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Small-Signal Analysis of the Constant Peak Current-Operated Flyback Converter in Frequency Foldback

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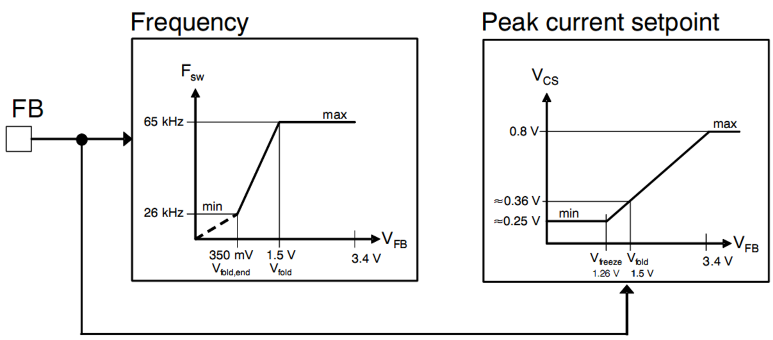
Reducing the Switching Frequency

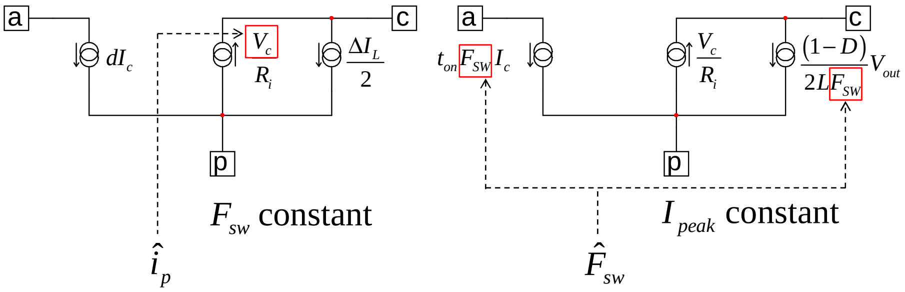
- ❑ New generation controllers reduce frequency in light load
- ❑ The peak current is frozen and frequency is controlled



- ❑ The relationship between $\hat{v}_{out}$ and $\hat{i}_p$ is known at constant F_{sw}
- ❑ What about the relationship linking $\hat{v}_{out}$ to $\hat{F}_{sw}$ at I_{peak} constant?

Where do We Start From?

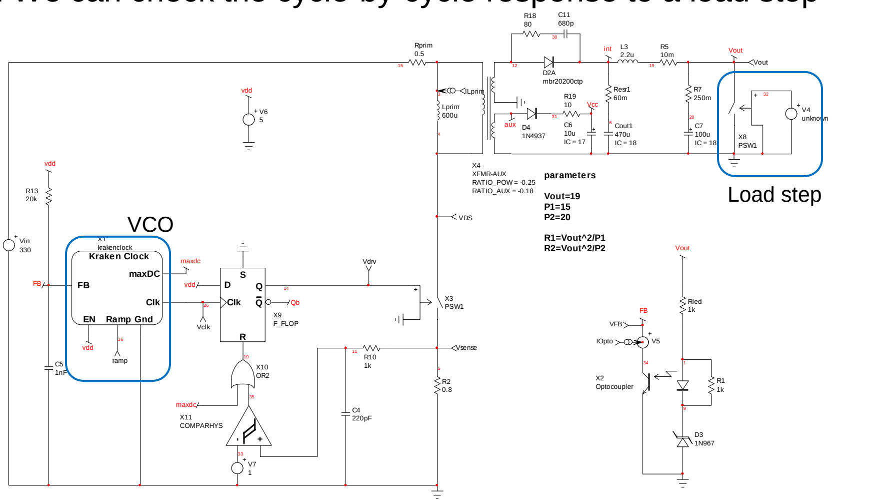
- ❑ We have a DCM peak-current mode control large-signal model
- ❑ The PWM switch controls the peak current at a fixed F_{sw}
- ❑ Why not fixing the peak current and controlling F_{sw}?



- ❑ The peak current is frozen and the frequency is controlled

A Transient Load Step Response First

- ❑ We can check the cycle-by-cycle response to a load step

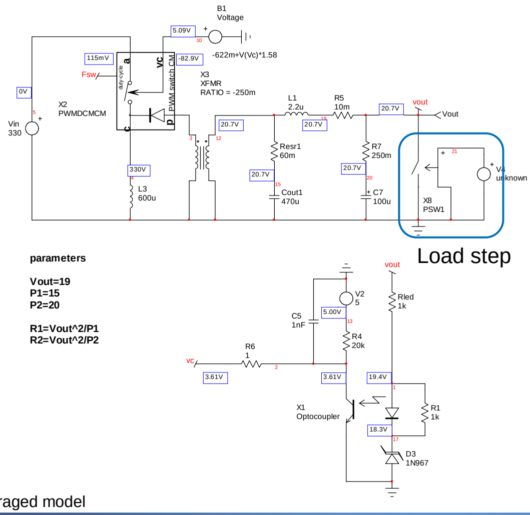


Cycle by cycle model

F_{sw} = 51 kHz

A Transient Load Step Response First

❑ And compare it to that of a modified PWM switch model



Averaged model

```
*
.param {Vc}=1
*
Bdc dcx 0 V =
{Vc}*V(Fsw)*10k/({Se}+(abs(v(a,c))*{Ri})/{L}+1u)
Xdc dcx dc limit params: clampH=0.99
clampL=7m
BVcp 6 p V=(V(dc)/(V(dc)+V(d2)))*V(a,p)
BIap a p I=(V(dc)/(V(dc)+V(d2)))*I(VM)
Bd2 d2X 0 V=(2*I(VM)*{L}-
v(a,c)*V(dc)^2*1/(V(Fsw)*10k)) / (
+v(a,c)*V(dc)*1/(V(Fsw)*10k)+1u )
Xd2 d2X dc d2 limit2
Rdum1 dc 0 1Meg
Rdum2 vc 0 1Meg
RS 7 c 1u
VM 6 7
*
.ENDS
```

