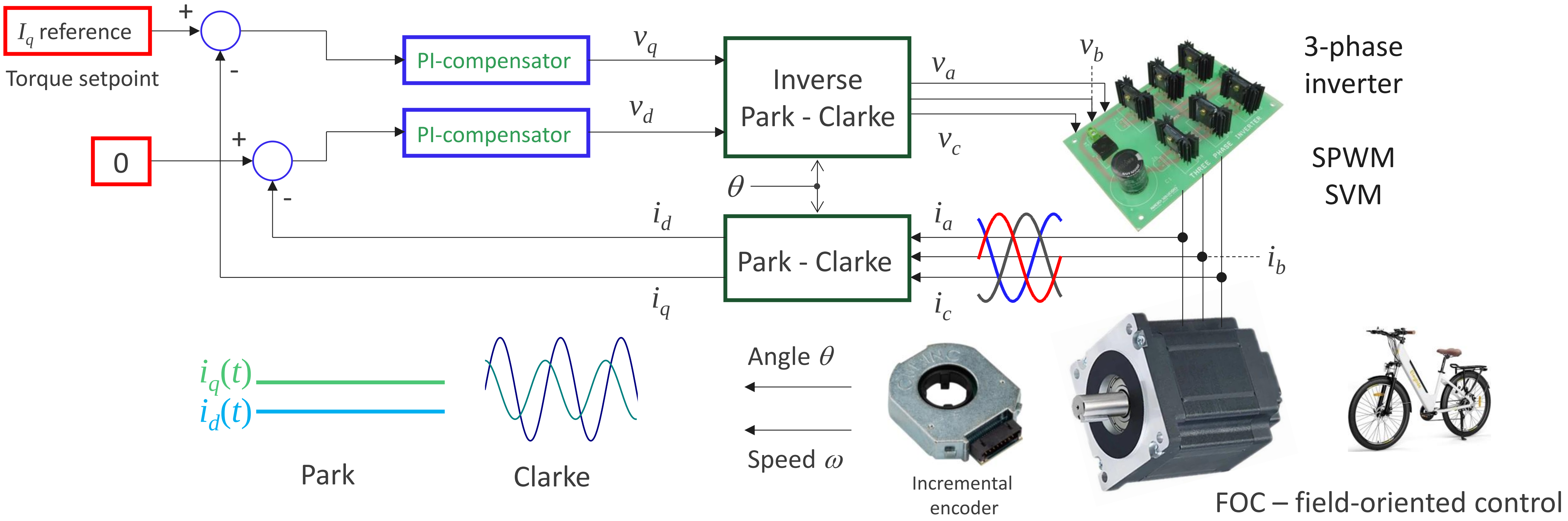


When the $d$ component is zeroed, the stator field is orthogonal to the rotor field.

An Introduction to BLDC Motor – 6-part series on YouTube from Melda Ulusoy

# A Practical Implementation

- A permanent magnet synchronous motor (PMSM) can be controlled by FOC
- Currents in phases are adjusted by the inverter to control the motor torque
- ✓ A control system zeroes the $d$ components while the $q$ input sets a reference value



# Agenda

- Symmetrical Components
- Clarke Transform
- Park Transform
- Power Processing
- Power Factor Correction

# Power Processing

- We have seen how the Clarke/Park's transforms are used in motor control
- The $dq0$ transform directly leads to $d$ and $q$ components

$$v_d = \sqrt{\frac{2}{3}}\left[v_a \cos(\theta) + v_b \cos\left(\theta - \frac{2\pi}{3}\right) + v_c \cos\left(\theta + \frac{2\pi}{3}\right)\right] \qquad i_d = \sqrt{\frac{2}{3}}\left[i_a \cos(\theta) + i_b \cos\left(\theta - \frac{2\pi}{3}\right) + i_c \cos\left(\theta + \frac{2\pi}{3}\right)\right]$$

$$v_q = \sqrt{\frac{2}{3}}\left[-v_a \sin(\theta) - v_b \sin\left(\theta - \frac{2\pi}{3}\right) - v_c \sin\left(\theta + \frac{2\pi}{3}\right)\right] \qquad i_q = \sqrt{\frac{2}{3}}\left[-i_a \sin(\theta) - i_b \sin\left(\theta - \frac{2\pi}{3}\right) - i_c \sin\left(\theta + \frac{2\pi}{3}\right)\right]$$

$$v_0 = \sqrt{\frac{1}{6}}\left[v_a + v_b + v_c\right] \qquad i_0 = \sqrt{\frac{1}{6}}\left[i_a + i_b + i_c\right]$$

- Average [W] and reactive [VAR] powers can be computed:

$$P = v_d i_d + v_q i_q + v_0 i_0 \quad [\mathrm{W}]$$

$$Q = v_q i_d - v_d i_q \quad [\mathrm{VAR}]$$

$$S = P + jQ \quad [\mathrm{VA}]$$

➡

In a PFC system:

✓ The loop sets $i_d$ to keep $V_{out}$ constant
✓ $i_q$ is set to 0

The $d$-axis is aligned with the a phase

# Computing the Angle

- For processing the $d$ and $q$ parameters, you need the $\theta$ angle
- It can be obtained by processing the individual phase voltages

Phase expressions

$$v_a(t)=V_{in}\sqrt{2}\sin(\omega t)$$

$$v_b(t)=V_{in}\sqrt{2}\left(\omega t-\frac{2\pi}{3}\right)$$

$$v_c(t)=V_{in}\sqrt{2}\left(\omega t-\frac{4\pi}{3}\right)$$

$$v_c(t)-v_b(t)=V_{in}\sqrt{2}\left[\sin\left(\omega t-\frac{4\pi}{3}\right)-\sin\left(\omega t-\frac{2\pi}{3}\right)\right]$$

$$v_c(t)-v_b(t)=V_{in}\sqrt{6}\cos(\omega t)$$

$$v_c(t)+v_b(t)=V_{in}\sqrt{2}\left[\sin\left(\omega t-\frac{4\pi}{3}\right)+\sin\left(\omega t-\frac{2\pi}{3}\right)\right]$$

$$v_c(t)+v_b(t)=-V_{in}\sqrt{2}\sin(\omega t)$$

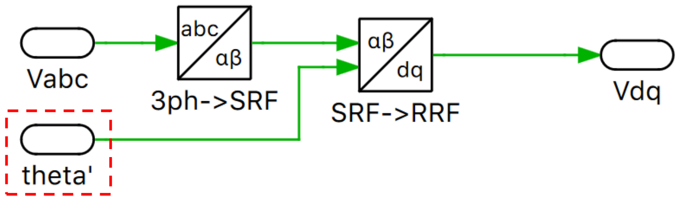
$$\Rightarrow \theta(t)=-\tan^{-1}\left(\frac{\sqrt{3}\left[v_c(t)+v_b(t)\right]}{v_c(t)-v_b(t)}\right)$$

- The practical implementation can be done this way, involving $v_a(t)$:

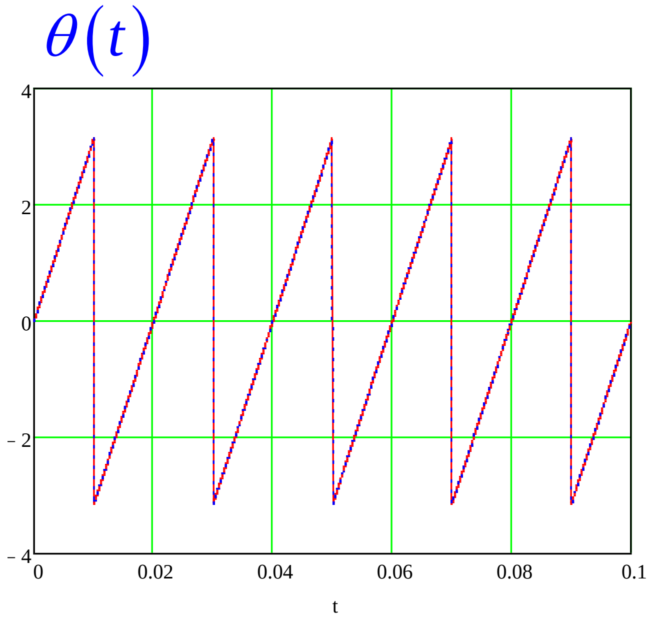
$$\theta(t)=-\tan^{-1}\left(-\frac{1}{\sqrt{3}}\frac{v_a(t)-2v_b(t)-2v_c(t)}{\left[v_b(t)-v_c(t)\right]}\right)$$

$$-\operatorname{atan2}\left[\left(v_c(t)-v_b(t)\right),\sqrt{3}\cdot\left(v_c(t)+v_b(t)\right)\right]$$

$$\operatorname{atan2}\left[-\frac{\sqrt{3}}{3}\cdot\left(v_b(t)-v_c(t)\right),\frac{\left(v_a(t)-2\cdot v_b(t)-2\cdot v_c(t)\right)}{3}\right]$$

