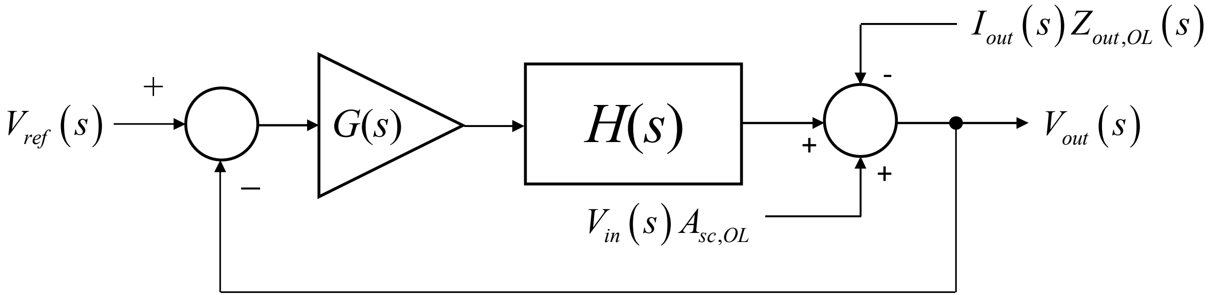
Christophe Basso
December 2020
Rev. 0.11
Typo on arg(TOL) found by Evan Zhang – corrected April 2024

Here is a typical block-diagram representation of a closed-loop system:



Based on this unity-return system, we can write:

$$V_{out}(s) = V_{ref}(s)\frac{T_{OL}(s)}{1+T_{OL}(s)} - I_{out}(s)\frac{Z_{out,OL}(s)}{1+T_{OL}(s)} + V_{in}\frac{A_{SC,OL}}{1+T_{OL}(s)}$$

In this expression, the input voltage and the output current are perturbations rejected by the system. The feedback actually enforces the ability of the open-loop converter to reject these perturbations. We can write:

$$Z_{out,CL}(s) = \frac{Z_{out,OL}(s)}{1+T_{OL}(s)}$$

$$A_{SC,CL}(s) = \frac{A_{SC,OL}}{1+T_{OL}(s)}$$

Where $Z_{out,CL}$ is the closed-loop output impedance and $A_{SC,CL}$ represents the closed-loop audio susceptibility or power supply rejection ratio (PSRR).

$$S(s) = \frac{1}{1+T_{OL}(s)}$$

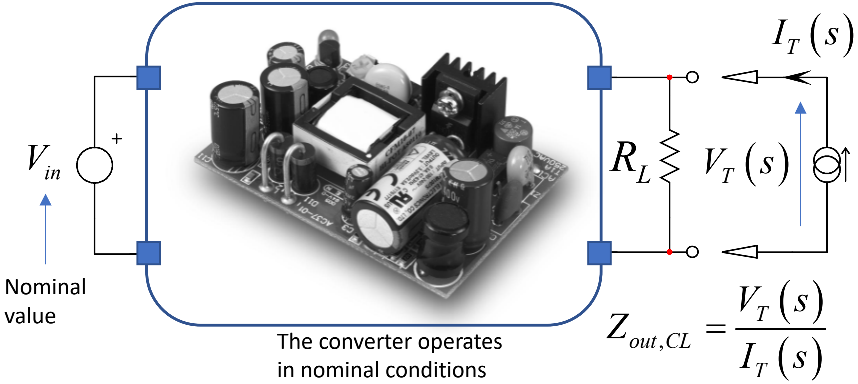
is the sensitivity function.

By measuring either the output impedance or the PSRR, we have a means to determine the open-loop gain T(s):

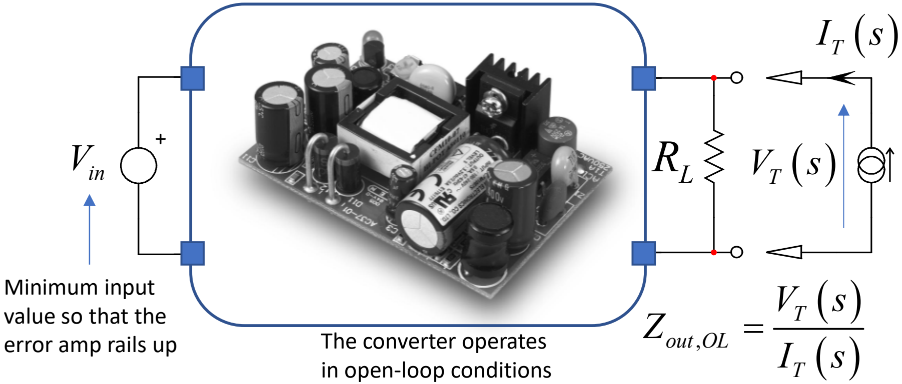
$$Z_{out,CL}(s) = \frac{Z_{out,OL}(s)}{1+T_{OL}(s)} \Rightarrow Z_{out,CL}(s)\left[1+T_{OL}(s)\right] = Z_{out,OL}(s) \Rightarrow T_{OL}(s) = \frac{Z_{out,OL}(s)-Z_{out,CL}(s)}{Z_{out,CL}(s)}$$

$$A_{SC,CL} = \frac{A_{SC,OL}}{1+T_{OL}(s)} \Rightarrow A_{SC,CL}(s)\left[1+T_{OL}(s)\right] = A_{SC,OL}(s) \Rightarrow T_{OL}(s) = \frac{A_{SC,OL}(s)-A_{SC,CL}(s)}{A_{SC,CL}(s)}$$

However, measuring the input voltage rejection in a high-gain system may be a perilous exercise considering the small amplitude collected on the power supply output. Measuring the open- and closed-loop output impedance is the way to go. Two measurements are required:



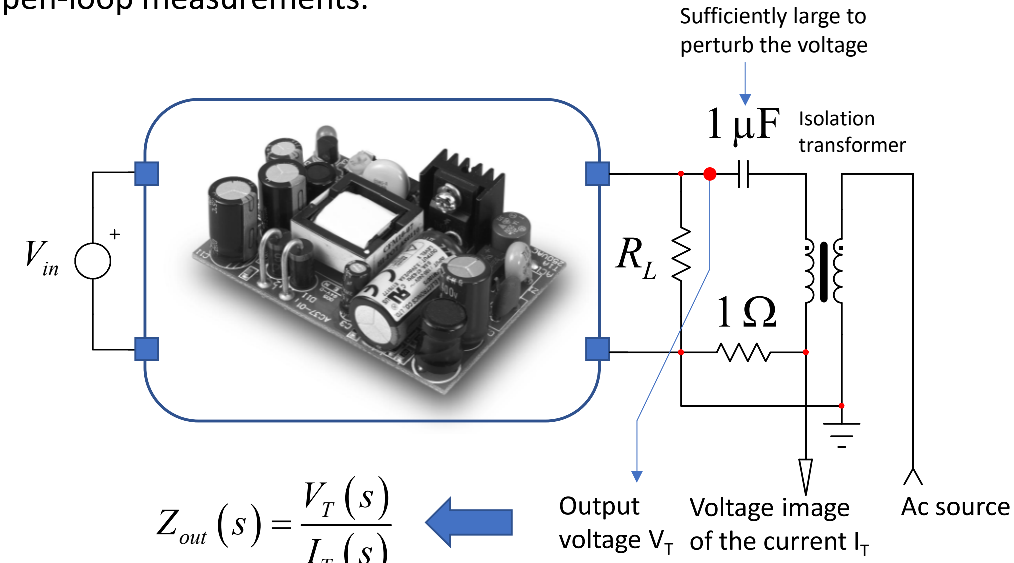
$$Z_{out,CL} = \frac{V_T(s)}{I_T(s)}$$



$$Z_{out,OL} = \frac{V_T(s)}{I_T(s)}$$

R. D. Middlebrook, *Measurement of Loop Gain in Feedback Systems*, International Journal of Electronics, vol. 38, no. 4, pp. 485-512 (1975)

Measuring the output impedance in closed-loop condition requires care and the adequate injection circuit. Also, one possible issue is to bring open-loop operation of the converter. The principle to make this happen is to force the error amplifier to rail up: once saturated, it can no longer react to the output stimulus and the converter runs in open-loop conditions. Unfortunately, many controllers host a protection circuitry which stops operations when the loop is lost. A way to overcome this is to disable the protection scheme when some dedicated pins are available. Also, some converters such as the flyback converter offer, in current-mode control, an open-loop output impedance equal to the load resistance paralleled with the output capacitor. Turning off the converter for measuring Z_{out} is thus the way to go for $Z_{out,OL}$. Each converter will thus require some study to determine the correct output impedance configuration for open-loop measurements.



$$Z_{out}(s) = \frac{V_T(s)}{I_T(s)}$$

R. Ridley, *Output Impedance Measurements and Loop Gains*, article [57], https://ridleyengineering.com/

Reconstructing the loop gain from the equation requires the manipulation of complex numbers:

$$Z_{out,OL} = x_{OL} + jy_{OL} \qquad Z_{out,CL} = x_{CL} + jy_{CL}$$

$$T_{OL}(s) = \frac{Z_{out,OL}(s) - Z_{out,CL}(s)}{Z_{out,CL}(s)}$$

$$\left|Z_{out,OL} - Z_{out,CL}\right| = \sqrt{\left(x_{OL} - x_{CL}\right)^2 + \left(y_{OL} - y_{CL}\right)^2}$$

$$20\log_{10}\left|T_{OL}\right| = 20\log_{10}\left(\frac{\left|Z_{out,OL} - Z_{out,CL}\right|}{\left|Z_{out,CL}\right|}\right) = 20\log_{10}\frac{\sqrt{\left(x_{OL} - x_{CL}\right)^2 + \left(y_{OL} - y_{CL}\right)^2}}{\sqrt{x_{CL}{}^2 + y_{CL}{}^2}}$$

$$\arg T_{OL} = \arg\left(Z_{out,OL} - Z_{out,CL}\right) - \arg\left(Z_{out,CL}\right)$$

$$\arg\left(Z_{out,OL} - Z_{out,CL}\right) = \tan^{-1}\left(\frac{y_{OL} - y_{CL}}{x_{OL} - x_{CL}}\right)$$

$$\arg\left(T_{OL}\right) = \tan^{-1}\left(\frac{y_{OL} - y_{CL}}{x_{OL} - x_{CL}}\right) - \tan^{-1}\left(\frac{y_{CL}}{x_{CL}}\right)$$

N. Tsukiji, Y. Kobori, H. Kobayashi, *Derivation of Loop Gain from Output Impedances in DC-DC Buck Converter*, Web download

In case magnitude and phase curves are only available, you'll have to reconstruct the imaginary points:

$$z = x + jy \quad \begin{cases} |z| = \sqrt{x^2 + y^2} \\ \angle z = \tan^{-1}\dfrac{y}{x} \end{cases}$$

First, convert the magnitude expressed in dB into an uncompressed value:

$G = 10^{\frac{G_{dB}}{20}}$ If G_{dB} is 45 dB then G is: $G = 10^{\frac{45}{20}} = 177.82$

Using the trigonometric form, we can obtain x and y easily:

$$z = G(\cos\varphi + j\sin\varphi) = x + jy$$

$$x = G \cdot \cos(\varphi) \qquad y = G \cdot \sin(\varphi)$$

in which φ is the phase delivered by the analyzer

$G_1 := 25$ dB

$G_2 := 10^{\frac{G_1}{20}} = 17.783$ $z := -95°$

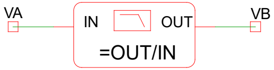
$x := G_2 \cdot \cos(z) = -1.55$

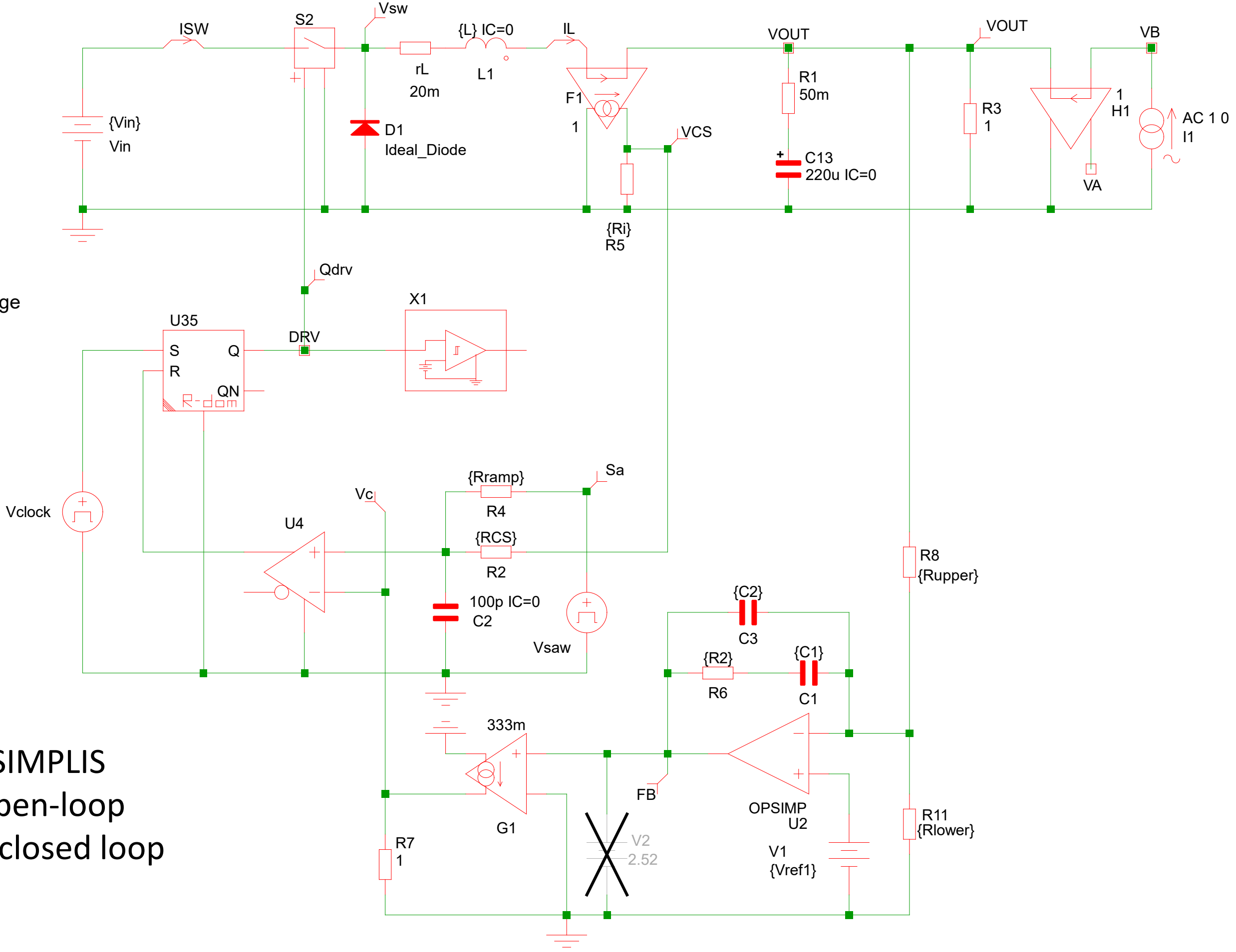
$y := G_2 \cdot \sin(z) = -17.715$

$\mathrm{atan2}(x, y) = -95°$ *atan2* accounts for x and y signs

$\sqrt{x^2 + y^2} = 17.783$

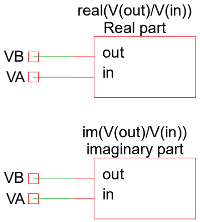
$$\arg(x + jy) = \begin{cases} \tan^{-1}(y/x), & x > 0, & \text{(right half-plane)}, \\ \tan^{-1}(y/x) + \pi, & x < 0, & \text{(left half-plane)}, \\ \pi/2, & x = 0, y > 0, & (+j\text{-axis}), \\ -\pi/2, & x = 0, y < 0, & (-j\text{-axis}), \\ \text{undefined}, & x = 0, y = 0, & \text{(origin)}. \end{cases}$$





In this compensated buck, extract the power stage magnitude and phase at the selected crossover frequency and via F11 enter these data.

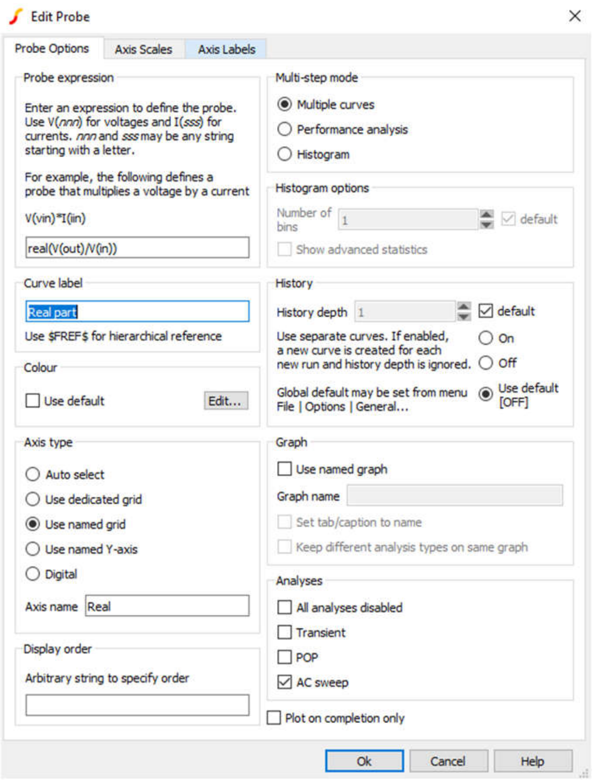
Christophe Basso.