Current frozen to $1/R_{sense}$

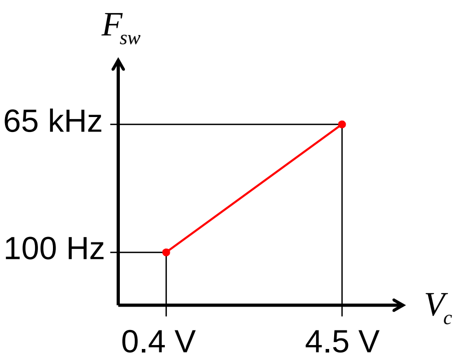
1 V = 10 kHz

Modified netlist of the PWM switch

F_{sw} = 50.9 kHz

You Need to Adjust the VCO Modulator

❑ The control voltage to the switching frequency is the VCO gain



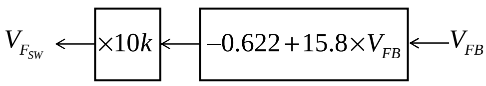
$$y = ax + b$$

$$y = \frac{\Delta F_{sw}}{\Delta V_c} = \frac{64.9k}{4.1} = 15.8\,\mathrm{kHz/V}$$

$$100 = ax + b = 0.4 \times 15.8k + b$$

$1/10000$ Scaling factor

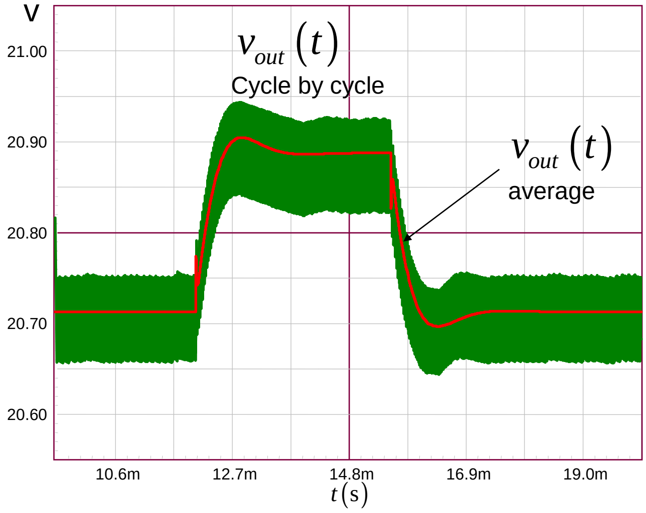
$$b = 0.10 - 0.4 \times 15.8 = -622\,\mathrm{mV}$$



VCO gain G_{VCO} is $15.8\,\mathrm{kHz/V}$

Compare the Transient Responses

❑ Responses are identical, the averaged version looks correct



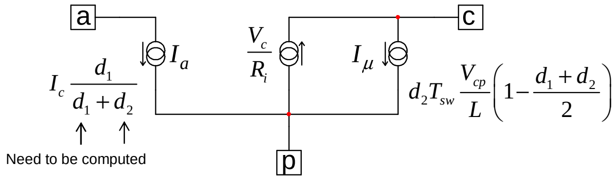
Keys of Small-Signal Analysis

❑ Rather than going full speed into small-signal analysis:

➢ break down the system into smaller parts

➢ run simplifications whenever you can

➢ go step by step and verify the answer always fits the original



Start from here, DCM model

"Switch-mode Power Supplies: SPICE Simulations and Practical Designs", C. Basso, McGraw-Hill 2008, page 161

Simplifying and Compacting the Model

❑ Compact the DCM model to suppress variables computing

$$I_a = I_c \frac{d_1}{d_1 + d_2} \qquad I_\mu = \frac{d_2 V_{cp}}{F_{sw} L_p}\left(1 - \frac{d_1 + d_2}{2}\right) \qquad d_1 = \frac{L_p V_c F_{sw}}{V_{ac} R_i} \qquad d_2 = \frac{2 I_c F_{sw} L_p}{d_1 V_{ac}} - d_1$$

substitute

"Let the craziness begin"

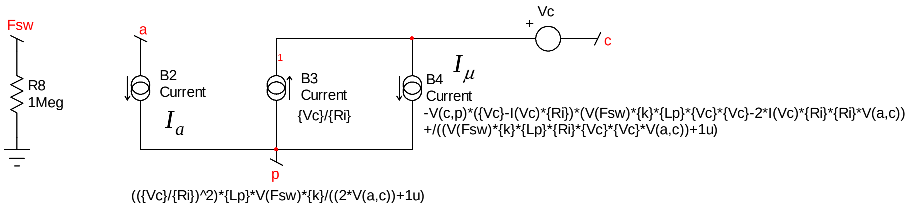
$$I_a = I_c \frac{d_1}{d_1 + d_2}$$

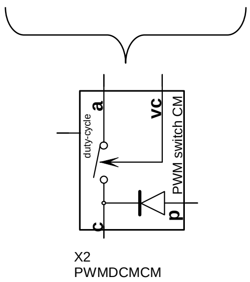
$$I_a = \frac{F_{sw} L_p V_c^{\,2}}{2 R_i^{\,2} V_{ac}}$$

$$I_\mu = -\frac{V_{cp}\left(V_c - I_c R_i\right)\left(F_{sw} L_p V_c^{\,2} - 2 I_c R_i^{\,2} V_{ac}\right)}{F_{sw} L_p R_i V_c^{\,2} V_{ac}}$$

The New Model Looks Simpler

❑ The equations no longer include a computed variable

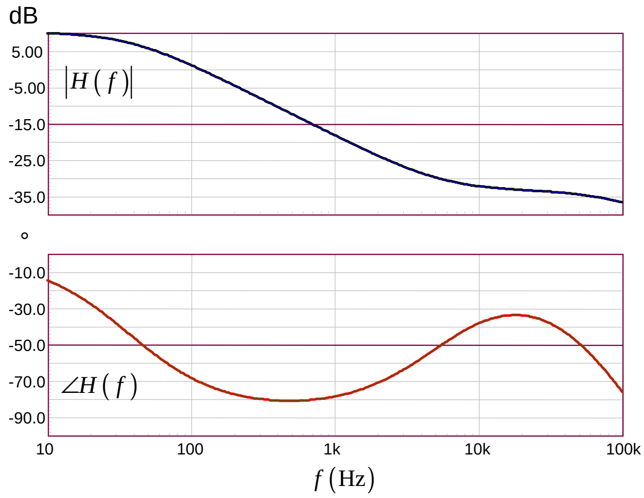




Check against the complete model

Ac Responses are Similar

❑ The curves perfectly superimpose, 1st step is ok



Second Step, Linearize the Sources

❑ To apply Laplace equations, we need linear elements
❑ Linearization can be done in different ways:

✓ perturb all equations with a small quantity (the "hat" notation)
✓ re-arrange the terms and collect dc and ac contributors
❖ can be tedious to re-arrange, you neglect cross products

$$V_1 = R_1 I_1 + D V_3$$

$$V_1 + \hat{v}_1 = R_1 \left(I_1 + \hat{i}_1\right) + \left(\hat{d} + D\right)\left(V_3 + \hat{v}_3\right)$$

$$V_1 = R_1 I_1 + D V_3 \quad \text{dc equation (bias point)}$$

$$\hat{v}_1 = R_1 \hat{i}_1 + \hat{d} V_3 + \underbrace{\boxed{\hat{d}\hat{v}_3}}_{\approx 0} + D\hat{v}_3 \quad \text{ac equation}$$