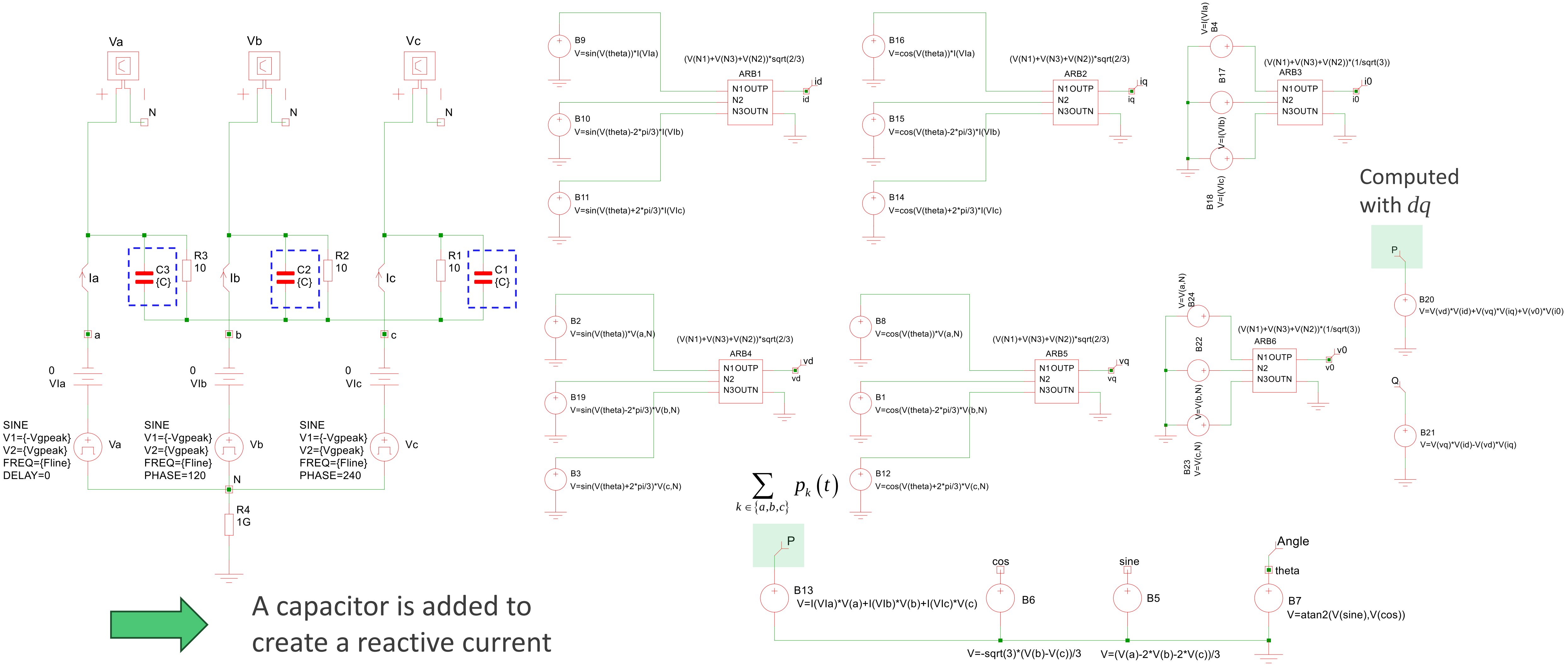


PLECS $dq0$ block needs $\theta$



# Simulating the $dq0$ Transform

- SIMetrix® with its in-line equations simulates a $dq0$ transform



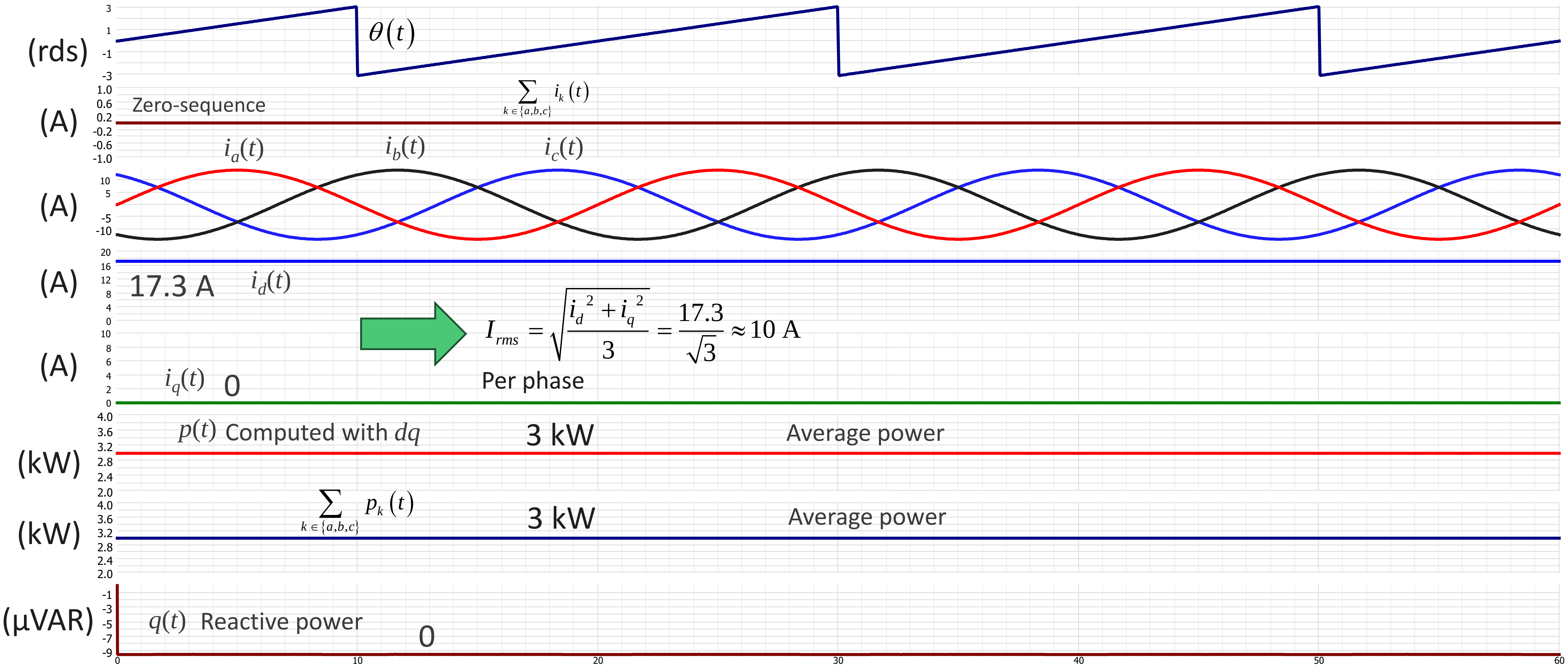
Computed with $dq$

$$\sum_{k\in\{a,b,c\}} p_k\left(t\right)$$

A capacitor is added to create a reactive current

# Balanced System – no Reactive Power

- 3-phase 100-V rms feeds 10-ohm loads, no capacitive component $I_{rms} = \sqrt{\frac{P_{tot}}{3R_L}} = 10\ \text{A}$

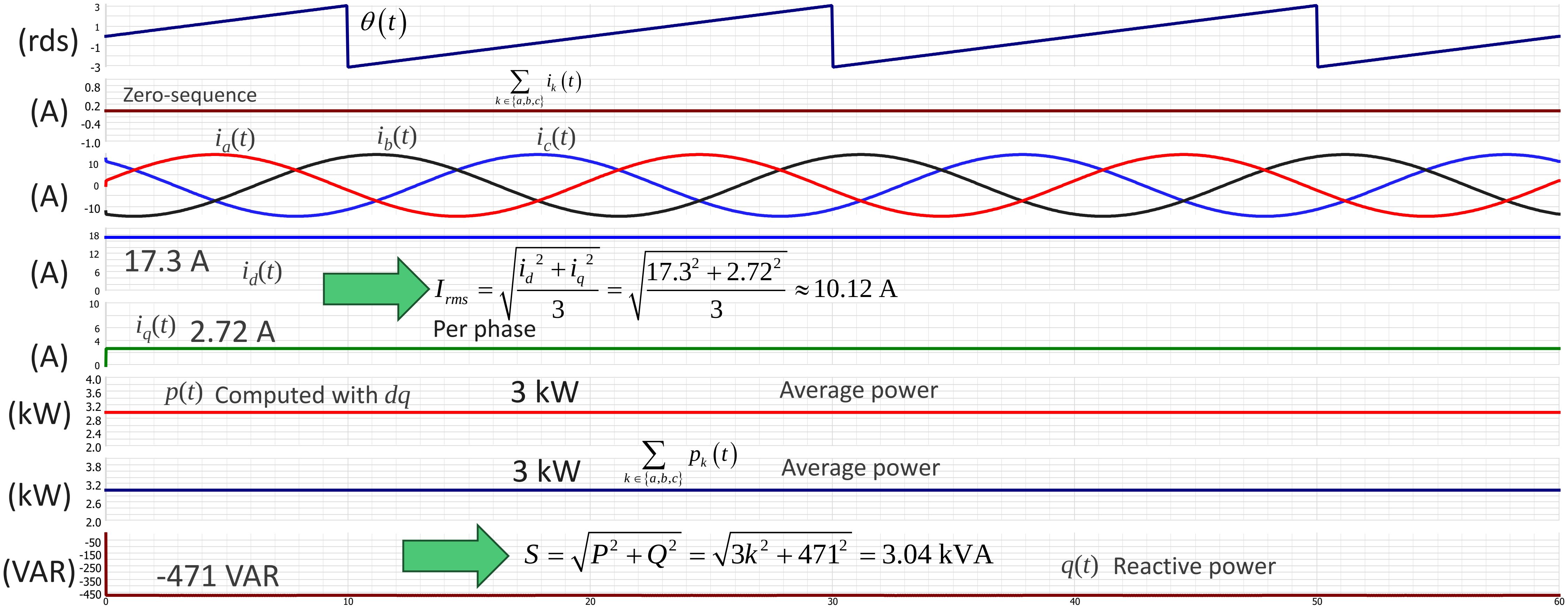


$$I_{rms} = \sqrt{\frac{{i_d}^2 + {i_q}^2}{3}} = \frac{17.3}{\sqrt{3}} \approx 10\ \text{A}$$

Per phase

# Balanced System – Reactive Power

- 3-phase 100-V rms feeds 10-ohm loads with 50-µF capacitors in parallel



$$I_{rms} = \sqrt{\frac{i_d^2 + i_q^2}{3}} = \sqrt{\frac{17.3^2 + 2.72^2}{3}} \approx 10.12\text{ A}$$

Per phase

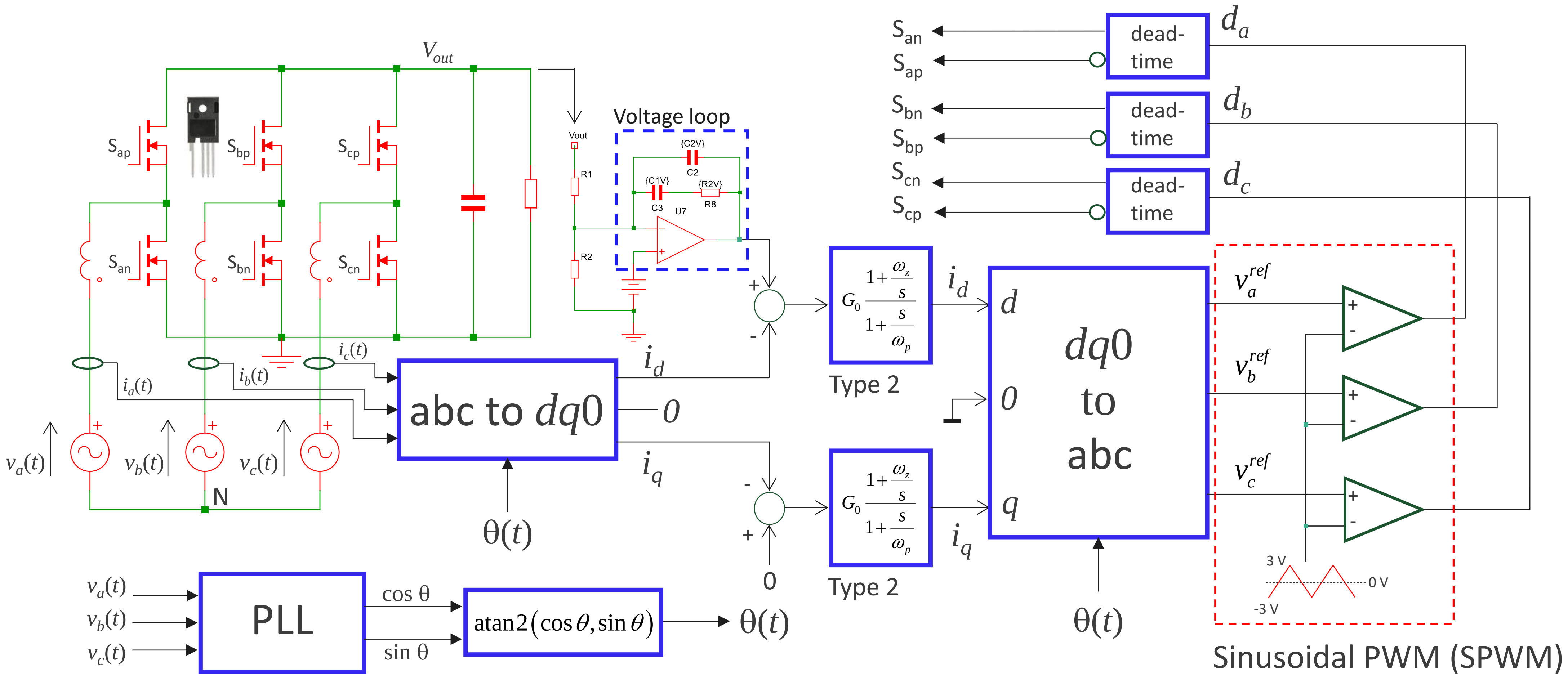
$$S = \sqrt{P^2 + Q^2} = \sqrt{3k^2 + 471^2} = 3.04\text{ kVA}$$

# Agenda

- Symmetrical Components
- Clarke Transform
- Park Transform
- Power Processing
- Power Factor Correction

# 3-Phase Boost Rectifier with SPWM

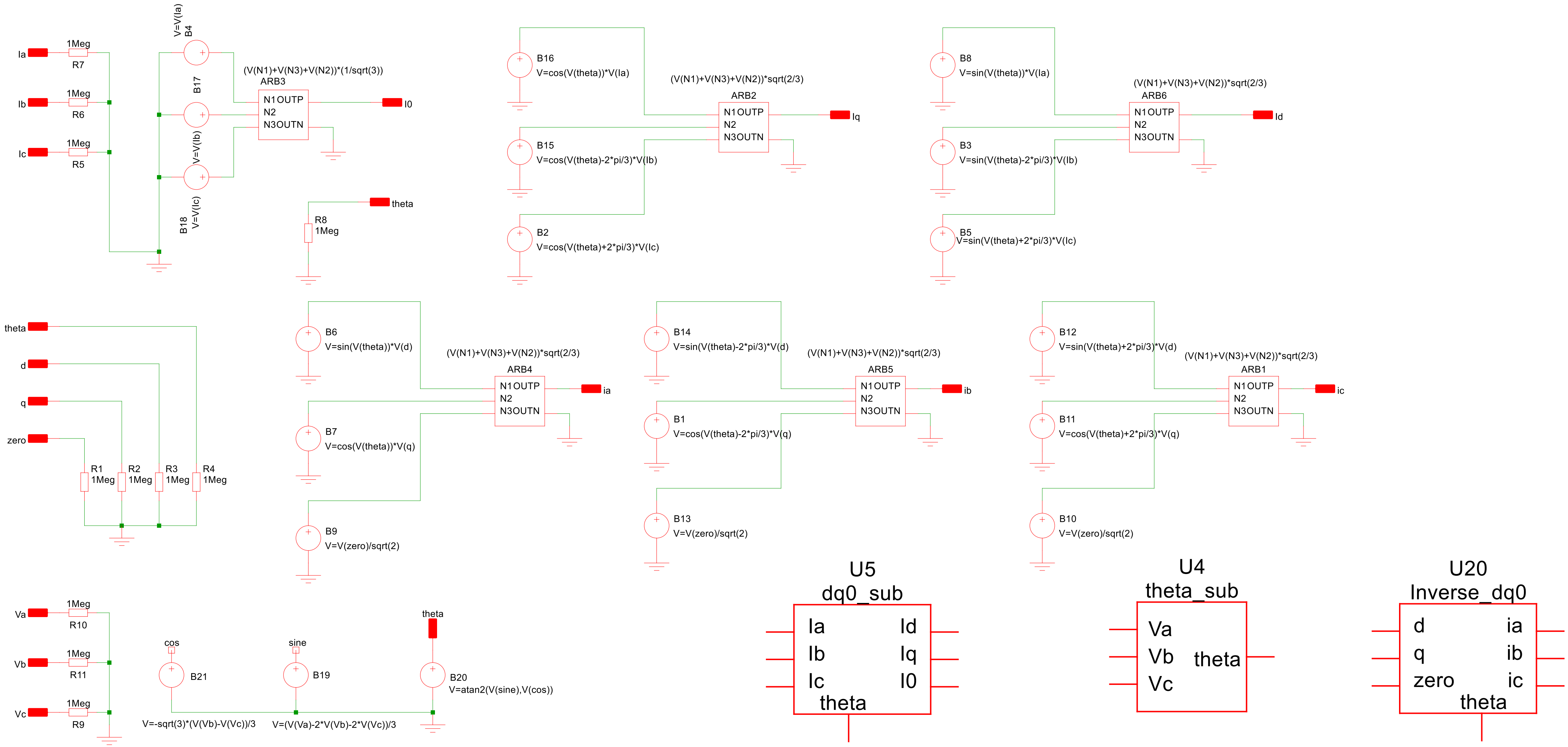
- A phase-locked loop (PLL) is used for extracting the angle $\theta$
- Each input current is sensed and used to compute $dq0$ variables with $\theta$