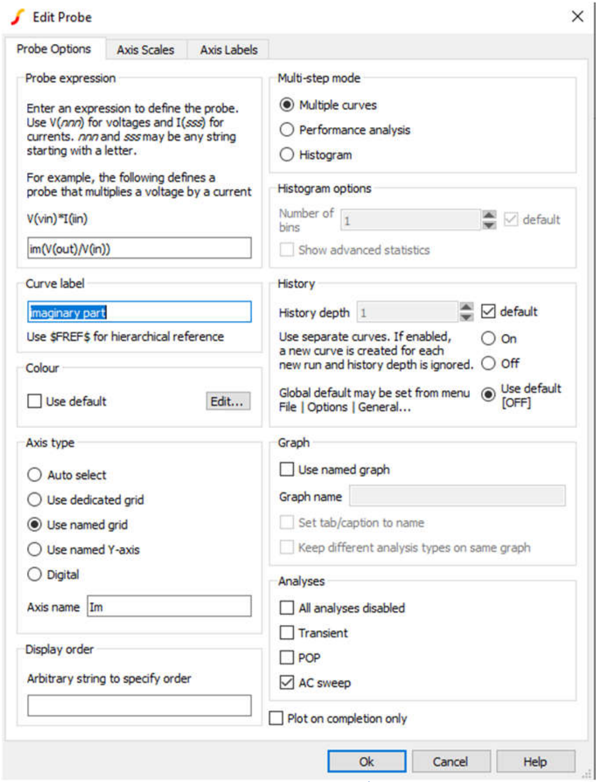


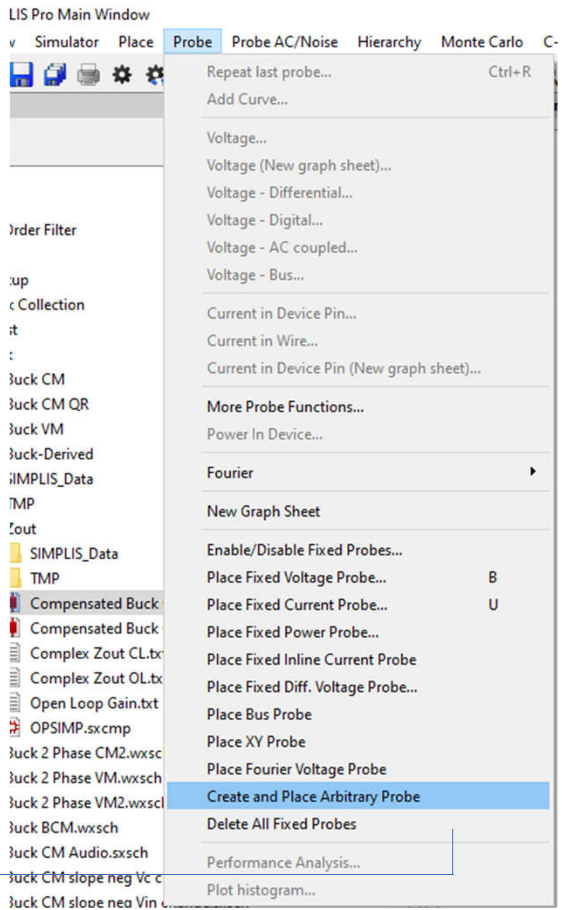
This is the simulation template in SIMPLIS

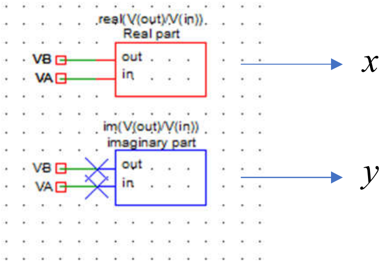
U_2 is disabled and V_2 is enabled: open-loop