Second Step, Linearize the Sources

- ❑ A second option is to calculate partial derivative coefficients

✓ the process can be automated by Mathcad®
✓ you only have ac terms, no sort needed

$$V_1(I_1, D, V_3) = R_1 I_1 + D V_3$$

$$dV_1 = \left(\frac{\partial V_1(I_1, D, V)}{\partial I_1} \right) \Bigg|_{D,V_3} dI_1 + \frac{\partial V_1(I_1, D, V)}{\partial D} \Bigg|_{I_1,V_3} dD + \frac{\partial V(I_1, D, V)_1}{\partial V_3} \Bigg|_{D,I_1} dV_3$$

$$\hat{v}_1 = \frac{\partial V_1(I_1, D, V)}{\partial I_1} \Bigg|_{D,V_3} \hat{i}_1 + \frac{\partial V_1(I_1, D, V)}{\partial D} \Bigg|_{I_1,V_3} \hat{d} + \frac{\partial V_1(I_1, D, V)}{\partial V_3} \Bigg|_{D,I_1} \hat{v}_3$$

$$\hat{v}_1 = R_1 \hat{i}_1 + D \hat{v}_3 + \hat{d} V_3$$

ac equation, no cross products

Second Step, Linearize the Sources

- ❑ Now, identify the variables in each source

$$I_a = \frac{F_{sw} L_p V_c^{\,2}}{2 R_i^{\,2} V_{ac}} \longrightarrow F_{sw},\ V_{ac}$$

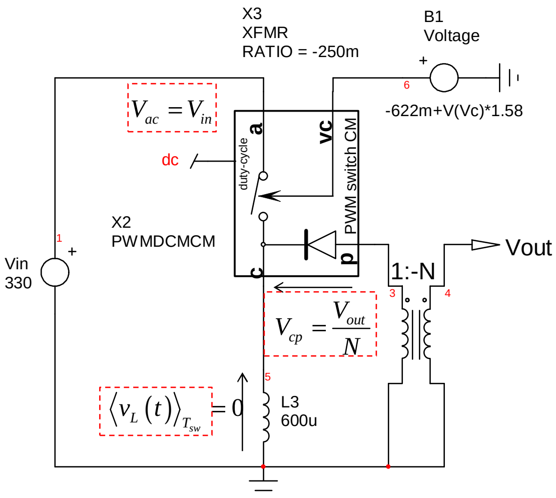
$$I_\mu = -\frac{V_{cp}\left(V_c - I_c R_i\right)\left(F_{sw} L_p V_c^{\,2} - 2 I_c R_i^{\,2} V_{ac}\right)}{F_{sw} L_p R_i V_c^{\,2} V_{ac}} \longrightarrow V_{cp},\ V_{ac},\ F_{sw},\ I_c$$

- ❑ 6 variables imply six partial derivatives, 6 coefficients
- ❑ You must identify these static variables first

➢ Look at the PWM switch configuration

Identify the Variables in the Schematic

- the average voltage across L_3 is 0: point c is grounded.



Sources Derivation

- We can now individually derive all these sources

$$I_a = \frac{F_{sw} L_p V_c^2}{2 R_i^2 V_{ac}} \qquad \hat{i}_a = \left(\frac{\partial I_a}{\partial F_{sw}}\right)_{V_{ac}} \hat{F}_{sw} + \left(\frac{\partial I_a}{\partial V_{ac}}\right)_{F_{sw}} \hat{v}_{ac}$$

$$k_1 = \frac{L_p V_c^2}{2 R_i^2 V_{ac}} \qquad k_2 = -\frac{F_{sw} L_p V_c^2}{2 R_i^2 V_{ac}^2}$$

$$I_\mu = -\frac{V_{cp}(V_c - I_c R_i)\left(F_{sw} L_p V_c^2 - 2 I_c R_i^2 V_{ac}\right)}{F_{sw} L_p R_i V_c^2 V_{ac}}$$

$$\hat{i}_\mu = \left(\frac{\partial I_\mu}{\partial V_{cp}}\right)_{I_c, F_{sw}, V_{ac}} \hat{v}_{cp} + \left(\frac{\partial I_\mu}{\partial I_c}\right)_{F_{sw}, V_{cp}, V_{ac}} \hat{i}_c + \left(\frac{\partial I_\mu}{\partial F_{sw}}\right)_{I_c, V_{cp}, V_{ac}} \hat{F}_{sw} + \left(\frac{\partial I_\mu}{\partial V_{ac}}\right)_{V_{cp}, I_c, F_{sw}} \hat{v}_{ac}$$

$$k_3 \qquad k_4 \qquad k_5 \qquad k_6$$

Sources Derivation

- Yes, Mathcad® or an equivalent software is of great help…

$$k_3 = -\frac{(V_c - I_c R_i)\left(F_{sw} L_p V_c^2 - 2 I_c R_i^2 V_{ac}\right)}{F_{sw} L_p R_i V_c^2 V_{ac}}$$

$$k_4 = \frac{V_{cp}\left(F_{sw} L_p V_c^2 - 2 I_c R_i^2 V_{ac}\right)}{F_{sw} L_p V_c^2 V_{ac}} + \frac{2 R_i V_{cp} (V_c - I_c R_i)}{F_{sw} L_p V_c^2}$$

$$k_5 = \frac{V_{cp}(V_c - I_c R_i)\left(F_{sw} L_p V_c^2 - 2 I_c R_i^2 V_{ac}\right)}{F_{sw}^2 L_p R_i V_c^2 V_{ac}} - \frac{V_{cp}(V_c - I_c R_i)}{F_{sw} R_i V_{ac}}$$

$$k_6 = \frac{V_{cp}(V_c - I_c R_i)\left(F_{sw} L_p V_c^2 - 2 I_c R_i^2 V_{ac}\right)}{F_{sw} L_p R_i V_c^2 V_{ac}^2} + \frac{2 I_c R_i V_{cp}(V_c - I_c R_i)}{F_{sw} L_p V_c^2 V_{ac}}$$

Evaluate all These Coefficients

- Select a converter at a certain operating point

$V_{out} = 21.1\text{ V}$

$R_{load} = 18\ \Omega$

$R_i = 0.8\ \Omega$

$V_c = 1\text{V}$

$V_{ac} = 330\text{V}$

$r_C = 0.06\Omega$

$C_{out} = 470\mu\text{F}$

$N_1 = 0.25$

$V_{cp} = \frac{V_{out}}{N_1} = 84.4\text{V}$

$L_p = 600\mu\text{H}$

$k_F = 10000$

$$F_{sw} = \frac{2 I_{out} R_i^2 V_{out}}{L_p V_c^2} = 52.8\text{kHz}$$

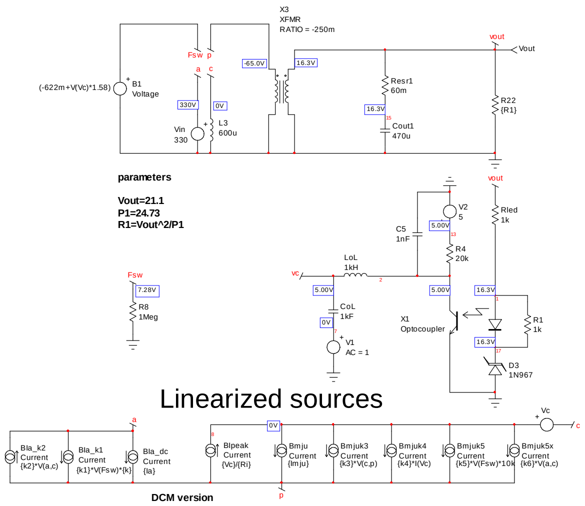
$$I_c = \frac{F_{sw} L_p V_c^2 \left(V_{ac} + V_{cp}\right)}{2 R_i^2 V_{ac} V_{cp}} = 0.368\text{A}$$