D. Boroyevich, *Modeling and Control of Three-Phase PWM Converters*, PECon 2008, Malaysia

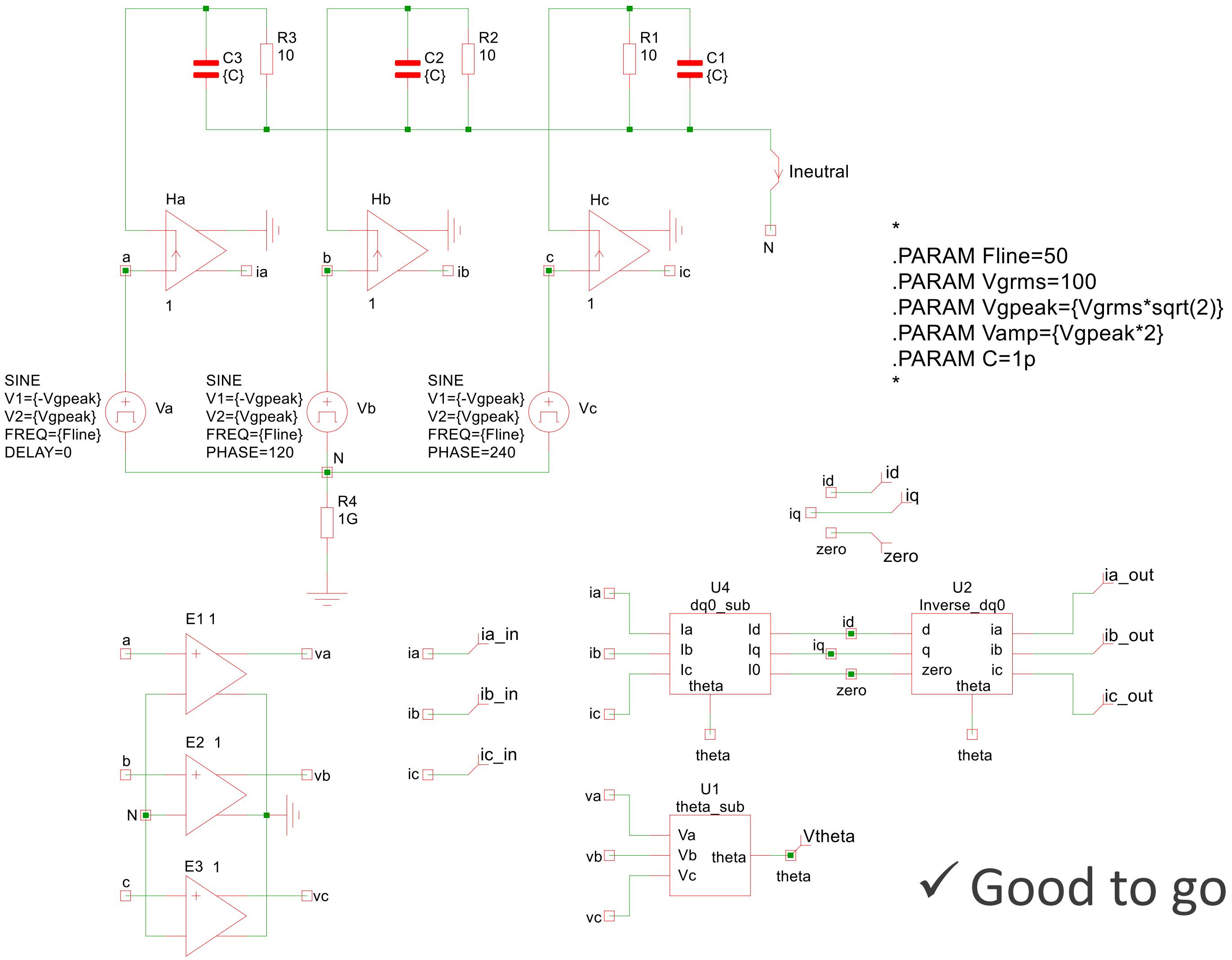
# Building Subcircuits Blocks

- Computing blocks are created in SIMetrix® and encapsulated



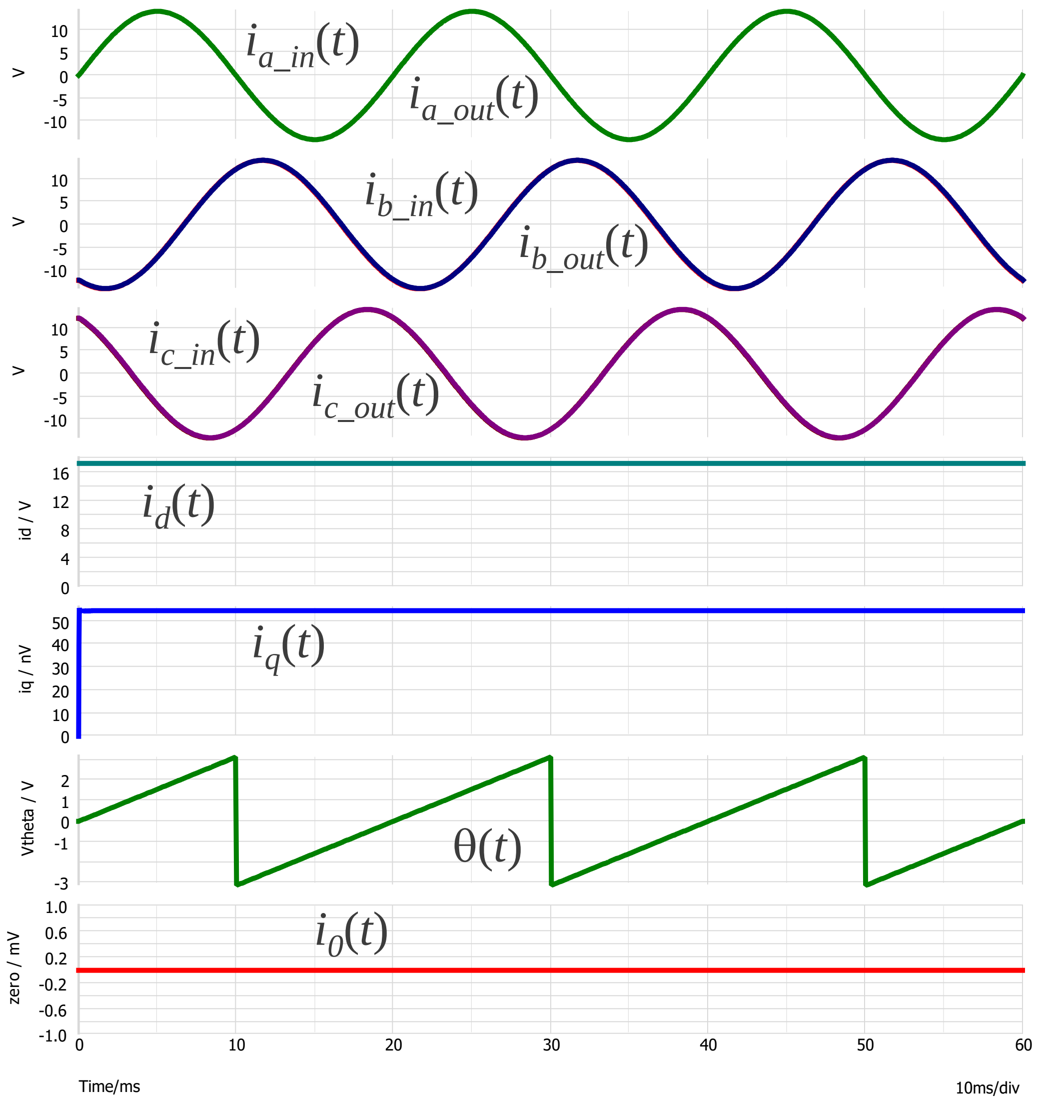
# Testing Subcircuits

- Before operating the blocks in a 3-phase circuit, verify the computed results

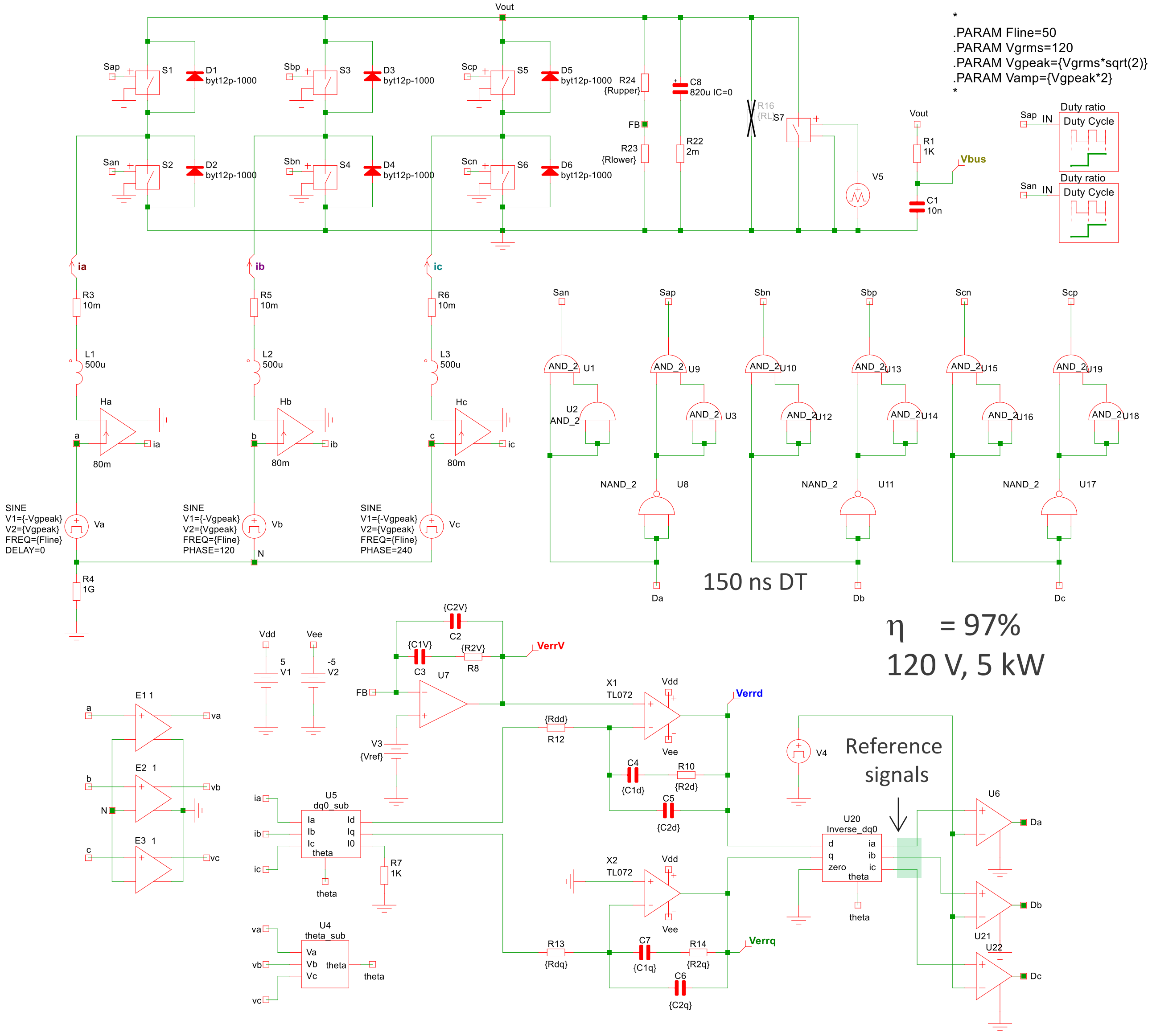


```
*
.PARAM Fline=50
.PARAM Vgrms=100
.PARAM Vgpeak={Vgrms*sqrt(2)}
.PARAM Vamp={Vgpeak*2}
.PARAM C=1p
*
```

✓ Good to go!



