Full-wave rectification, bulk capacitor calculations

Chris Basso – January 2009

This short paper shows how to calculate the bulk capacitor value based on ripple specifications and evaluate the rms current that crosses it.

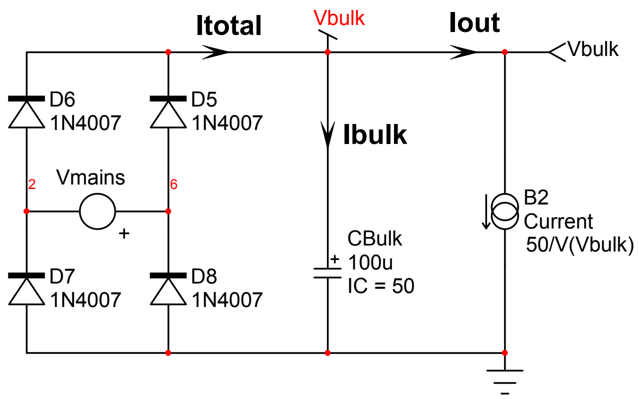

Figure 1: a classical full-wave rectification.

The bulk capacitor peak current can be estimated as follows:

$$I_{C_{bulk},peak} = C_{bulk}\frac{dV_{C_{bulk}}(t)}{dt} = C_{bulk}\frac{d\left(V_{peak}\sin\left(2\pi F_{line}t_1\right)\right)}{dt} = 2\pi F_{line}C_{bulk}V_{peak}\cos\left(2\pi F_{line}\Delta t\right) \quad (1)$$

What we need is to derive the time at which the refueling of the capacitor from V_{min} to V_{peak} starts. This is exactly where the input signal reaches V_{min} (neglecting the diodes drops):

$$\Delta t = \frac{\sin^{-1}\left(\frac{V_{min}}{V_{peak}}\right)}{2\pi F_{line}} \quad (2)$$

The total refueling time from V_{min} to V_{peak} is therefore a fourth of the line period (the distance from 0 to the line voltage peak occurrence) minus (2):

$$t_1 = \frac{1}{4F_{line}} - \frac{\sin^{-1}\left(\frac{V_{min}}{V_{peak}}\right)}{2\pi F_{line}} \quad (3)$$

Now that the capacitor peak current is known, we need to calculate the diode peak current $I_{d,peak}$. This peak is nothing else than the capacitor peak current added with the load current. However, as shown by Figure 1, the load current is not a continuous current. This is because as the switching power supply maintains a constant output power, the ripple superimposed on the bulk capacitor permanently changes the operating point of the power supply, imposing a variable drawn current. As shown in Figure 2, this load current $I_{load}(t)$ is rather distorted but can be approximated as a rectified sinusoidal signal. Therefore, the average or dc current drawn by the switching converter is:

$$I_{load,avg} \approx \frac{2F_{line}P_{out}}{\eta}\int_0^{1/2F_{line}} \frac{1}{V_{min} + \left(V_{peak} - V_{min}\right)\sin\left(2\pi F_{line}t\right)} \tag{4}$$

Solving this equation is a complex process. Rather, we can calculate the maximum load current that occurs at the minimum bulk voltage:

$$I_{load,\max} = \frac{P_{out}}{\eta V_{min}} \tag{5}$$

The diode peak current is therefore immediately found by:

$$I_{d,peak} = I_{C_{bulk},peak} + I_{load,\max} \tag{6}$$

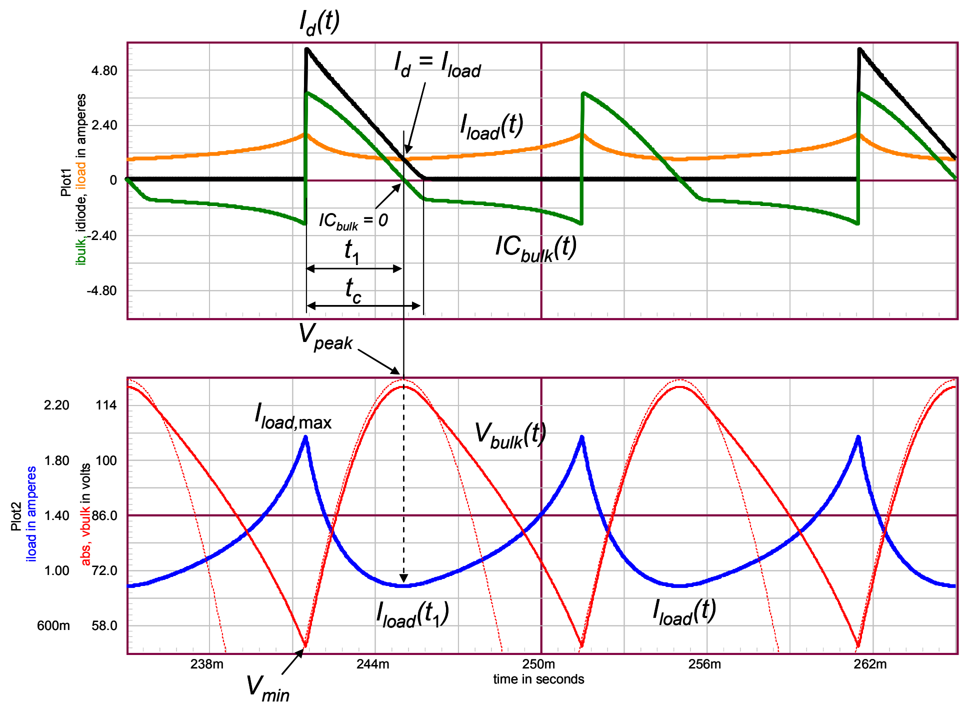

Figure 2: bulk capacitor refueling waveforms.

As shown in Figure 2, the capacitor current reaches zero when the diode current equals that of the load. This occurs exactly at the top of the sinusoidal input voltage (where the derivative is null). The time at which this event takes place is known, this is t_1 calculated in (3). Therefore, the diode current down slope can be approximated by:

$$S_{diode} = \frac{I_{d,peak} - I_{load}\left(t_1\right)}{t_1} \tag{7}$$

Where $I_{load}(t_1)$ represents the minimum load current when the sine wave is at its top:

$$I_{load}\left(t_1\right) = I_{load,\min} = \frac{P_{out}}{\eta V_{max}} \tag{8}$$

Once the slope is known, we can calculate the time taken to go from $I_{d,peak}$ to 0, leading to the total diode conduction time t_c:

$$t_c = \frac{I_{d,peak}}{S_{diode}} \tag{9}$$

From this value, it becomes possible to calculate the load average current in a simpler manner than in (4):

$$I_{load,avg} = \frac{I_{d,peak} t_c}{2} \frac{1}{\frac{T_{line}}{2}} = I_{d,peak} t_c F_{line} \tag{10}$$

Where F_{line} represents the mains frequency.

We can show that the rms current in the capacitor can be put under the following form:

$$I_{C_{bulk,rms}} = I_{load,avg} \sqrt{\frac{2}{3F_{line} t_c} - 1} \tag{11}$$

The diode rms current is derived as follows:

$$I_{d,rms} = \frac{I_{load,avg}}{\sqrt{3F_{line} t_c}} \tag{12}$$

Each pair of conducting diodes sees half of the load average current:

$$I_{d,avg} = \frac{I_{load,avg}}{2} \tag{13}$$

The total input rms current comes easily as:

$$I_{d,rms,total} = \frac{I_{load,avg}\sqrt{2}}{\sqrt{3F_{line} t_c}} \tag{14}$$

Design example:

Suppose we need the following specifications:

V_{in} is 85 V rms, 50 Hz
V_{peak} is therefore 120 V
V_{min} is chosen to 50 V (targeted ripple)
P_{out} = 90 W
η is 86%

1. calculate the bulk capacitor value:

$$C_{bulk} = \frac{2P_{out}\left(\frac{1}{4F_{line}} + \frac{\sin^{-1}\left(\frac{V_{min}}{V_{peak}}\right)}{2\pi F_{line}}\right)}{\eta\left(V_{peak}^{2} - V_{min}^{2}\right)} = 112\ \mu F \tag{15}$$