U_2 is enabled while V_2 is disabled: closed loop

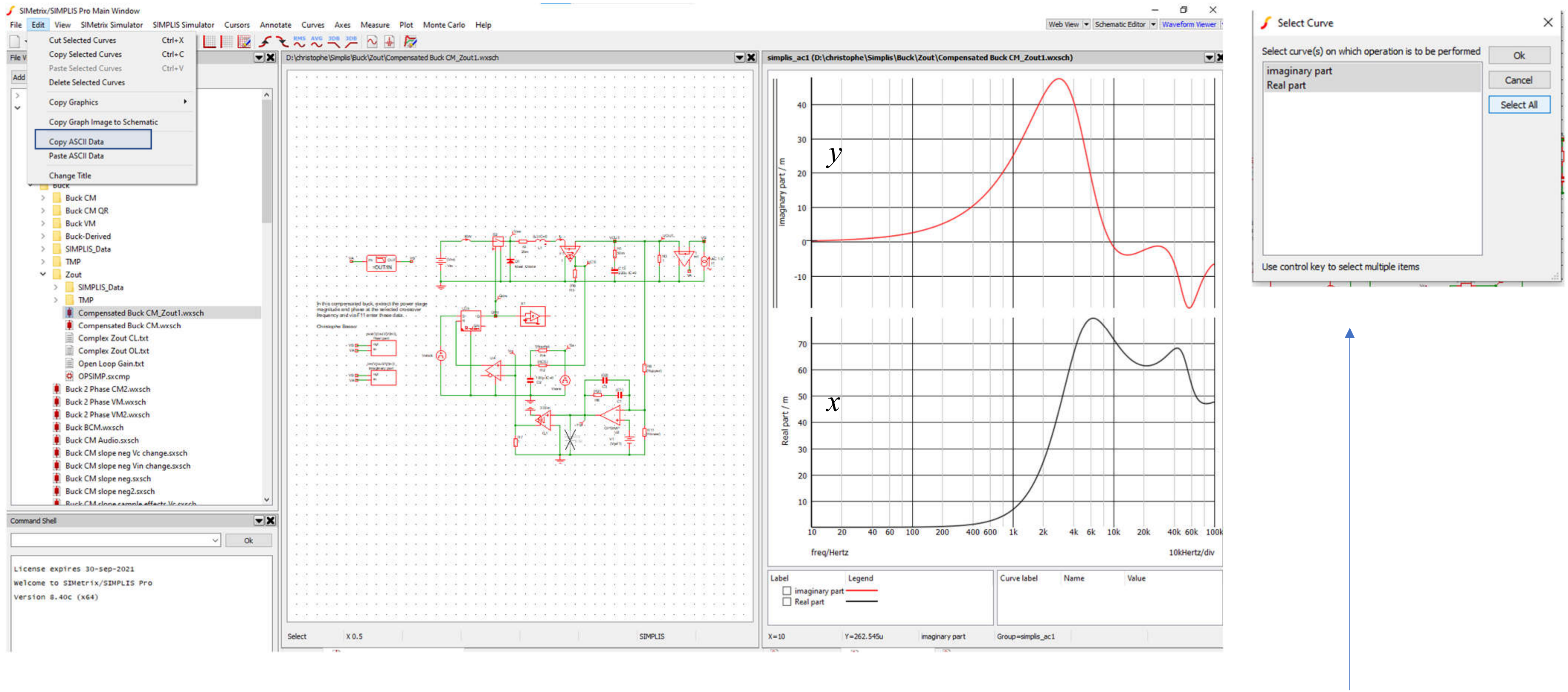




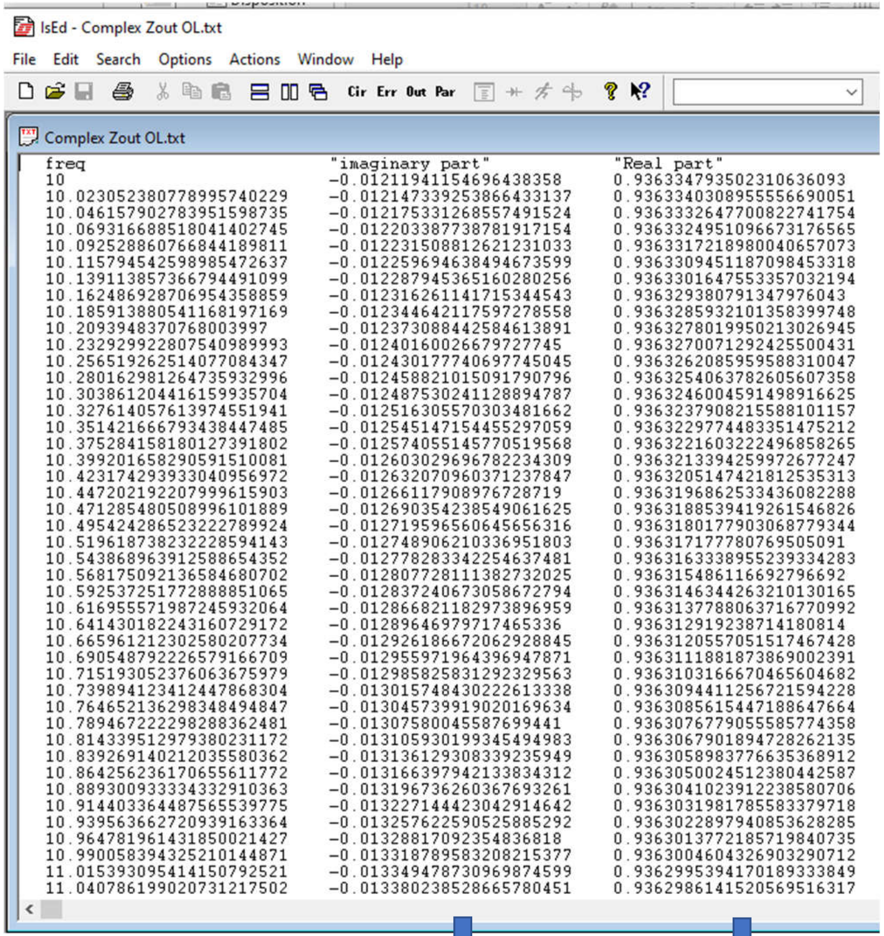




You need to place two specific probes to extract the real and imaginary parts.



Once the simulation is complete, you have to export the real and imaginary data in a text file located in the same directory as the future Mathcad file.



freq	"imaginary part"	"Real part"
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10.046157902783951598735	−0.012175331268557491524	0.9363332647700822741754
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10.162486928706954358859	−0.012316261141715344543	0.936329380791347976043
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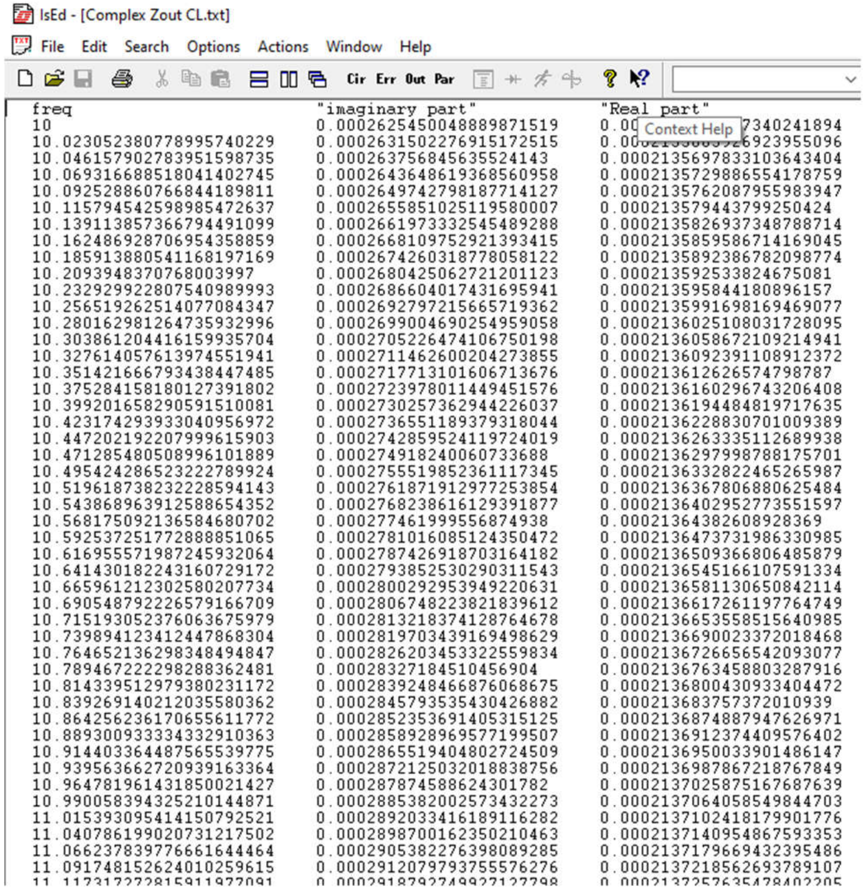

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[illegible]	[illegible]	[illegible]

Mathcad array name → $Z_{outOL}^{\langle 1\rangle} \rightarrow y_{OL}$ $Z_{outOL}^{\langle 2\rangle} \rightarrow x_{OL}$