$$I_a = \frac{F_{sw} L_p V_c^2}{2 R_i^2 V_{ac}} = 0.075\text{A}$$

$$I_\mu = 0.882\text{A}$$

Test the Coefficient Values with the Sources

❑ Capture a new schematic with the linearized sources



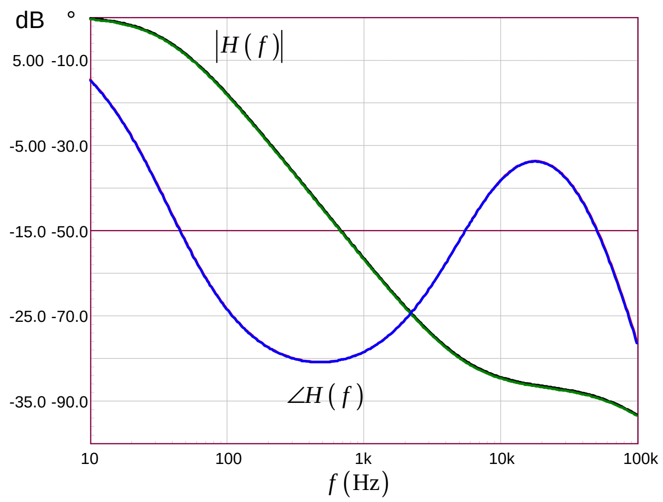
parameters

Vc=1
Lp=600u
Ri=0.8
Lp=600u
k=10k
N=250m
Fsw=52.766k
Iout=P1/Vout
Vac=330
Vcp=Vout/N
Ia=Fsw*Lp*Vc^2/(2*Ri^2*Vac)
k1=(Lp*Vc^2)/(2*Ri^2*Vac)
k2=(Fsw*Lp*Vc^2)/(2*Ri^2*Vac^2)
AA=Fsw*Lp*Vc^2*Vac+Fsw*Lp*Vc^2*Vcp
BB=2*Ri^2*Vac*Vcp
Ic=AA/BB
A=-Vcp*(Vc-Ic*Ri)*(Fsw*Lp*Vc^2-2*Ic*Ri^2*Vac)
B=Fsw*Lp*Ri*Vc^2*Vac
Imju=A/B
AAA=-(Vc-Ic*Ri)*(Fsw*Lp*Vc^2-2*Ic*Ri^2*Vac)
BBB=Fsw*Lp*Ri*Vc^2*Vac
k3=AAA/BBB
k4A=Vcp*(Fsw*Lp*Vc^2-2*Ic*Ri^2*Vac)
k4B=Fsw*Lp*Vc^2*Vac
k4C=2*Ri*Vcp*(Vc-Ic*Ri)/(Fsw*Lp*Vc^2)
k4=(k4A/k4B)+k4C
k5A=Vcp*(Vc-Ic*Ri)*(Fsw*Lp*Vc^2-2*Ic*Ri^2*Vac)
k5B=Fsw^2*Lp*Ri*Vc^2*Vac
k5C=Vcp*(Vc-Ic*Ri)/(Fsw*Ri*Vac)
k5=(k5A/k5B)-k5C
k6A=Vcp*(Vc-Ic*Ri)*(Fsw*Lp*Vc^2-2*Ic*Ri^2*Vac)
k6B=Fsw*Lp*Ri*Vc^2*Vac^2
k6C=2*Ic*Ri*Vcp*(Vc-Ic*Ri)/(Fsw*Lp*Vc^2*Vac)
k6=(k6A/k6B)+k6C

Mathcad® coefficients

Responses with Previous Models are Similar

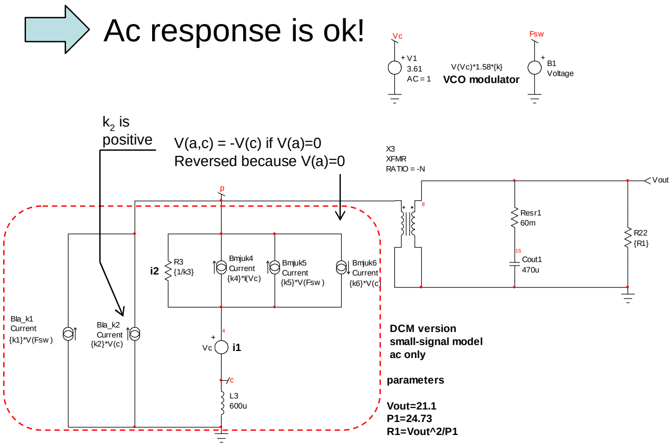
❑ The curves perfectly superimpose, 2nd step is ok



Combine and Arrange the Sources

- ❑ Now, re-arrange the sources in a more convenient way

Ac response is ok!