# The Complete Simulation



```
*
.PARAM Fline=50
.PARAM Vgrms=120
.PARAM Vgpeak={Vgrms*sqrt(2)}
.PARAM Vamp={Vgpeak*2}
*
```

SINE
V1={-Vgpeak}
V2={Vgpeak}
FREQ={Fline}
DELAY=0

SINE
V1={-Vgpeak}
V2={Vgpeak}
FREQ={Fline}
PHASE=120

SINE
V1={-Vgpeak}
V2={Vgpeak}
FREQ={Fline}
PHASE=240

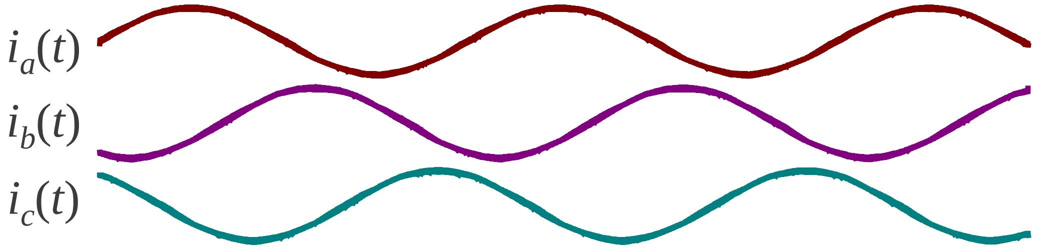
150 ns DT

η = 97%
120 V, 5 kW

Reference signals

- There are three compensators in this circuit
- The voltage loop is compensated for a low crossover
- The inner current loops are faster ≈ 1-2 kHz
- The converter delivers 5 kW from a 120-V ac input
- $V_{out}$ = 400 V

THD ≈4.5%



```
.param GfcV=20 ; magnitude at crossover *
.param PSV=-90 ; phase lag at crossover *
*
* Enter Design Goals Information Here *
*
.param fcV=20 ; targeted crossover *
.param PMV=60 ; choose phase margin at crossover *
*
* Enter the Values for Vout and Bridge Bias Current *
*
.param Vout=400
.param Pout=5k
.param RL={(Vout)^2/(Pout)}
.param Ibias=100u
.param Vref=2.5
.param Rlower={Vref/Ibias}
.param Rupper={(Vout-Vref)/Ibias}
```

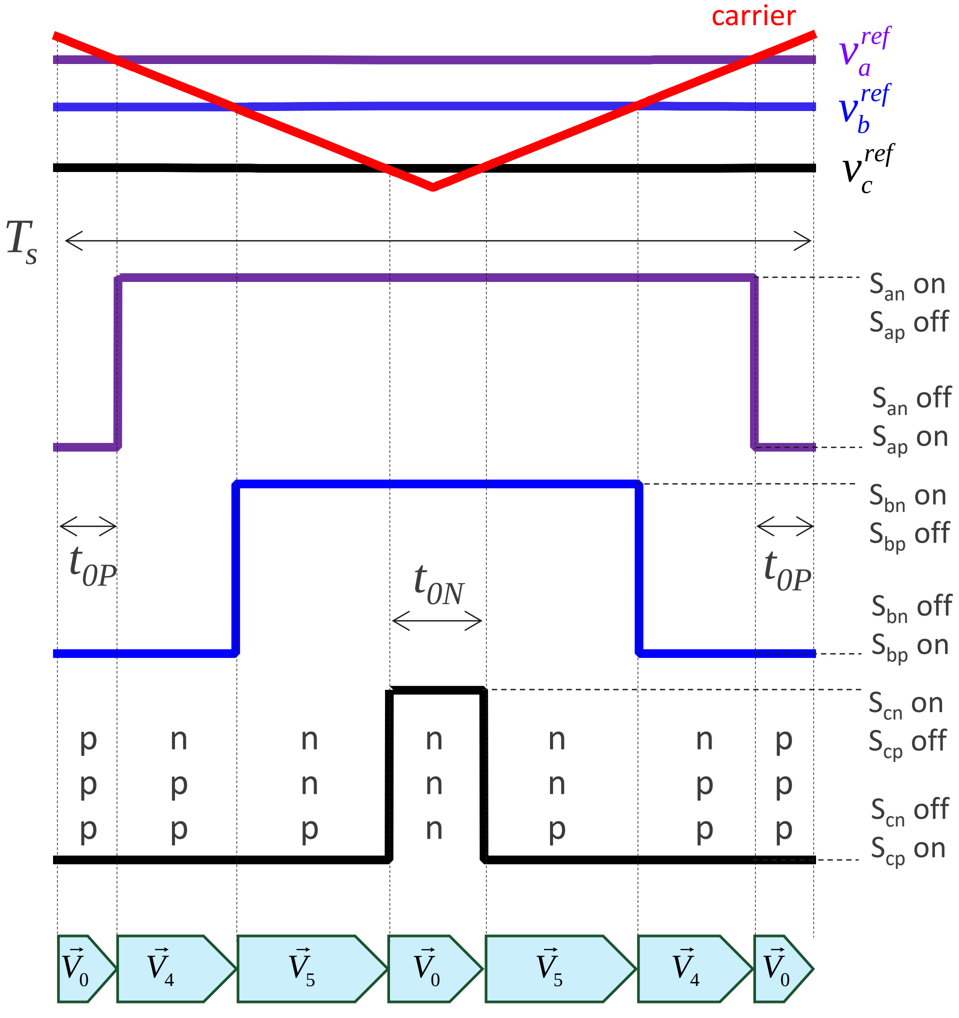
```
*
* Do not edit the below lines *
.param boostV={PMd-PSd-90}
.param GV={10^(-GfcV/20)}
.param kV={tan((boostV/2+45)*pi/180)}
.param fpV={fcV*kV}
.param fzV={fcV/kV}
.param
C2V={1/(2*pi*fcV*GV*kV*Rupper)}
.param C1V={C2V*(kV^2-1)}
.param R2V={kV/(C1V*2*pi*fcV)}
```

Type 2 voltage loop compensation

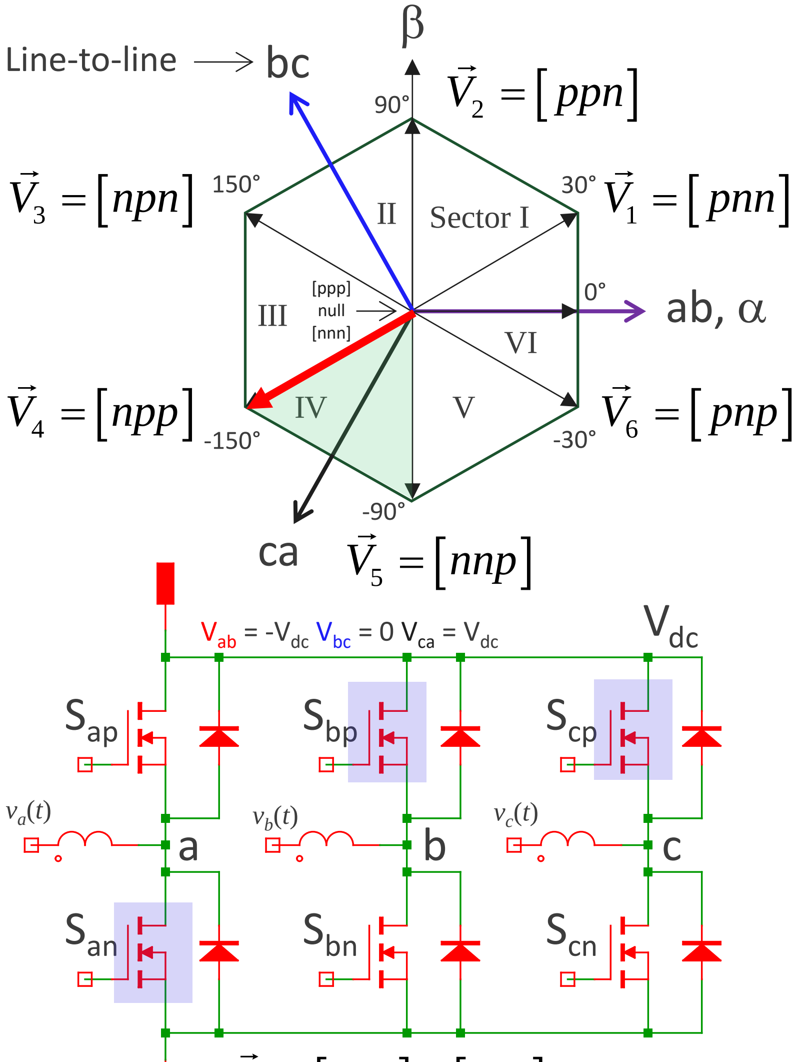
Simulation time for 200 ms: 5 mn

# Switching Pattern at Steady State

- This is a symmetrical center-based three-phase modulation like SVM2



Sector IV



$$\vec{V}_4 = [npp] = [011]$$

- The idea is to generate a vector $\vec{V}_{ref}$ whose position and magnitude depends on adjacent vectors.
- Here, $\vec{V}_{ref}$ is generated by weighting the time during which vectors $\vec{V}_4$ and $\vec{V}_5$ are active.
- The switch pattern in example shows the realization of vector $\vec{V}_4$.

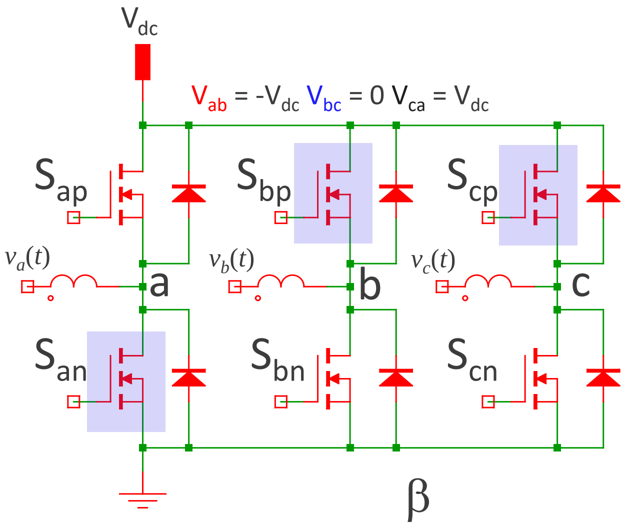
$$\vec{V}_{ref} = \rho \cdot e^{j\theta}$$

Space vector in $\alpha\beta$

$$\vec{V}_0 = [ppp] = [nnn]$$

# Vectors Hexagon for Star or Delta Configuration

- The hexagon represents the six possible vectors for a star or a delta configuration

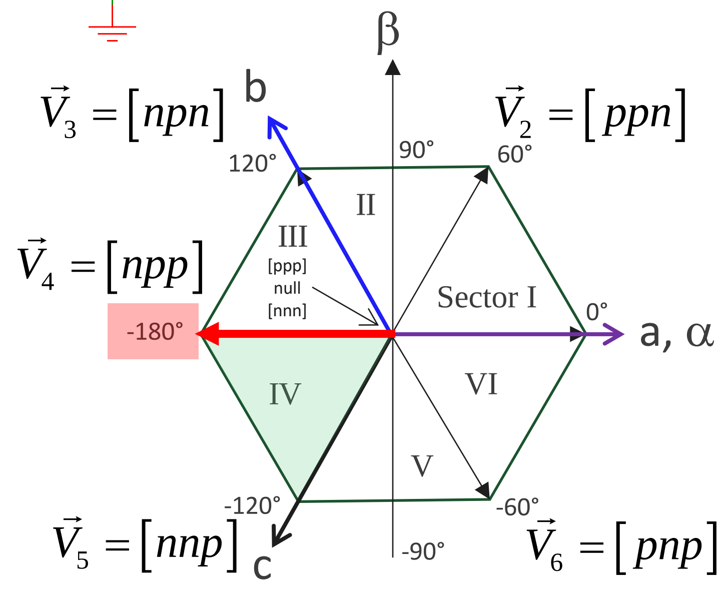


$V_{ab}$ = -$V_{dc}$
$V_{bc}$ = 0
$V_{ca}$ = $V_{dc}$

Vector $V_4$ is synthesized in αβ

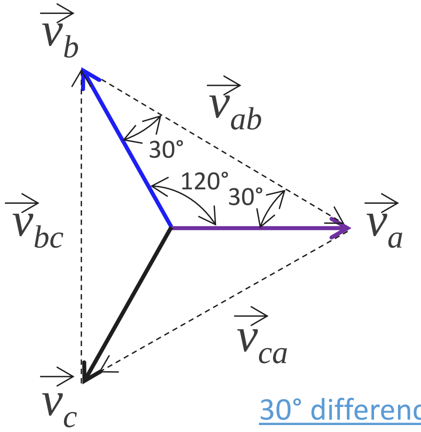
$$\sqrt{\frac{2}{3}}\cdot\begin{pmatrix}1 & -\frac{1}{2} & -\frac{1}{2}\\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}\end{pmatrix}\cdot\begin{pmatrix}-V_{dc}\\ 0\\ V_{dc}\end{pmatrix}\rightarrow\begin{pmatrix}-\frac{\sqrt{2}\cdot\sqrt{3}\cdot V_{dc}}{2}\\ -\frac{\sqrt{2}\cdot V_{dc}}{2}\end{pmatrix}\longrightarrow\begin{array}{c}\alpha\\ \beta\end{array}\begin{pmatrix}-\sqrt{\frac{3}{2}}\cdot V_{dc}\\ -\frac{1}{\sqrt{2}}\cdot V_{dc}\end{pmatrix}$$

$$\rho = V_{dc}\cdot\sqrt{\left[-\left(\sqrt{\frac{3}{2}}\right)\right]^2+\left[-\left(\frac{1}{\sqrt{2}}\right)\right]^2} = \sqrt{2}\cdot V_{dc} \qquad \theta := \operatorname{atan2}\left[-\sqrt{\frac{3}{2}},-\left(\frac{1}{\sqrt{2}}\right)\right] = \boxed{-150\cdot°}$$