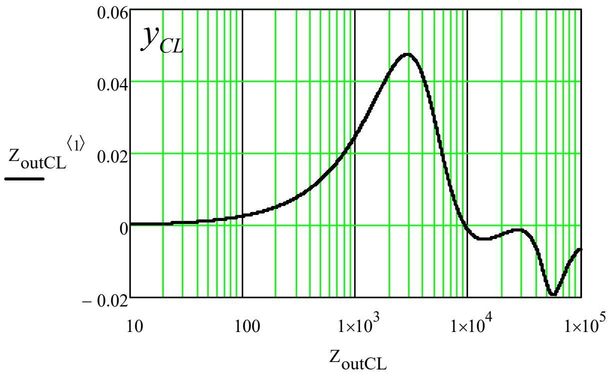
Frequency counter

$Z_{outCL}^{\langle 1\rangle} \rightarrow y_{CL}$ $Z_{outCL}^{\langle 2\rangle} \rightarrow x_{CL}$

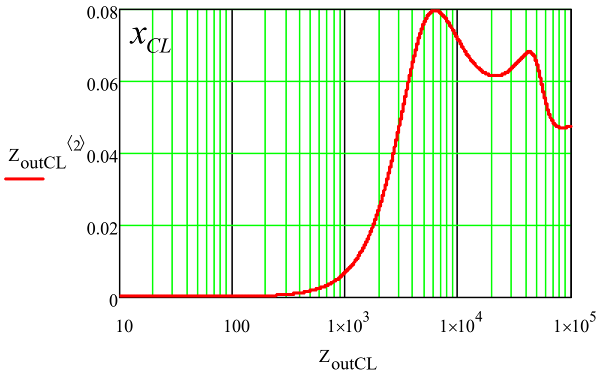
$Z_{outOL} := \text{READPRN("Complex Zout OL.txt")}$

$Z_{outCL} := \text{READPRN("Complex Zout CL.txt")}$

This makes Mathcad read the .TXT files

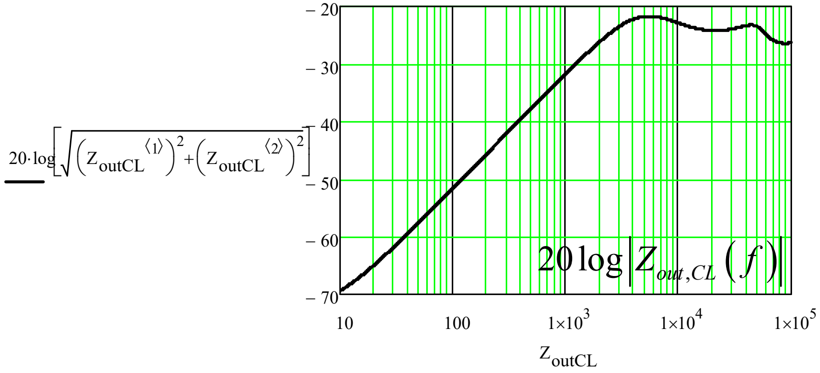


Imaginary part of the closed-loop impedance

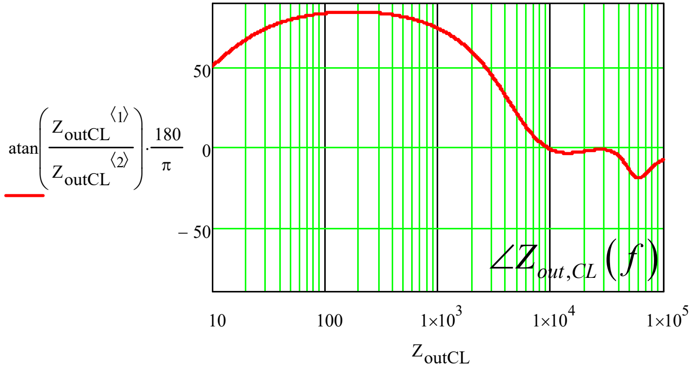


Real part of the closed-loop impedance

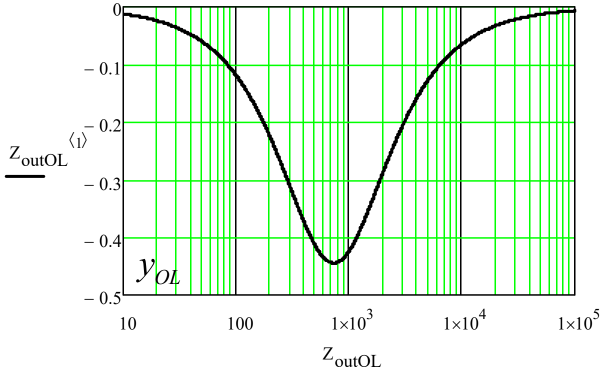
Check this is the same shape as in the SIMPLIS simulation. Same for the reconstruction of the magnitude and phase response obtained from x and y.



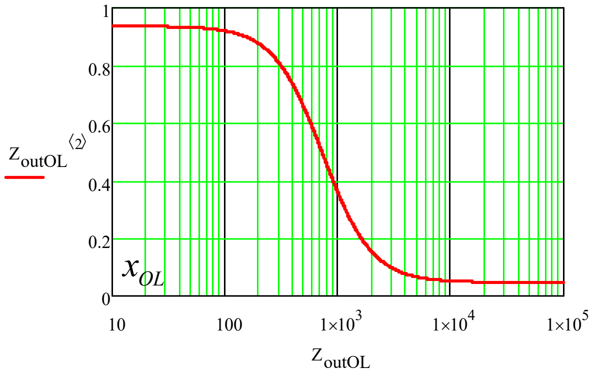
Magnitude of closed-loop output impedance in dBohms