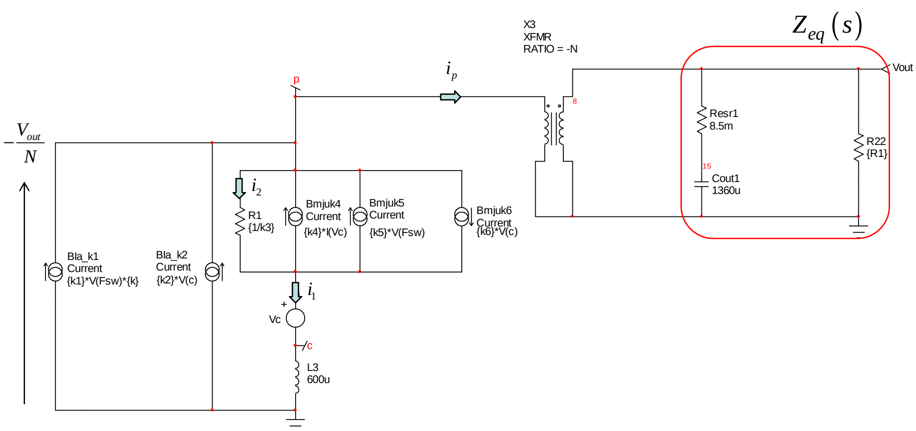
Linearized PWM switch

parameters

Vc=1
Lp=600u
Ri=0.8
Lp=600u
k=10k
N=250m
Fsw=52.766k
Iout=P1/Vout
Vac=330
Vcp=Vout/N
Ia=Fsw*Lp*Vc^2/(2*Ri^2*Vac)
k1=(Lp*Vc^2)/(2*Ri^2*Vac)
k2=(Fsw*Lp*Vc^2)/(2*Ri^2*Vac^2)
AA=Fsw*Lp*Vc^2*Vac+Fsw*Lp*Vc^2*Vcp
BB=2*Ri^2*Vac*Vcp
Ic=AA/BB
A=-Vcp*(Vc-Ic*Ri)*(Fsw*Lp*Vc^2-2*Ic*Ri^2*Vac)
B=Fsw*Lp*Ri*Vc^2*Vac
Imju=A/B
AAA=-(Vc-Ic*Ri)*(Fsw*Lp*Vc^2-2*Ic*Ri^2*Vac)
BBB=Fsw*Lp*Ri*Vc^2*Vac
k3=AAA/BBB
Rk3=1/k3
k4A=Vcp*(Fsw*Lp*Vc^2-2*Ic*Ri^2*Vac)
k4B=Fsw*Lp*Vc^2*Vac
k4C=2*Ri*Vcp*(Vc-Ic*Ri)/(Fsw*Lp*Vc^2)
k4=(k4A/k4B)+k4C
k5A=Vcp*(Vc-Ic*Ri)*(Fsw*Lp*Vc^2-2*Ic*Ri^2*Vac)
k5B=Fsw^2*Lp*Ri*Vc^2*Vac
k5C=Vcp*(Vc-Ic*Ri)/(Fsw*Ri*Vac)
k5=(k5A/k5B)-k5C
k6A=Vcp*(Vc-Ic*Ri)*(Fsw*Lp*Vc^2-2*Ic*Ri^2*Vac)
k6B=Fsw*Lp*Ri*Vc^2*Vac^2
k6C=2*Ic*Ri*Vcp*(Vc-Ic*Ri)/(Fsw*Lp*Vc^2*Vac)
k6=(k6A/k6B)+k6C

Go for Mesh and Node Analysis

- ❑ Express the current and voltage in the primary side



Mesh and Node Analysis

❑ KCL: the sum of currents arriving at a node equals the sum of currents leaving the node:

$$k_4 i_1 + k_5 V(F_{sw}) + i_1 = i_2 + sk_6 L_p i_1 \qquad i_2 = i_1 + i_1 k_4 + k_5 V(F_{sw}) - sL_p i_1 k_6$$

$$i_1 = \frac{-\frac{V_{out}}{N_1} - R_1 i_2}{sL_p}$$

$$i_1(s) = -\frac{V_{out}(s) + N_1 R_1 i_1(s) + N_1 R_1 i_1(s) k_4 + N_1 R_1 k_5 V(F_{sw}) - sL_p R_1 i_1(s) k_6 N_1}{L_p N_1 s}$$

❑ solve for i_1:

$$i_1(s) = -\frac{V_{out}(s) + N_1 R_1 k_5 V(F_{sw})}{N_1 R_1 (1 + k_4) + sL_p N_1 (1 - k_6 R_1)}$$

Mesh and Node Analysis

❑ Apply similar technique to get the primary current:

$$i_p(s) = k_1 V(F_{sw}) + k_2 i_1(s) sL_p - i_1(s)$$

$$i_p(s) = \frac{V_{out}(s) - L_p V_{out}(s) k_2 s + N_1 R_1 V(F_{sw})(k_1 + k_5 + k_1 k_4) + sL_p N_1 V(F_{sw})(k_1 - R_1 k_1 k_6 - R_1 k_2 k_5)}{N_1 (R_1 + R_1 k_4 + sL_p - sL_p R_1 k_6)}$$

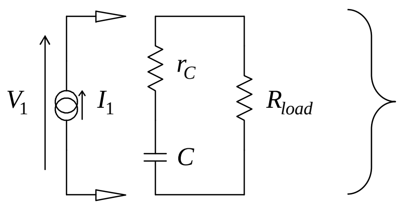
$$i_{out} = -\frac{i_p}{N_1} \qquad V_{out} = i_{out} Z_{eq}$$

$$V_{out}(s) = \frac{V_{out}(s) - L_p V_{out}(s) k_2 s + N_1 R_1 V(F_{sw})(k_1 + k_5 + k_1 k_4) + sL_p N_1 V(F_{sw})(k_1 - R_1 k_1 k_6 - R_1 k_2 k_5)}{N_1^2 (R_1 + R_1 k_4 + sL_p - sL_p R_1 k_6)} Z_{eq}(s)$$

$$\frac{V_{out}(s)}{V(F_{sw})} = \frac{\left(N_1 R_1 (k_1 + k_5 + k_1 k_4) + sL_p N_1 (k_1 - R_1 k_1 k_6 - R_1 k_2 k_5)\right)}{N_1^2 (sR_1 L_p k_6 - sL_p - R_1 k_4 - R_1) - Z_{eq}(s) + sL_p k_2 Z_{eq}(s)} Z_{eq}(s)$$

Fast Analytical Techniques

- ❑ Fast analytical techniques unveil Z_{eq} in a second!



1st-order system

$$Z_{eq}(s) = Z_0 \frac{1+s/\omega_z}{1+s/\omega_p}$$

1. In dc, open the capacitor: $Z_0 = R_{load}$

2. What prevents the excitation I_1 from reaching the output V_1?

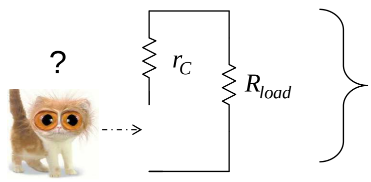
➢ A short-circuit between r_C and *C*:

$$r_C + \frac{1}{sC} = \frac{sr_C C + 1}{sC} = 0 \quad \Rightarrow \quad \omega_z = \frac{1}{r_C C}$$

Fast Analytical Techniques

- ❑ Get the time constant by putting the excitation to zero:

➢ open the current source and look at the cap. driving R



$$\tau = (r_C + R_{load})C$$

- ❑ The equivalent impedance is therefore:

$$Z_{eq}(s) = R_{load} \frac{1+sr_C C}{1+sC(r_C + R_{load})}$$

You cannot beat equation-solving by inspection!

Almost There…

- Develop the expression with $Z_{eq}(s)$, cry and re-arrange:

$$H(s)=H_0\frac{N(s)}{D(s)} \qquad H_0=-G_{VCO}\frac{N_1R_{load}R_1(k_1+k_5+k_1k_4)}{R_{load}+N_1^2R_1(1+k_4)}=\frac{V_{out}}{2F_{sw}}G_{VCO}$$

$$N(s)=\left(1+sL_p\left(\frac{k_1-R_1k_1k_6-R_1k_2k_5}{R_1(k_1+k_5+k_1k_4)}\right)\right)(1+sr_CC) \qquad D(s)=1+as+bs^2$$

$$a=C_{out}\left(R_{load}+r_C-\frac{R_{load}^2}{R_{load}+N_1^2R_1(1+k_4)}\right)+L_p\left(\frac{N_1^2-R_{load}k_2-N_1^2R_1k_6}{R_{load}+N_1^2R_1(1+k_4)}\right)$$

$$b=L_pN_1^2C_{out}\left(\frac{R_{load}+r_C-\frac{R_{load}}{N_1^2}k_2r_C-R_1R_{load}k_6-R_1k_6r_C}{R_{load}+N_1^2R_1(1+k_4)}\right)$$