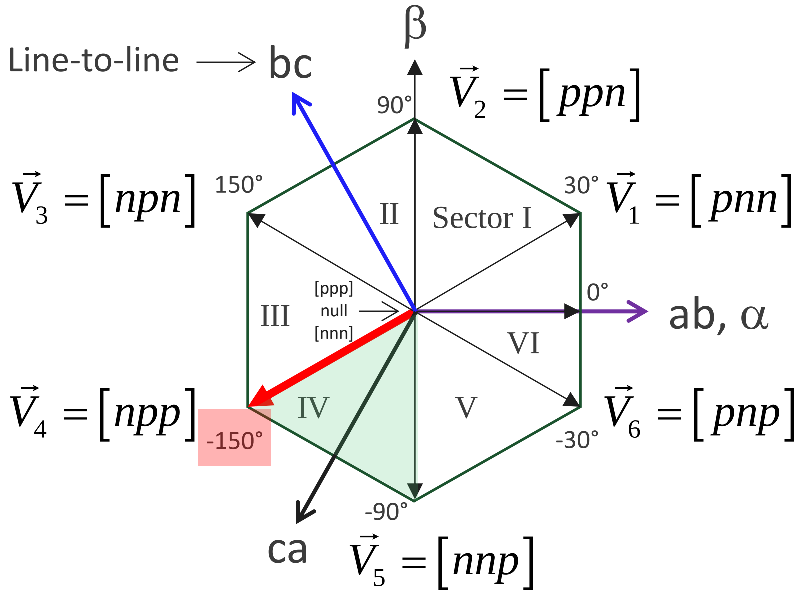


Star configuration

- Accounting for the 30° phase difference between delta and star:



30° difference in star delta systems

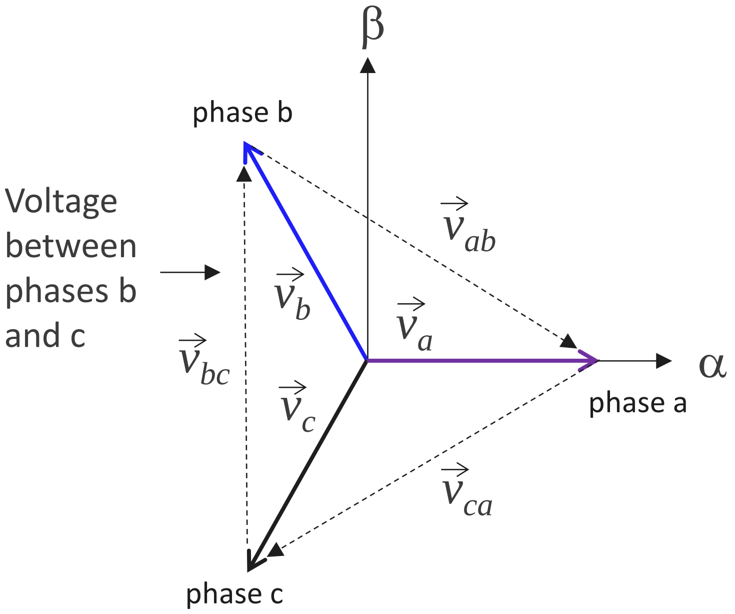


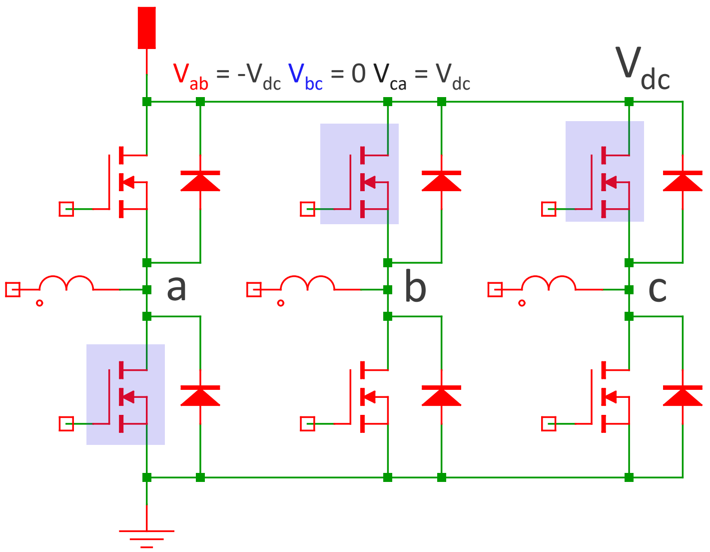
Delta configuration

# Synthetizing the Six Vectors

- We want to impose voltages between points a, b and c with the six switches
- $\vec{v}_{ab}$, $\vec{v}_{bc}$ and $\vec{v}_{ca}$ can take on one of the three values: $-V_{dc}$, 0 or $V_{dc}$

✓ you add or subtract these vectors to obtain the vector imposed by the switches

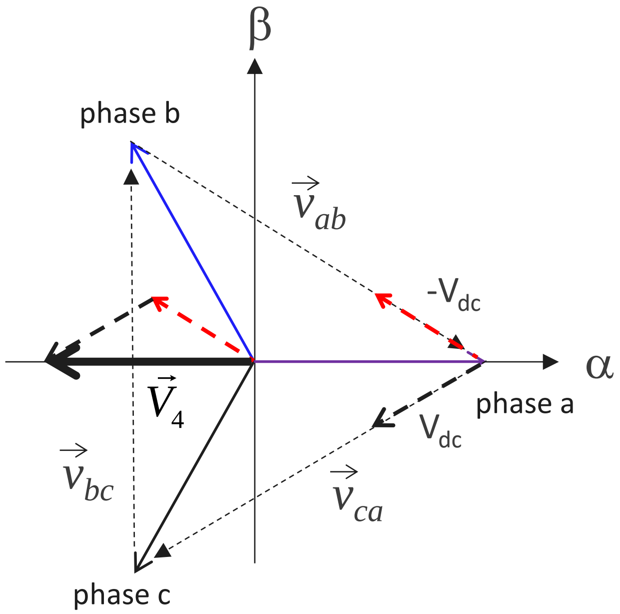




$V_{ab} = -V_{dc}$

$V_{bc} = 0$

$V_{ca} = V_{dc}$

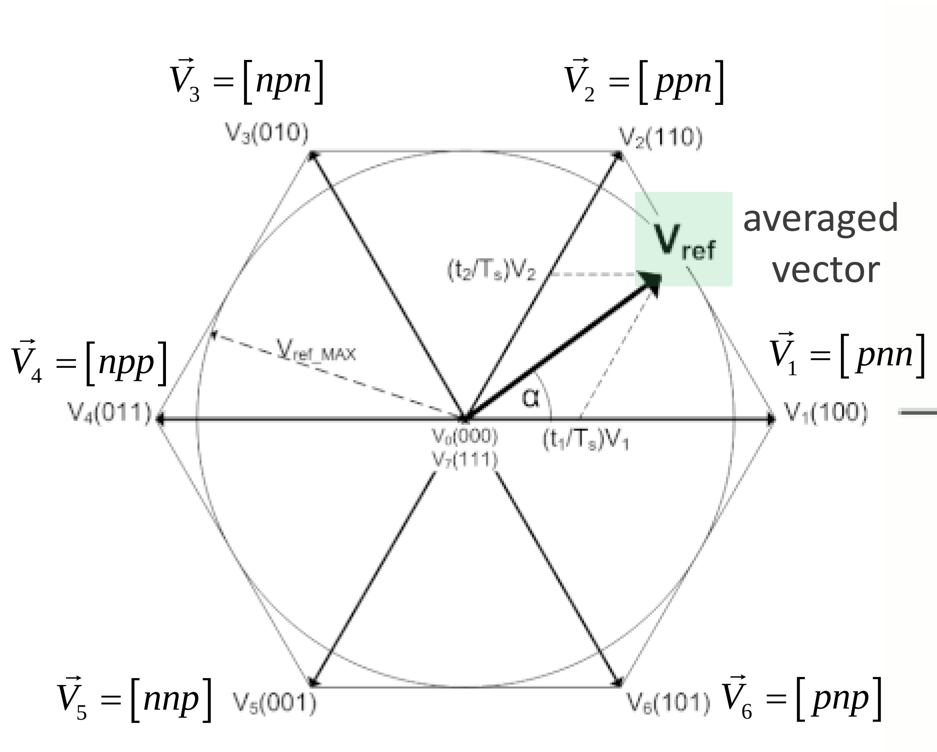


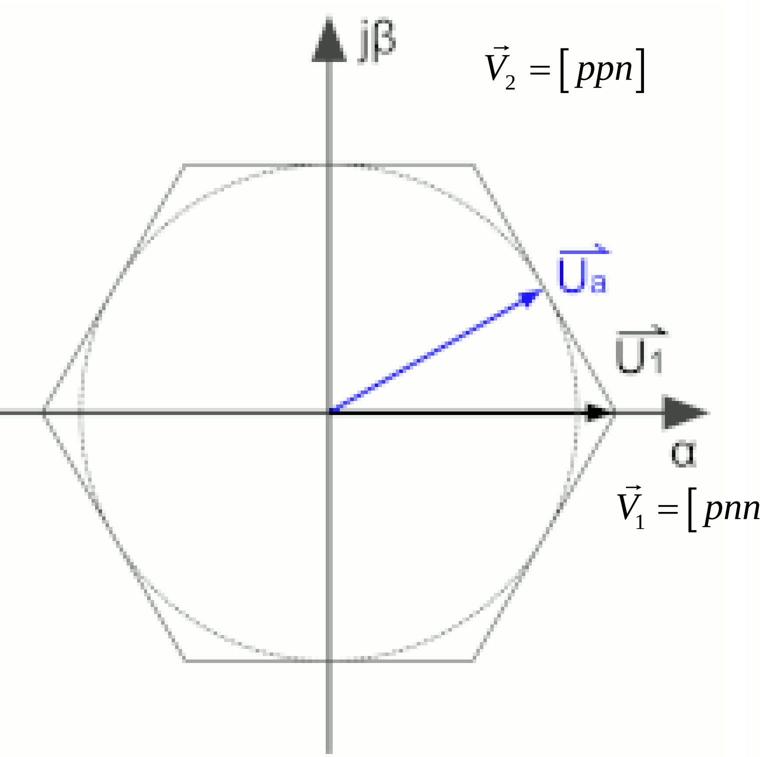
- Vector $V_4$ is created by summing the two vectors of $-V_{dc}$ and $V_{dc}$ magnitudes

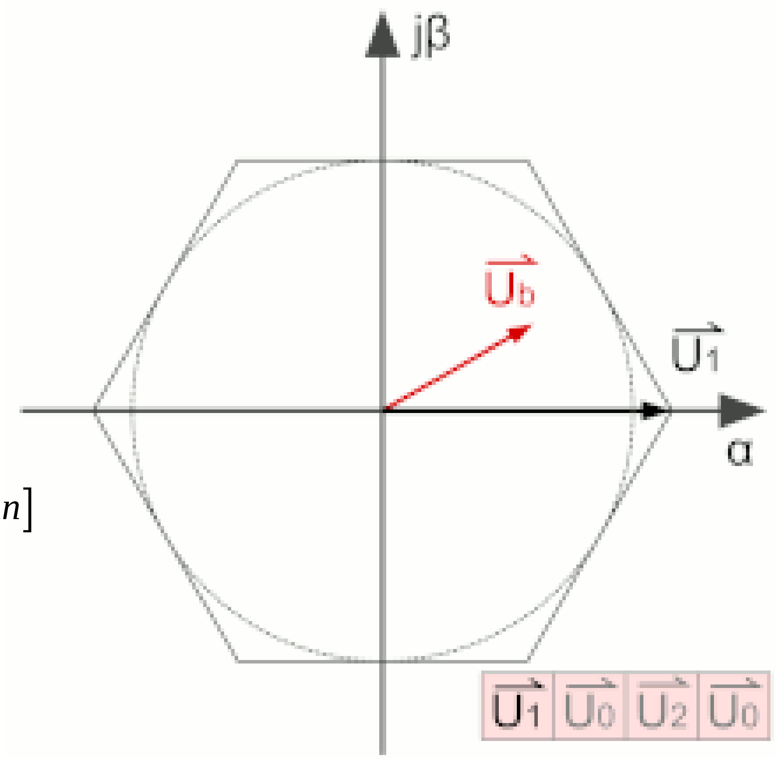
J. Schönberger, *Space Vector Control of a Three-Phase Rectifier using PLECS®*

# Averaging the Position

- The reference vector is created by alternating between the two adjacent vectors
- The magnitude is adjusted by inserting a deadtime with vectors 0: [000] or [111]







- Vector $\overrightarrow{V_{ref}}$ is synthesized by selecting the adjacent vectors in the considered sector

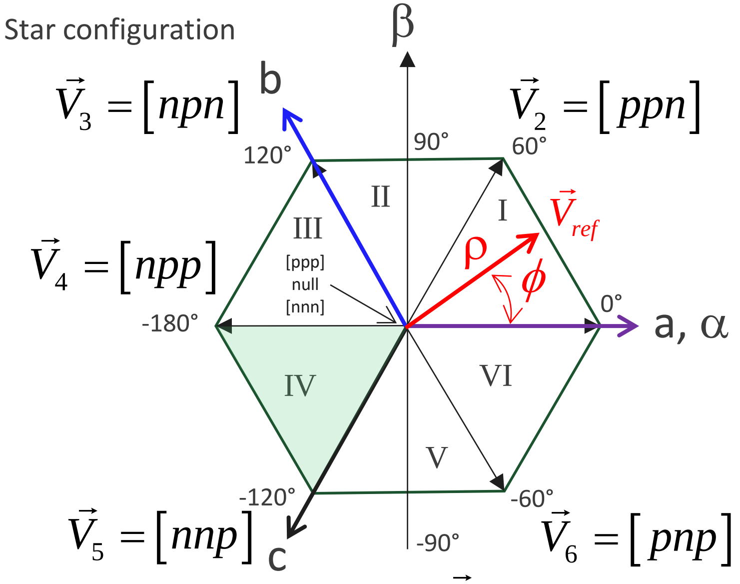
- The angle is determined by the time duration of each vector in the sector