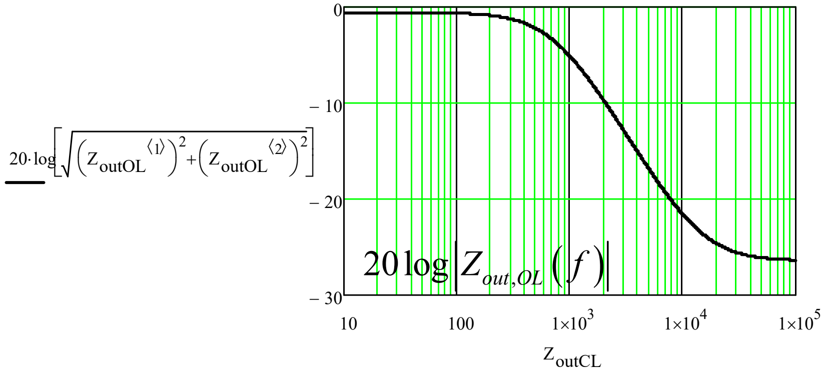
Phase of closed-loop output impedance



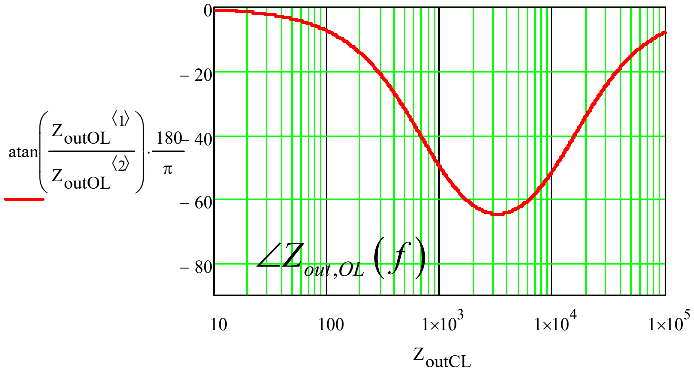
Imaginary part of the open-loop impedance