A Few More Minutes, Keep the Faith…

- Put the denominator under a second-order form and identify

$$D(s)=1+as+bs^2=1+\frac{s}{\omega_0Q}+\left(\frac{s}{\omega_0}\right)^2$$

$$f_0=\frac{1}{2\pi\sqrt{L_pC_{out}}}\sqrt{\frac{\frac{R_{load}}{N_1^2}+R_1(1+k_4)}{R_{load}+r_C-\frac{R_{load}}{N_1^2}k_2r_C-R_1k_6(R_{load}+r_C)}}=1.75\,\text{kHz}$$

$$Q=\frac{1}{\omega_0\left(C_{out}\left(R_{load}+r_C-\frac{R_{load}^2}{R_{load}+N_1^2R_1(1+k_4)}\right)+L_p\left(\frac{N_1^2-R_{load}k_2-N_1^2R_1k_6}{R_{load}+N_1^2R_1(1+k_4)}\right)\right)}=0.021$$

A Few More Minutes, Keep the Faith…

- ❑ Extract the zeros:

$$f_{z_1} = \frac{1}{2\pi L_p \left(\frac{k_1 - R_1 (k_1 k_6 - k_2 k_5)}{R_1 (k_1 + k_5 + k_1 k_4)} \right)} = -487\,\text{kHz} \qquad f_{z_2} = \frac{1}{2\pi r_C C_{out}} = 5.6\,\text{kHz}$$

↑ RHPZ

- ❑ Extract the low-frequency poles:

$$f_{p_1} = \frac{1}{2\pi C_{out} \left(R_{load} + r_C - \frac{R_{load}^2}{R_{load} + N_1^2 R_1 (1+k_4)} \right)} = \frac{1}{2\pi C_{out} \left(R_{load} + r_C - \frac{R_{load}}{2} \right)} \approx \frac{1}{\pi R_{load} C_{out}}$$

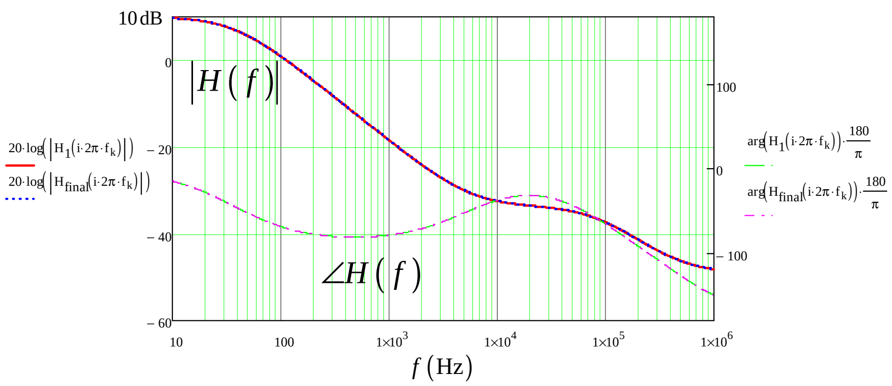
$r_C \ll R_{load}$

$$f_{p_1} = \frac{1}{\pi C_{out} (R_{load} + r_C)} = 37.73\,\text{Hz}$$

$$f_{p_2} = \frac{1}{2\pi L_p \left(\frac{N_1^2 - R_{load} k_2 - N_1^2 R_1 k_6}{R_{load} + N_1^2 R_1 (1+k_4)} \right)} = 152\,\text{kHz}$$

Final Lap, Compare the ac Plots

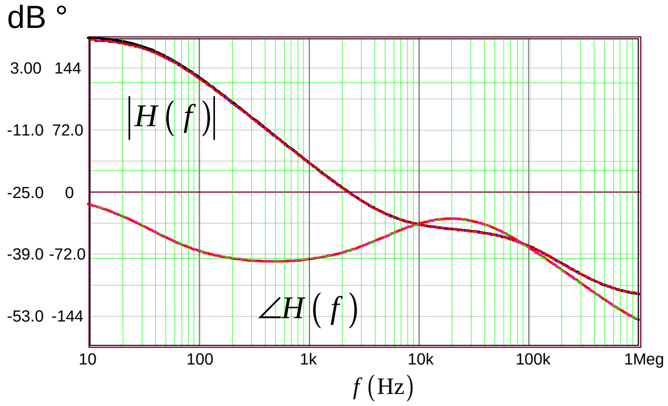
- ❑ Compare the original equation and its re-arranged form:



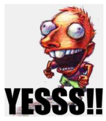
- ❑ It confirms the derivation is correct!

The Final Test: SPICE vs Mathcad

- ❑ If all is well, the curves must perfectly superimpose

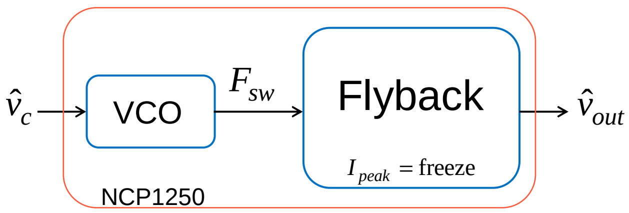


➢ If not, there is a hidden mistake: chase it (good luck)



The Final Transfer Function

- ❑ The current-mode flyback converter is operated in DCM
- ❑ The peak current is fixed and F_{sw} is controlled
- ❑ What is the simplified transfer function?

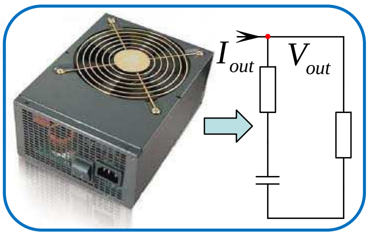


$$H(s) = \frac{\hat{v}_{out}}{\hat{v}_c} = H_0 \frac{1 + s/\omega_z}{1 + s/\omega_p} = \frac{V_{out}}{2F_{sw}} G_{VCO} \frac{1 + s r_C C_{out}}{1 + s \frac{R_{load}}{2} C_{out}}$$

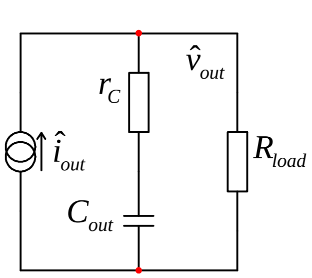
A Faster Way?