2. calculate Δt with (2):

$$\Delta t = \frac{\sin^{-1}\left(\frac{V_{min}}{V_{peak}}\right)}{2\pi F_{line}} = 1.368\ ms \tag{16}$$

3. calculate the bulk capacitor peak current with (1)

$$I_{C_{bulk,peak}} = 2\pi F_{line} C_{bulk} V_{peak} \cos\left(2\pi F_{line}\Delta t\right) = 3.84\ A \tag{17}$$

4. calculate the total charging time t_1:

$$t_1 = \frac{1}{4F_{line}} - \frac{\sin^{-1}\left(\frac{V_{min}}{V_{peak}}\right)}{2\pi F_{line}} = 3.632\ ms \tag{18}$$

5. calculate the load peak and minimum current values :

$$I_{load,\max} = \frac{P_{out}}{\eta V_{min}} = 2.09\ A \tag{19}$$

$$I_{load,\min} = \frac{P_{out}}{\eta V_{max}} = 0.872\ A \tag{20}$$

6. calculate the diode peak current:

$$I_{d,peak} = I_{C_{bulk,peak}} + I_{load,\max} = 3.84 + 2.09 = 5.93\ A \tag{21}$$

7. calculate the diode current down slope from the peak value to t_1:

$$S_{diode} = \frac{I_{d,peak} - I_{load}\left(t_1\right)}{t_1} = \frac{5.93 - 0.872}{3.632m} \approx 1.4\ kA/s \tag{22}$$

8. Derive the diode total conduction time t_c:

$$t_c = \frac{I_{d,peak}}{S_{diode}} = \frac{5.93}{1400} = 4.23\ ms \quad (23)$$

9. the average dc current can now be found :

$$I_{load,avg} = I_{d,peak} t_c F_{line} = 1.26\ A \quad (24)$$

10. from which we can get the bulk capacitor rms current:

$$I_{C_{bulk,rms}} = I_{load,avg} \sqrt{\frac{2}{3F_{line}t_c} - 1} = 1.72\ A \quad (25)$$

11. then each diode rms current:

$$I_{d,rms} = \frac{I_{load,avg}}{\sqrt{3F_{line}t_c}} = 1.58\ A \quad (26)$$

12. plus each diode average current:

$$I_{d,avg} = \frac{I_{load,avg}}{2} = 0.63\ A \quad (27)$$

13. and finally, the total rms input current:

$$I_{d,rms,total} = \frac{I_{load,avg}\sqrt{2}}{\sqrt{3F_{line}t_c}} = 2.23\ A \quad (28)$$

Comparison between the calculated values and the simulated values

Below is an array comparing the results given by a simulator and the one obtained from the analytical derivations (same input voltage and minimum bulk voltage):

C_{bulk} = 112 µF, P_{out} = 90 W

	Calculated	simulated	Error
$I_{in,rms}$	2.23 A	2.1 A	6.2%
$I_{d,peak}$	5.9 A	5.67 A	4%
$IC_{bulk,rms}$	1.84 A	1.71 A	7.6%
$I_{d,avg}$	0.63 A	0.593 A	6%

C_{bulk} = 249 µF, P_{out} = 200 W

	Calculated	simulated	Error
$I_{in,rms}$	4.9 A	4.66 A	5%
$I_{d,peak}$	13.2 A	12.5 A	5.6%
$IC_{bulk,rms}$	4.1 A	3.8 A	7.9%

$I_{d,avg}$	1.4 A	1.3 A	7.7%

C_{bulk} = 622 µF, P_{out} = 500 W

	Calculated	simulated	Error
$I_{in,rms}$	12.4 A	11.6 A	6.9%
$I_{d,peak}$	33 A	30.1 A	9%
$IC_{bulk,rms}$	10.2 A	9.5 A	7.4%
$I_{d,avg}$	3.5 A	3.3 A	6%