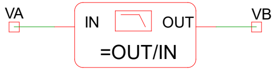
Real part of the open-loop impedance

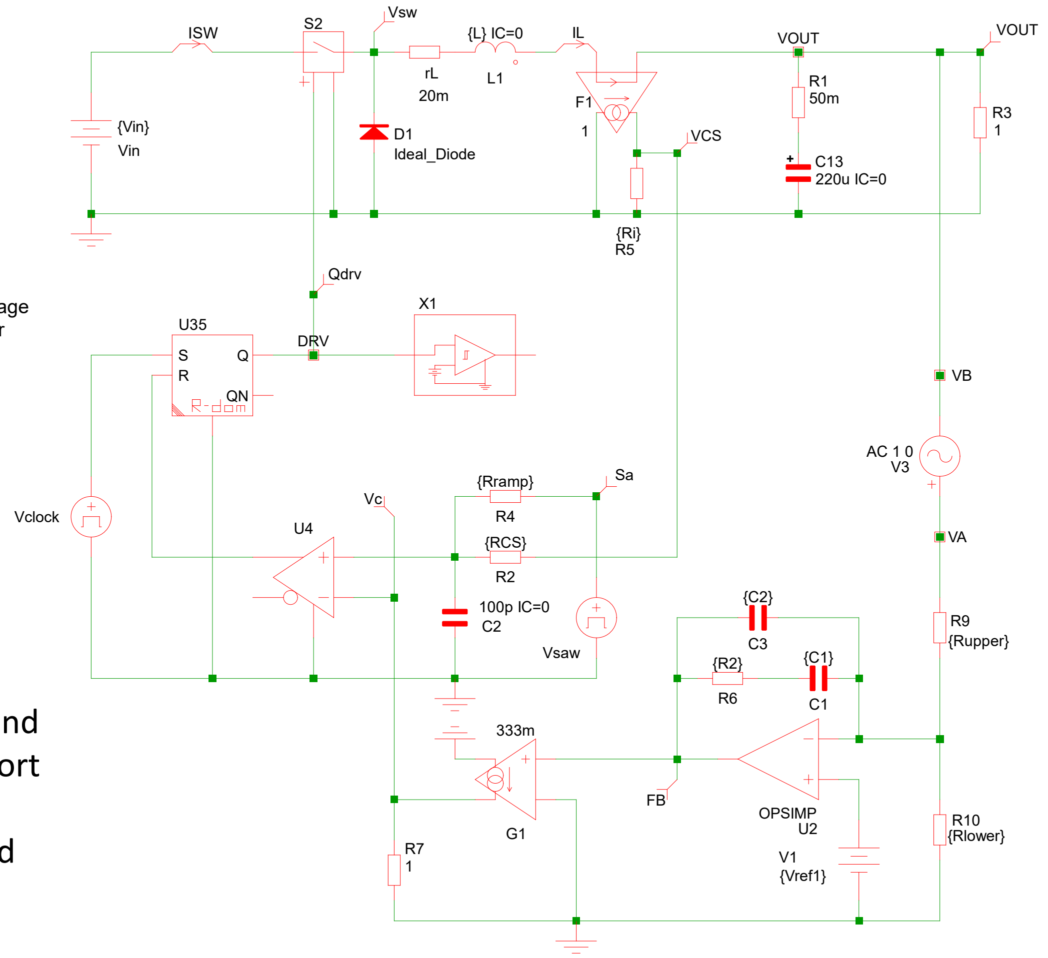


Magnitude of open-loop output impedance in dBohms



Phase of open-loop output impedance

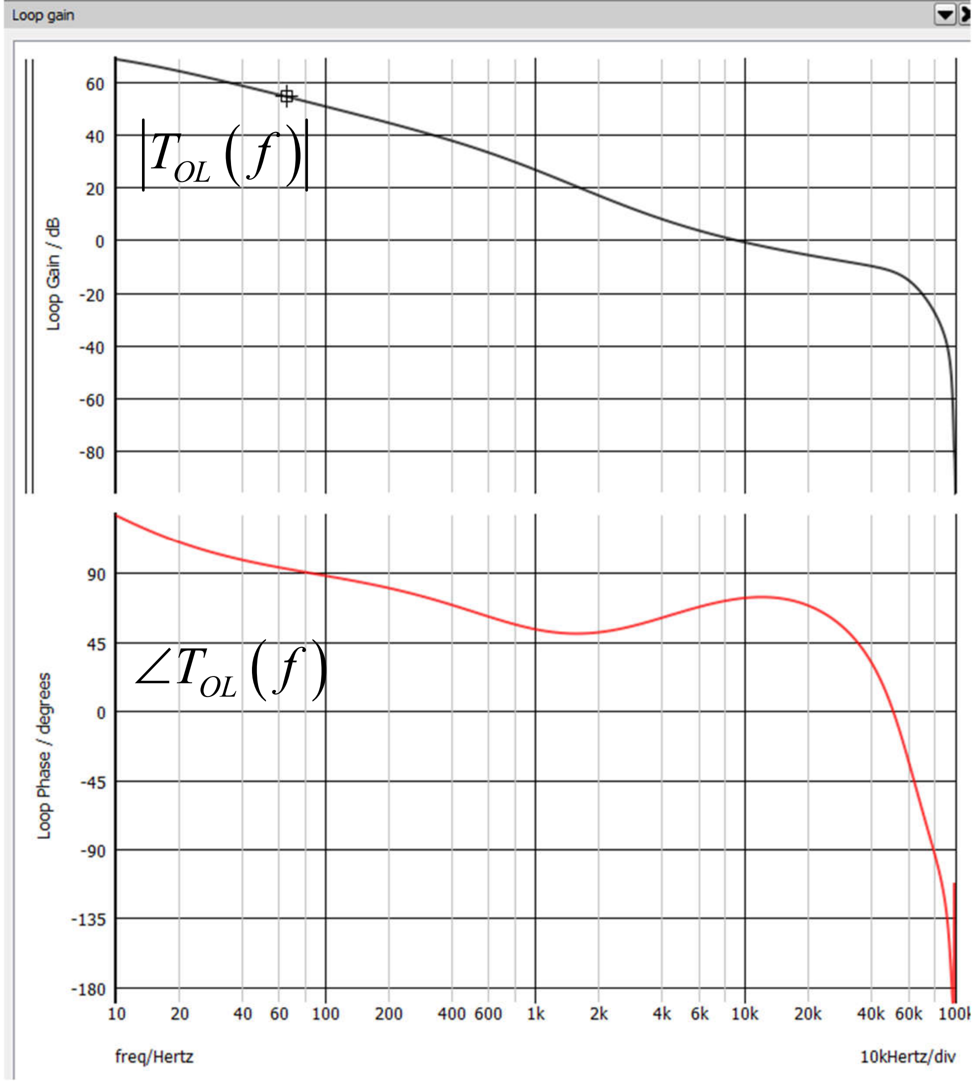




In this compensated buck, extract the power stage magnitude and phase at the selected crossover frequency and via F11 enter these data.

Christophe Basso.

Now run the complete buck converter and extract the compensated loop gain. Export the magnitude and phase response in a .TXT file to later import them in Mathcad as previously done.



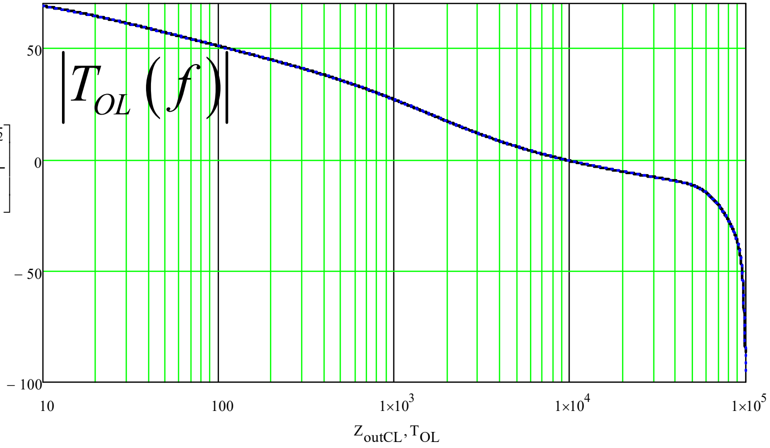
These are the phase and magnitude delivered by SIMPLIS in this open-loop gain measurement configuration. Now export the data into a .TXT file for a treatment in Mathcad.

Magnitude reconstruction from Z_{out}

$$20\cdot\log\left[\frac{\sqrt{\left(Z_{outOL}^{\langle 1\rangle}-Z_{outCL}^{\langle 1\rangle}\right)^2+\left(Z_{outOL}^{\langle 2\rangle}-Z_{outCL}^{\langle 2\rangle}\right)^2}}{\sqrt{\left(Z_{outCL}^{\langle 1\rangle}\right)^2+\left(Z_{outCL}^{\langle 2\rangle}\right)^2}}\right]$$

$$T_{OL}^{\langle 1\rangle}$$

SIMPLIS magnitude import



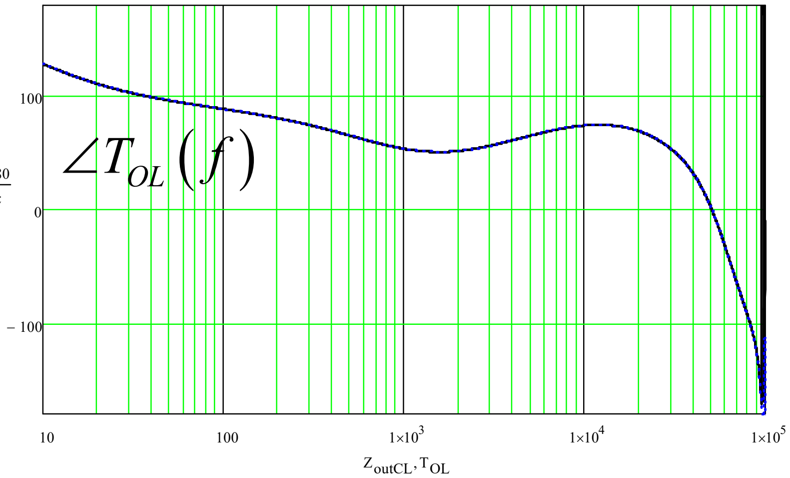
Open-loop phase reconstruction and comparison with true open-loop phase measurement

Phase reconstruction from Z_{out}

$$\left[\text{angle}\left(Z_{outOL}^{\langle 2\rangle}-Z_{outCL}^{\langle 2\rangle}, Z_{outOL}^{\langle 1\rangle}-Z_{outCL}^{\langle 1\rangle}\right)-\pi-\left(\text{atan2}\left(Z_{outCL}^{\langle 2\rangle}, Z_{outCL}^{\langle 1\rangle}\right)\right)\right]\cdot\frac{180}{\pi}$$

$$T_{OL}^{\langle 2\rangle}$$

SIMPLIS phase import



As you can see, the reconstruction is excellent and there is no deviation between the real measurement and the one using the output impedance measurements. Bench measurements will obviously differ considering noise and deviation at higher frequencies.