- The magnitude is adjusted by the insertion of a deadtime via $\overrightarrow{V_0}$

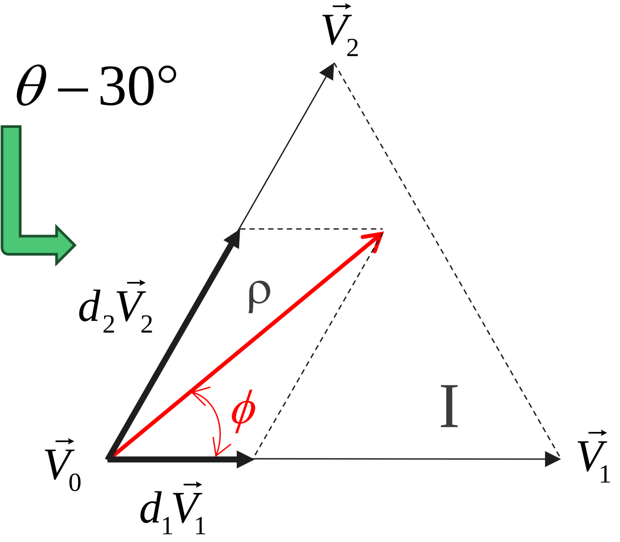
*Méthode du vecteur spatial*, Wikipedia

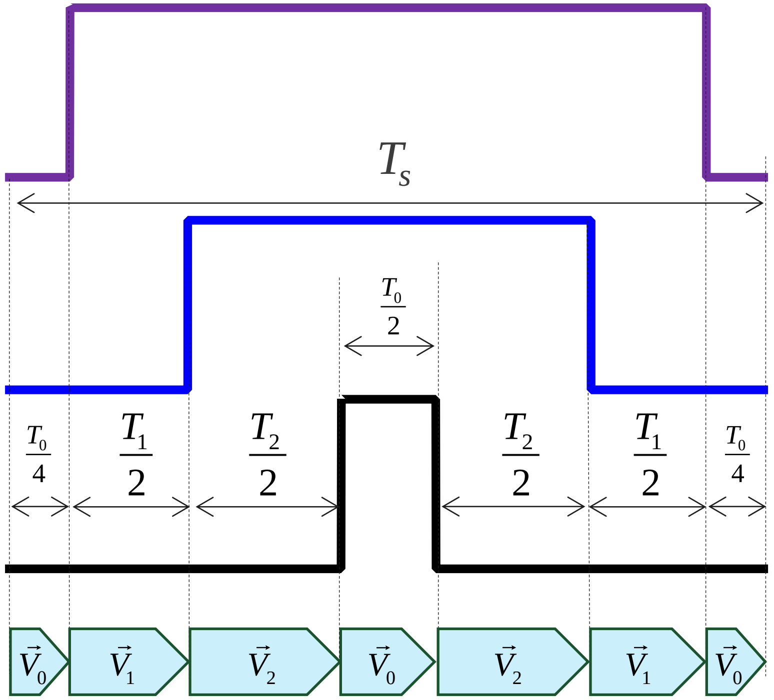
# Dwell Times Calculations

- Determine switching pattern time duration for generating the reference vector



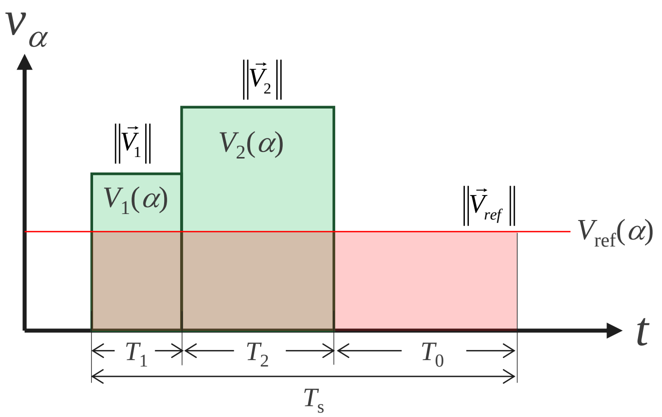
$$\phi = \theta - 30°$$





$$d_1 = \frac{T_1}{T_s} = \frac{\sqrt{3}}{V_{dc}}\left\|\vec{V}_{ref}\right\|\sin\left(60° - \phi\right)$$

$$d_2 = \frac{T_2}{T_s} = \frac{\sqrt{3}}{V_{dc}}\left\|\vec{V}_{ref}\right\|\sin\left(\phi\right)$$



$$\int_0^{T_s}\vec{V}_{ref}\,dt = \int_0^{T_1}\vec{V}_1 dt + \int_{T_1}^{T_2}\vec{V}_2 dt + \int_{T_1+T_2}^{T_s}\vec{V}_0 dt$$

$$\left\|\vec{V}_{ref}\right\|T_s = \left\|\vec{V}_1\right\|T_1 + \left\|\vec{V}_2\right\|T_2 + \left\|\vec{V}_0\right\|T_0$$

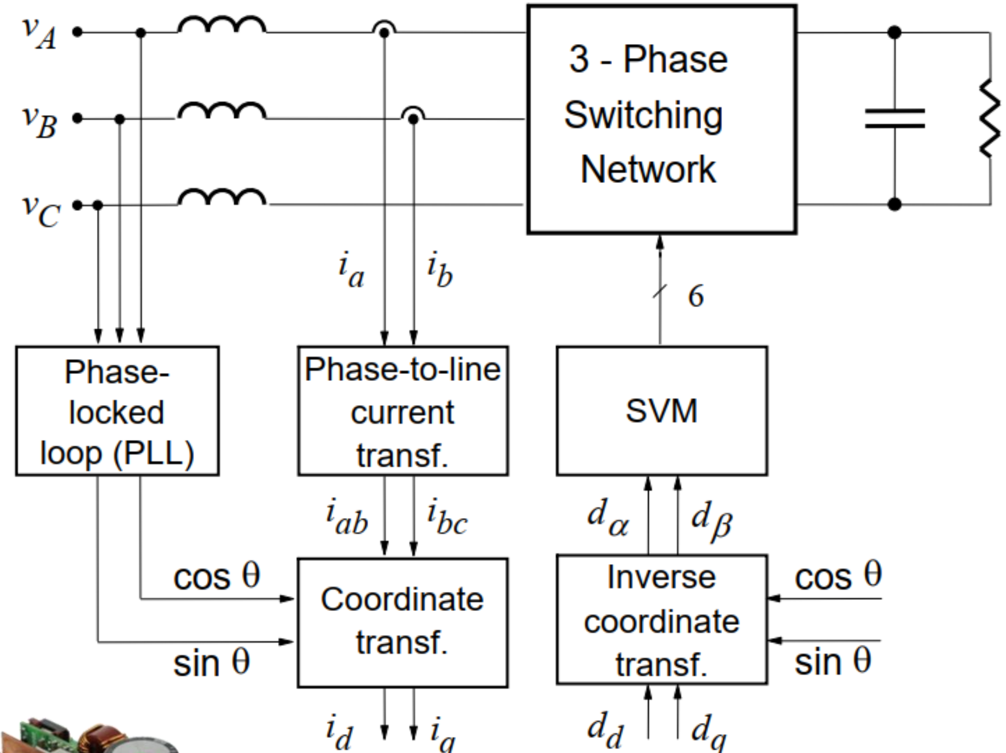
$$T_s = T_1 + T_2 + T_0 \rightarrow 1 = d_1 + d_2 + d_0$$

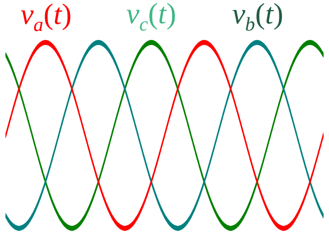
- $\vec{V}_{\text{ref}}$ is synthetized in the $\alpha\beta$ frame and will then be converted in variable setpoints $v_a$, $v_b$ and $v_c$

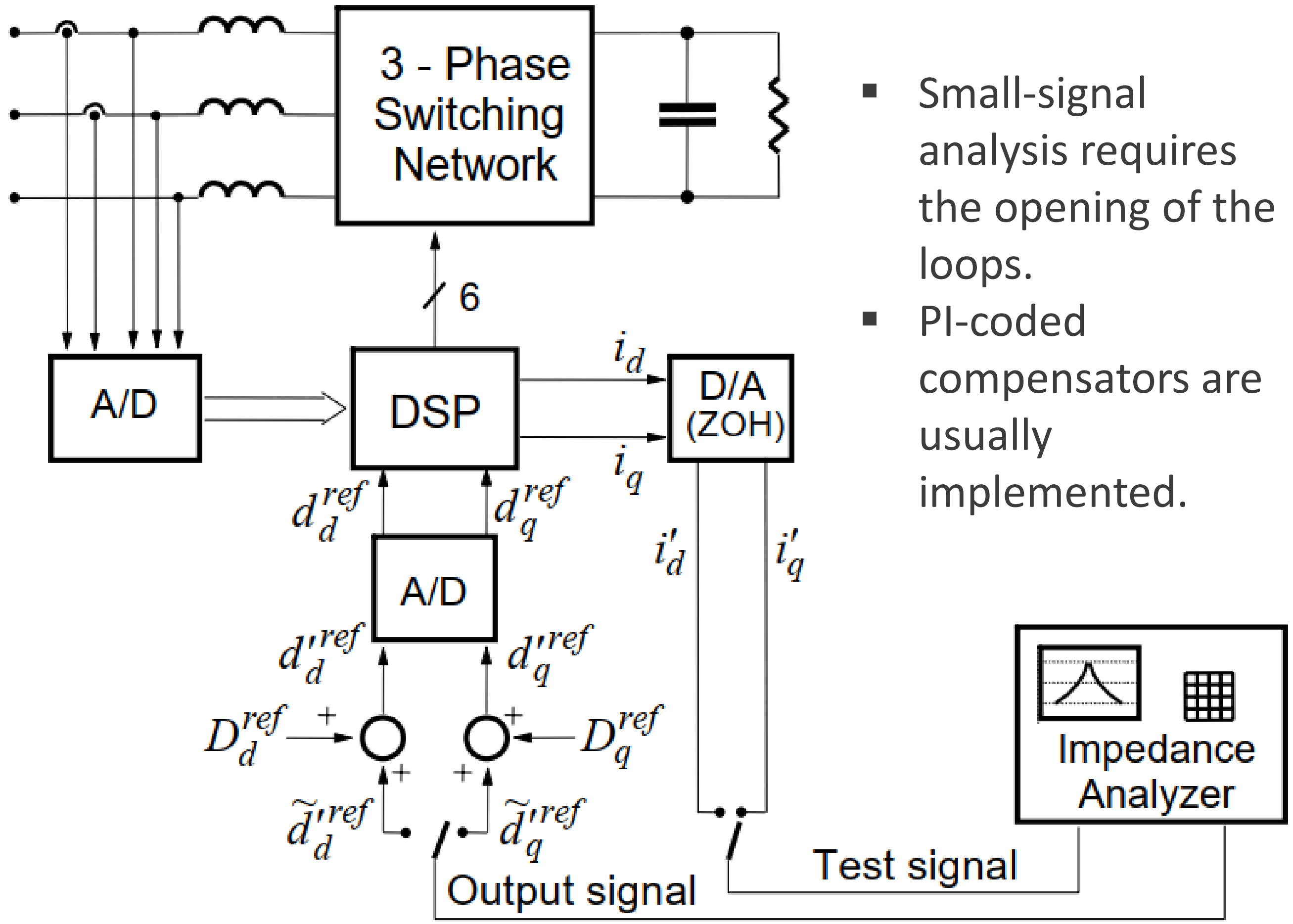
S. Hiti, *Modeling and Control of Three-Phase PWM Converters*, Ph. D Thesis, Virginia TECH, 1995

# 3-Phase Boost Rectifier with SVM

- In this example, $dq$ references are transformed into $\alpha\beta$ coordinates before SVM