Selecting a normalized value for the bulk capacitor

If equation (15) gives us a capacitor value, we are likely going to select a different value based on the rms current requirement as well as the available normalized values. Let us suppose we have selected a 150-µF capacitor further to a recommendation of 112 µF. How do we rescale all the calculated parameters? The first thing is to compute the new minimum bulk voltage, V_{min}. Unfortunately, the process is complex because the current consumed by the load as V_{bulk} goes down, permanently changes since the power supply keeps P_{out} constant. There is no close form solution and we must use a numerical approach to obtain the value we are looking for. Let's discover the steps. The energy accumulated in the capacitor during its charge equals that released during its discharge time t_d from V_{peak} to V_{min}. Therefore, we have:

$$0.5C_{bulk}\left(V_{peak}{}^{2}-V_{min}{}^{2}\right)=\frac{P_{out}}{\eta}t_{d} \qquad (29)$$

The discharge time t_d can be evaluated by looking at the below figure and combining the various time events:

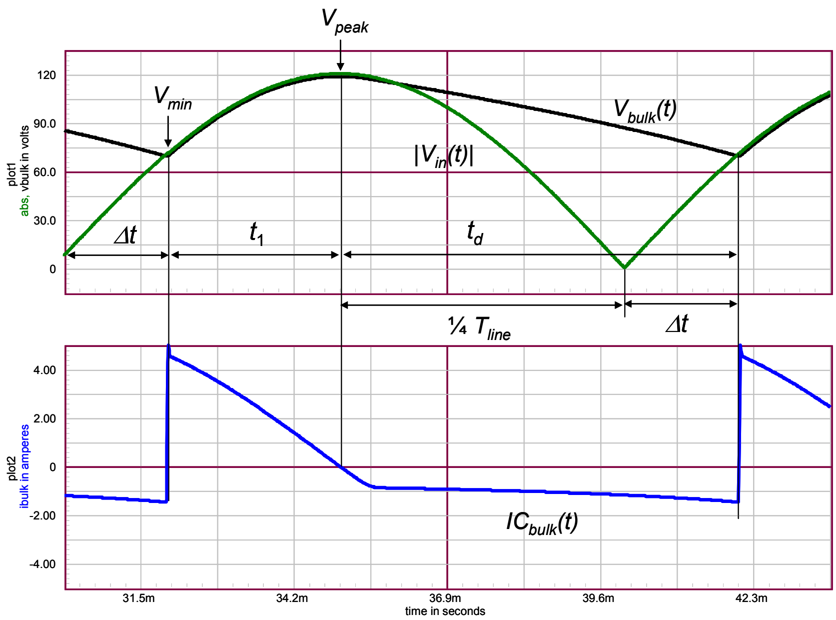

Figure 3: the charge/discharge cycle can be broken down in three different time durations.

It has actually been already derived in (15). We can see from Figure 3 that the discharge time t_d actually equals the time to go from the peak voltage to 0 (1/4th of the input period) plus Δt. Otherwise stated:

$$t_d = \frac{1}{4F_{line}} + \frac{\sin^{-1}\left(\frac{V_{min}}{V_{peak}}\right)}{2\pi F_{line}} = \frac{\pi + 2\sin^{-1}\left(\frac{V_{min}}{V_{peak}}\right)}{4\pi F_{line}} \tag{30}$$

We can now substitute t_d definition into (29) and obtain the final equation:

$$0.5C_{bulk}\left(V_{peak}{}^{2} - V_{min}{}^{2}\right) = \frac{P_{out}}{\eta}\frac{\pi + 2\sin^{-1}\left(\frac{V_{min}}{V_{peak}}\right)}{4\pi F_{line}} \tag{31}$$

This equation does not have a closed-form solution. One easy way to find its solution is to plot both sides of the equation and check the crossing point. Another option is to plot the difference between the right and left sides and check when it reaches zero as V_{min} is incremented from 0 to V_{peak}:

$$0.5C_{bulk}\left(V_{peak}{}^{2} - V_{min}{}^{2}\right) - \frac{P_{out}}{\eta}\frac{\pi + 2\sin^{-1}\left(\frac{V_{min}}{V_{peak}}\right)}{4\pi F_{line}} = 0 \tag{32}$$

We have entered the data into Mathcad® and we obtained a curve showing the zeroing of the curve for V_{min} = 68 V (Figure 4).