- ❑ If you are in a hurry and looking for a faster way:

- ➢ Formulate the output current expression
- ➢ Differentiate the expression to its variables
- ➢ Draw an equivalent schematic and solve the equations



A DCM/CCM current-mode converter



Simplified linearized model

The Founding Equation is Well Known

- ❑ A flyback converter running in DCM obeys:

$$P_{out} = I_{out}V_{out} = \frac{1}{2}L_p I_{peak}{}^2 F_{sw} \rightarrow I_{peak} = \frac{V_c}{R_i} \rightarrow I_{out}V_{out} = \frac{1}{2}L_p\left(\frac{V_c}{R_i}\right)^2 F_{sw}$$

$$I_{out} = \frac{L_p\left(\frac{V_c}{R_i}\right)^2 F_{sw}}{2V_{out}}$$

Two variables

$$\hat{i}_{out} = \left(\frac{\partial I_{out}\left(F_{sw}, V_{out}\right)}{\partial V_{out}}\right)_{F_{sw}} \hat{v}_{out} + \left(\frac{\partial I_{out}\left(F_{sw}, V_{out}\right)}{\partial F_{sw}}\right)_{V_{out}} \hat{F}_{sw}$$

First Expression Comes Easily

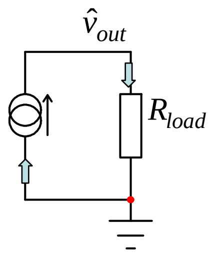
- Rework the individual coefficients to a simpler form

$$\left(\frac{\partial I_{out}\left(F_{sw},V_{out}\right)}{\partial V_{out}}\right)_{F_{sw}}\hat{v}_{out}=-\frac{F_{sw}L_p}{2V_{out}^2}\left(\frac{V_c}{R_i}\right)^2\hat{v}_{out}\Rightarrow g_m\cdot\hat{v}_{out}=\frac{1}{R}\hat{v}_{out}$$

$$g_m \rightarrow \left[\Omega^{-1}\right]$$

$$-\frac{F_{sw}L_p}{2V_{out}^2}\left(\frac{V_c}{R_i}\right)^2=-\frac{\frac{1}{2}I_{peak}^2F_{sw}L_p}{V_{out}^2}$$

$$-\frac{P_{out}}{V_{out}^2}=-\frac{V_{out}I_{out}}{V_{out}V_{out}}=-\frac{1}{R_{load}}$$

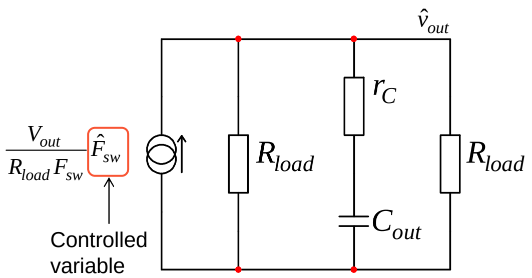


Second Expression is Also Simple

- Identify the power in the equation, simplify…

$$\left(\frac{\partial I_{out}\left(F_{sw},V_{out}\right)}{\partial F_{sw}}\right)_{V_{out}}\hat{F}_{sw}=\frac{L_p}{2V_{out}}\left(\frac{V_c}{R_i}\right)^2\hat{F}_{sw}$$

$$\frac{L_p}{2V_{out}}\left(\frac{V_c}{R_i}\right)^2\hat{F}_{sw}=\frac{P_{out}}{F_{sw}V_{out}}\hat{F}_{sw}=\frac{V_{out}}{R_{load}}\frac{V_{out}}{F_{sw}V_{out}}\hat{F}_{sw}=\frac{V_{out}}{R_{load}F_{sw}}\hat{F}_{sw}$$



Can't beat such a simple schematic!

The Transfer Function is Immediate

❑ The transfer function is now straightforward

$$\hat{v}_{out}(s) = \hat{F}_{sw}(s)\frac{V_{out}}{R_{load}F_{sw}}\boxed{R_{load} \parallel Z_{eq}(s)} \longrightarrow \approx \frac{R_{load}}{2}\frac{1+sr_C C}{1+sC\left(\frac{R_{load}}{2}\right)}$$

$$\hat{v}_{out}(s) = \hat{v}_c(s)k_F G_{VCO}\frac{V_{out}}{2F_{sw}}\frac{1+sr_C C}{1+sC\left(\frac{R_{load}}{2}\right)}$$

❑ Put it under the normalized form

$$H(s) = \frac{\hat{v}_{out}}{\hat{v}_c} = H_0\frac{1+s/\omega_z}{1+s/\omega_p} = \frac{V_{out}}{2F_{sw}}k_F G_{VCO}\frac{1+sr_C C_{out}}{1+s\frac{R_{load}}{2}C_{out}}$$

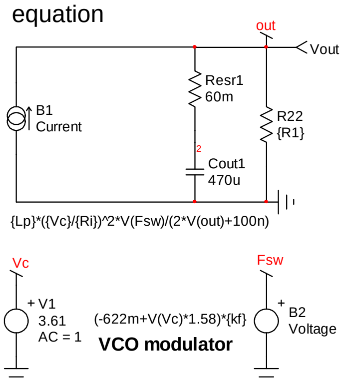
❑ No RHPZ and no high-frequency pole prediction

❑ Good for low-frequency analysis only, but good enough for us!

Simulation Shows the Differences

❑ SPICE only manipulates linear equations

Large-signal equation

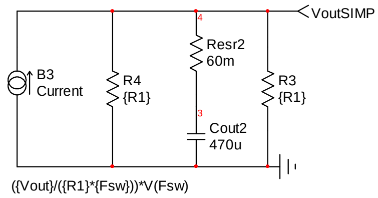


parameters

Vc=1
Lp=600u
Ri=0.8
Lp=600u
kf=10k

Vout=21.1
P1=24.73
R1=Vout^2/P1
Fsw=50.8k

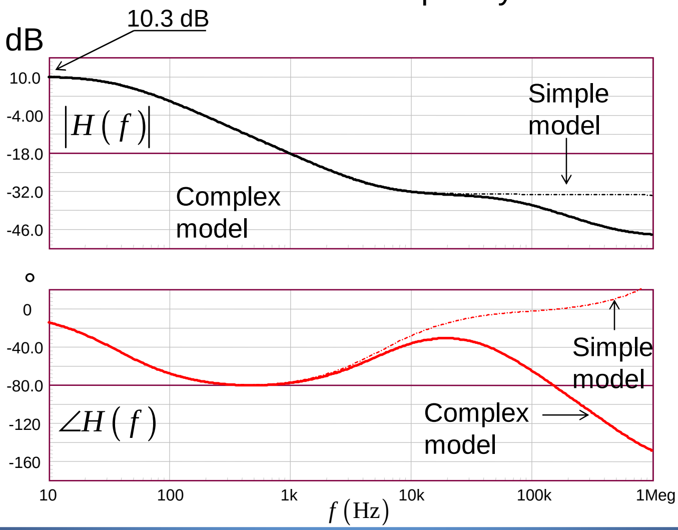
Small-signal equation



$$20\log_{10}|H_0| = 20\log_{10}\left(\frac{V_{out}}{2F_{sw}}k_F G_{VCO}\right) = 20\log_{10}\left(\frac{21.1}{2\times 50.8k}10k\cdot 1.58\right) = 10.3\text{dB}$$