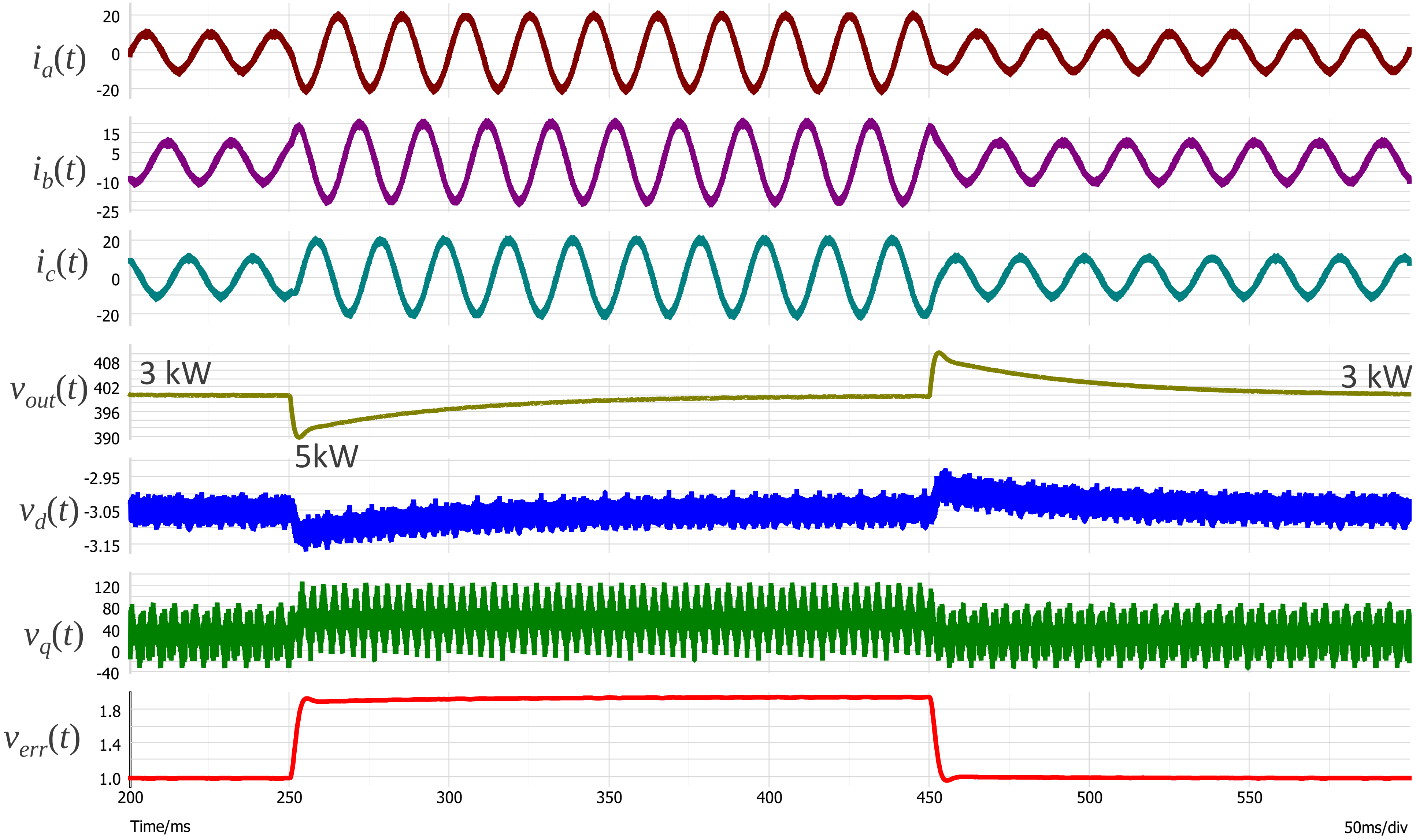




- Small-signal analysis requires the opening of the loops.
- PI-coded compensators are usually implemented.

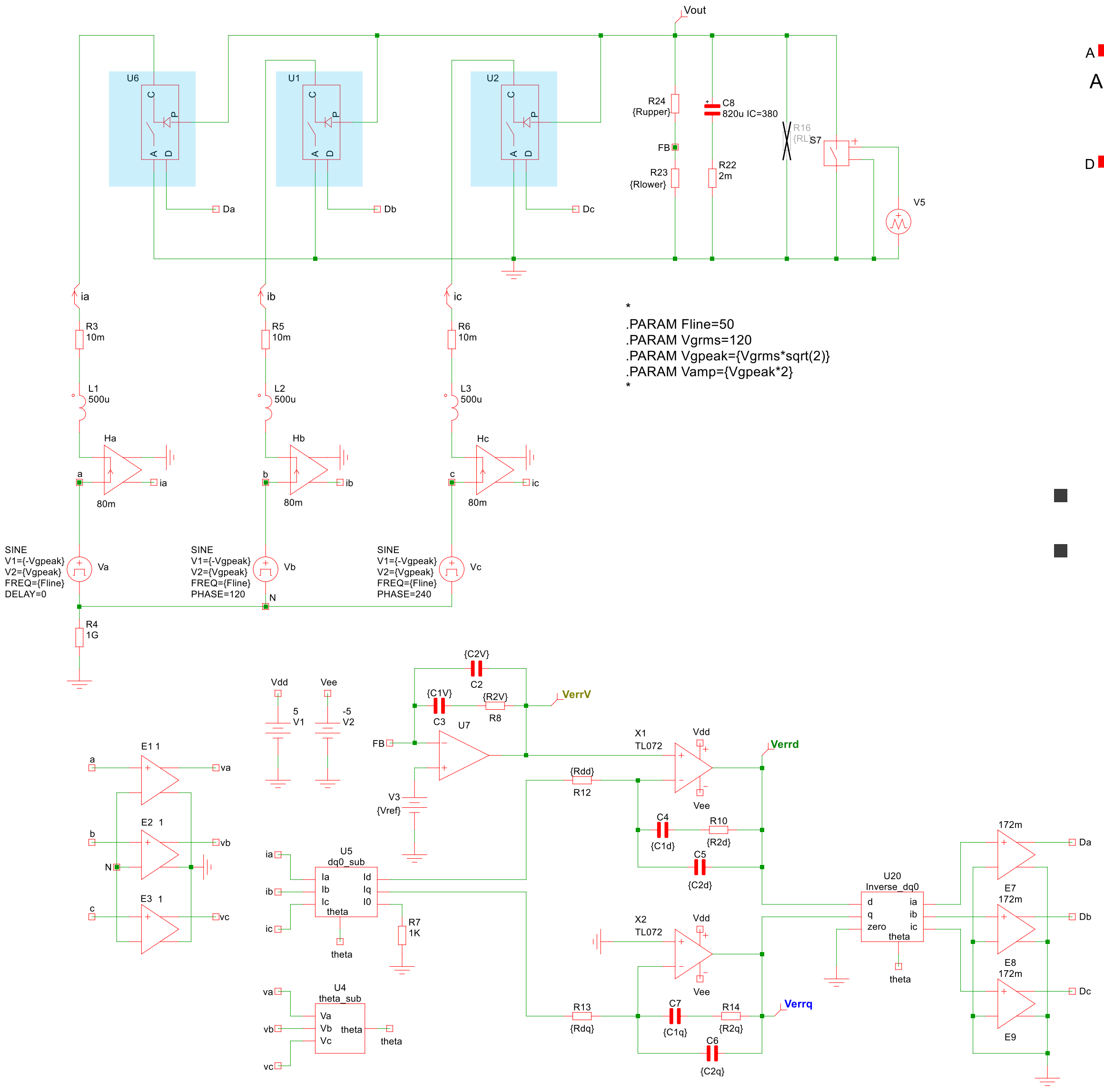
D. Boroyevich, *Modeling and Control of Three-Phase PWM Converters*, PECon 2008, Malaysia

# Transient Response of the SPWM Rectifier

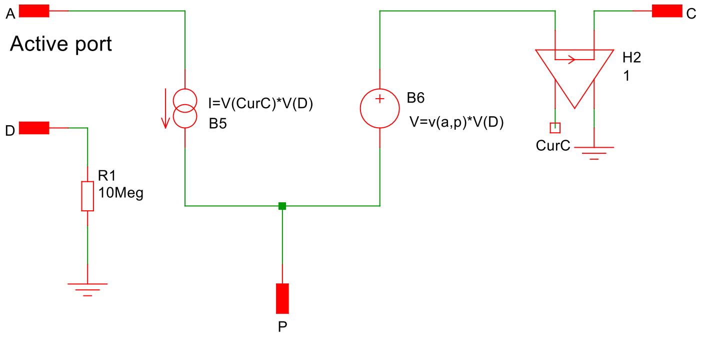


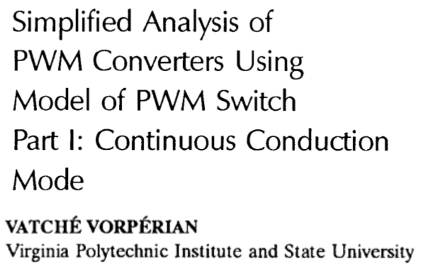
- The power is stepped from 3 kW to 5 kW in a few µs
- The response is good and does not show oscillations
- The $d$ and $q$ compensated signals react quickly

# Average Model of the 3-Phase PFC



```
*
.PARAM Fline=50
.PARAM Vgrms=120
.PARAM Vgpeak={Vgrms*sqrt(2)}
.PARAM Vamp={Vgpeak*2}
*
```

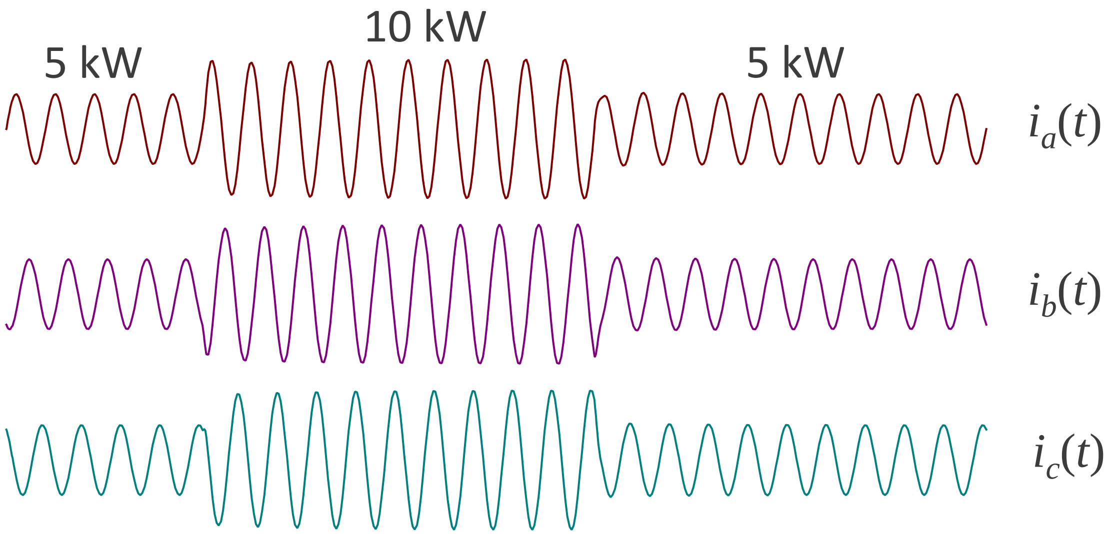



Simplified Analysis of
PWM Converters Using
Model of PWM Switch
Part I: Continuous Conduction
Mode

VATCHÉ VORPÉRIAN
Virginia Polytechnic Institute and State University

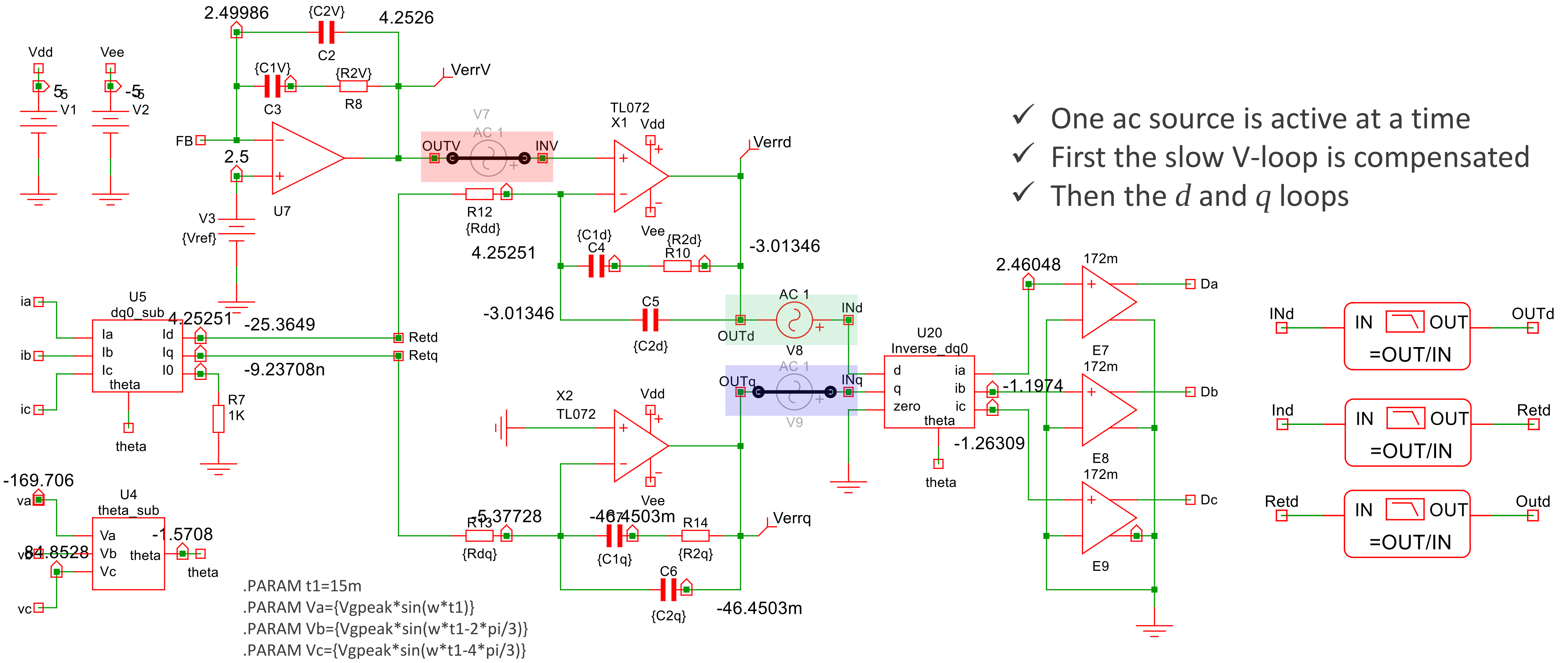
V. Vorpérian, *Simplified analysis of PWM converters using model of PWM switch. Continuous conduction mode*, in *IEEE Transactions on Aerospace and Electronic Systems*, vol. 26, no. 3,, May 1990