Having the minimum voltage, we can calculate the time needed to reach V_{min} using (2):

$$\Delta t = \frac{\sin^{-1}\left(\frac{V_{min}}{V_{peak}}\right)}{2\pi F_{line}} = 1.95\ ms \tag{33}$$

Then the capacitor charging time (equation (3)):

$$t_1 = \frac{1}{4F_{line}} - \frac{\sin^{-1}\left(\frac{V_{min}}{V_{peak}}\right)}{2\pi F_{line}} = 3.049\ ms \tag{34}$$

Then go to point 3. above and roll out the calculations. The following simulated data have been compared to the numerical values based on various capacitor and power selections:

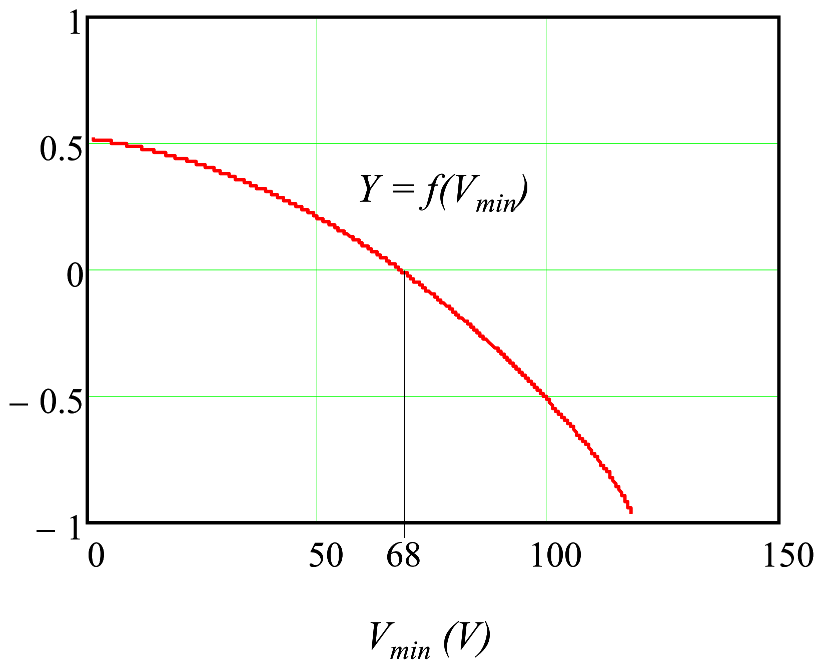

Figure 4: the graph of equation (32) shows a minimum voltage of 68 V.

C_{bulk} = 150 µF, P_{out} = 90 W

	Calculated	simulated	Error
V_{min}	67.8 V	69 V	1.7%
$I_{in,rms}$	2.1 A	2.095 A	0.23%
$I_{d,peak}$	6.1 A	6.22 A	1.9%
$IC_{bulk,rms}$	1.97 A	1.78 A	1.06%
$I_{d,avg}$	0.54 A	0.54 A	0% (yes!)

C_{bulk} = 330 µF, P_{out} = 200 W

	Calculated	simulated	Error
V_{min}	67.3 V	67.7 V	0.6%
$I_{in,rms}$	4.67 A	4.69 A	0.4%
$I_{d,peak}$	13.5 A	13.1 A	3%
$IC_{bulk,rms}$	4 A	3.97 A	0.75%
$I_{d,avg}$	1.21 A	1.23 A	1.6%

C_{bulk} = 1000 µF, P_{out} = 500 W

	Calculated	simulated	Error
V_{min}	76.56 V	76.5 V	0.7%
$I_{in,rms}$	11.67 A	12 A	2.75%
$I_{d,peak}$	35.8 A	35.57 A	0.64%
$IC_{bulk,rms}$	10.18 A	10.4 A	2.1%
$I_{d,avg}$	2.85 A	2.98 A	4.3%

As one can judge, the agreement is rather good.