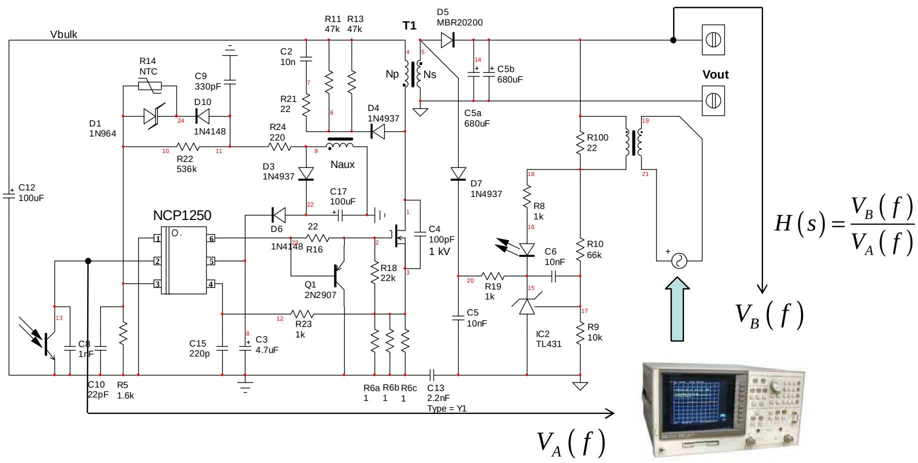
Simulation Shows the Differences

- ❑ Ac results are identical at low frequency



Test Fixture

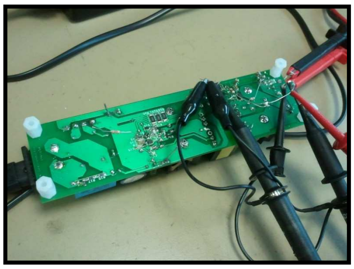
- ❑ Plant frequency response with a NCP1250
- ❑ Load is reduced to force DCM where I_{peak} is frozen



Connections

- ❑ Connect probe grounds to a quiet point: opto emitter is ok
- ❑ Use an isolated current-limited dc-supply
- ❑ Short primary and secondary grounds

100 V dc

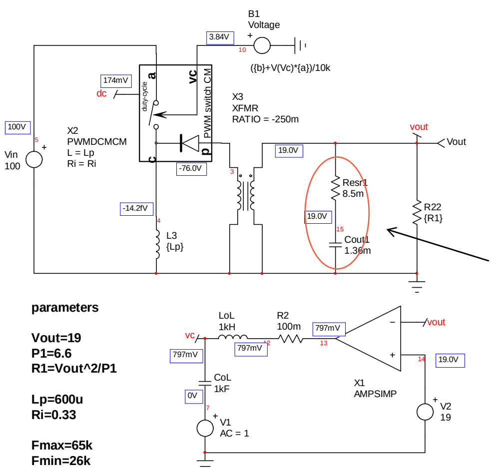


Power supply under test

$H(f)$

Injection transformer

Run the SPICE Simulation



parameters

Vout=19
P1=6.6
R1=Vout^2/P1

Lp=600u
Ri=0.33

Fmax=65k
Fmin=26k
Vfold=1.5
Vmin=0.35
a2=(Fmax-Fmin)/(Vfold-Vmin)
a=27.75k
b=Fmin-Vmin*a

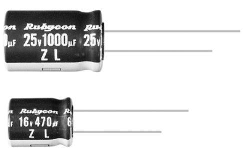
VCO slope measured to 27.8 kHz/V

- ❑ Check operating points:

$$F_{sw} = 38.6\,\text{kHz}(\text{bench})\ 38.4\,\text{kHz}(\text{SPICE})$$

$$V_{FB} = 0.71\,\text{V}(\text{bench})\,0.79\,\text{V}(\text{SPICE})$$

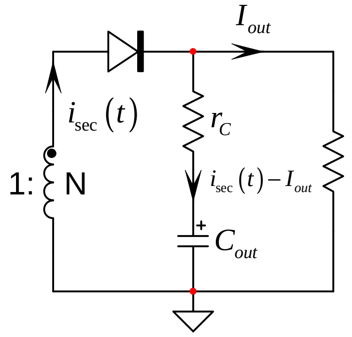
Main contributor to errors is the capacitor ESR

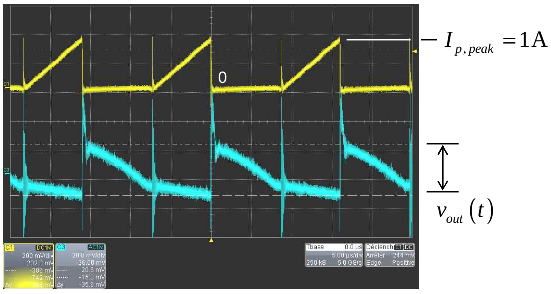


ZL series Rubycon

Approximate ESR Extraction

❑ A triangular output shows that ESR is the main contributor





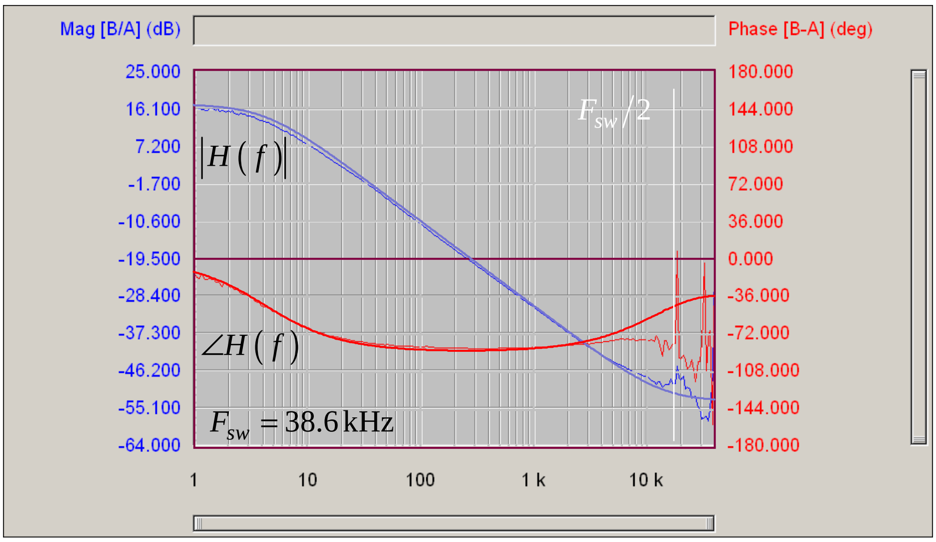
$$I_{sec,peak} = \frac{I_{p,peak}}{N} = \frac{1}{0.25} = 4\,\text{A}$$

$$V_{out,peak} \approx I_{sec,peak} r_C$$

$$r_C \approx \frac{V_{out,pp}}{I_{sec,peak}} = \frac{35m}{4} = 8.75\,\text{m}\Omega$$

DS says 11.5 mΩ

Plant Frequency Response



Conclusion

❑ The transfer function in DCM frequency foldback has been derived

❑ It can be approximated in low frequency as a 1^{st} order system

❑ Stability in this mode is not at stake with current designs

❑ Besides the comprehensive PWM switch approach, the linearized output current gives good results too.

❑ Bench measurements confirm the low-frequency results, high-frequency points start to diverge as we approach the switching frequency.