- I have replaced the switching cell by the VM PWM switch model
- Pulse-width modulators are replaced by small-signal gains



# Control-to-Output Transfer Functions

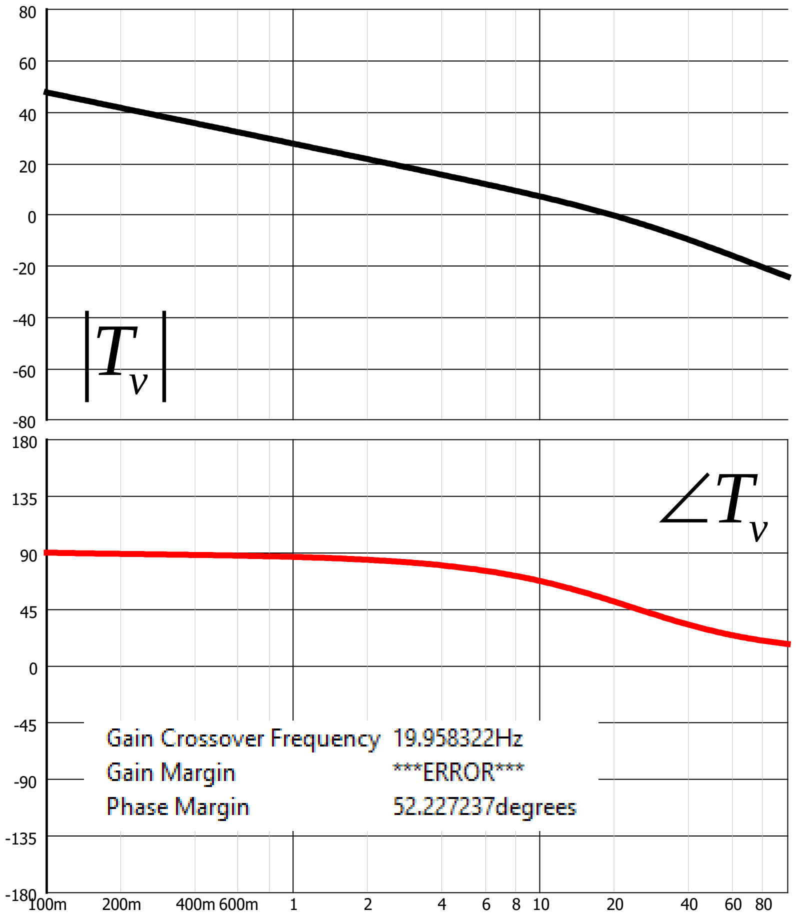
- Three loops: $V_{out}$ to be regulated at 400 V and the $d$ and $q$ paths
- The voltage loop will be closed with a 20-Hz crossover while $f_c$ for $d$ and $q$ will be 2 kHz



- ✓ One ac source is active at a time
- ✓ First the slow V-loop is compensated
- ✓ Then the $d$ and $q$ loops

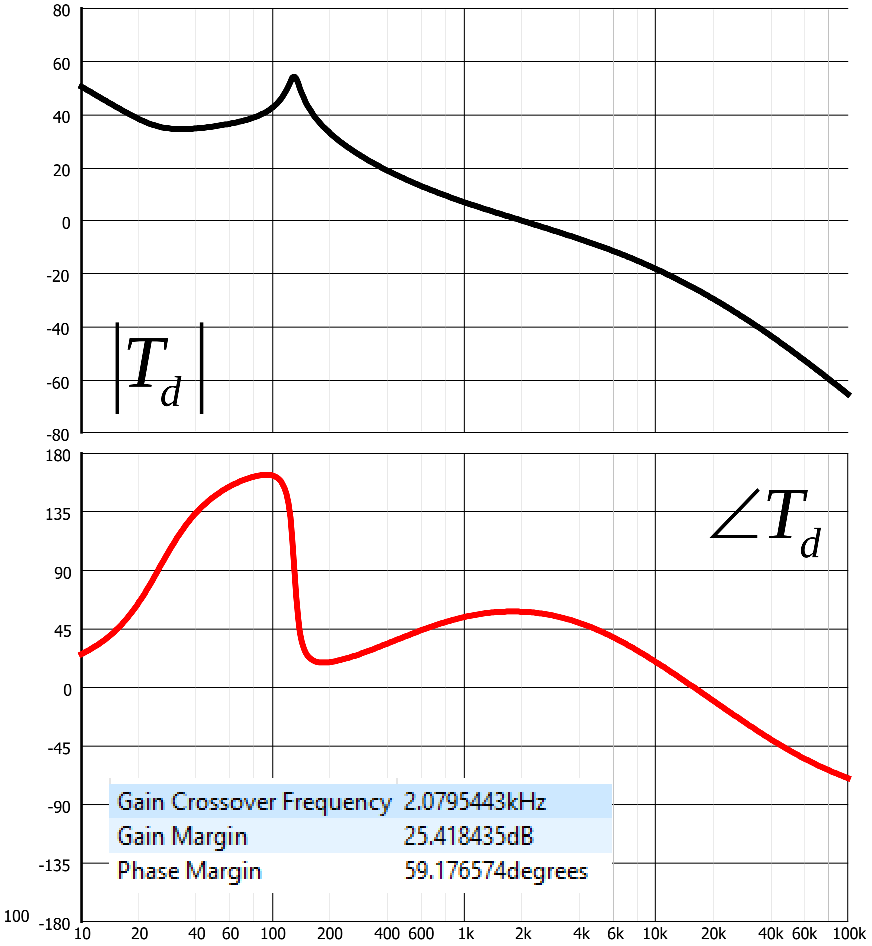
# Compensation Strategies

- The three loops are individually compensated with adequate phase and gain margins



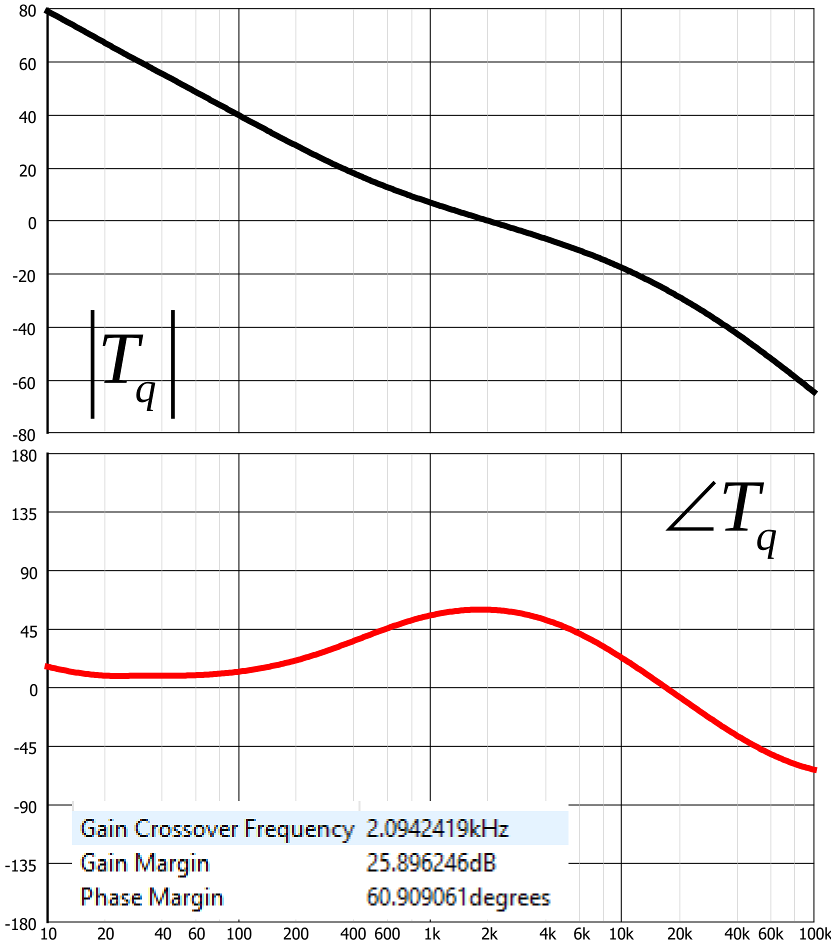
| | |
|---|---|
| Gain Crossover Frequency | 19.958322Hz |
| Gain Margin | ***ERROR*** |
| Phase Margin | 52.227237degrees |

Voltage loop – $f_c$ = 20 Hz



| | |
|---|---|
| Gain Crossover Frequency | 2.0795443kHz |
| Gain Margin | 25.418435dB |
| Phase Margin | 59.176574degrees |

*d* loop – $f_c$ = 2 kHz

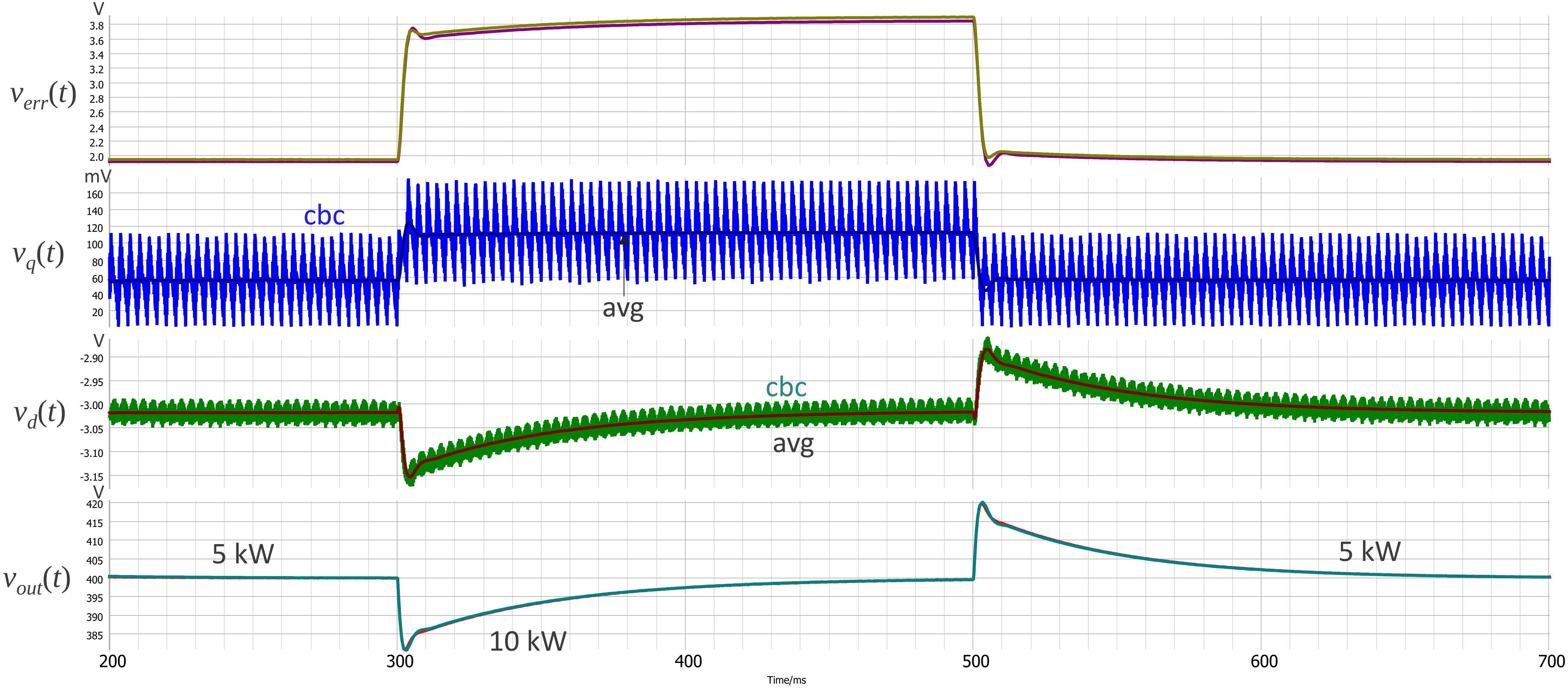


| | |
|---|---|
| Gain Crossover Frequency | 2.0942419kHz |
| Gain Margin | 25.896246dB |
| Phase Margin | 60.909061degrees |

*q* loop – $f_c$ = 2 kHz

# Comparison between Models

- Excellent correlation between cycle-by-cycle and average model waveforms



# Conclusion

- Tri-phase electrical networks are generated from spinning machines
- Modeling and controlling a three-variable system is complicated

✓ Clarke’s equations transform a 3-phase network into a 2-component frame

✓ Park’s equations transform the 2 ac components into dc variables, d and q

➢ Regulation on static variables is simpler and various techniques apply

- Numerous modulation schemes exist such as SPWM and SVM
- Average modeling helps extracting ac responses for loop control
- Implementation is rarely analogue but done via DSPs or micro-controllers