

# Designing Control Loops for Linear and Switching Power Supplies: A Tutorial Guide

Christophe Basso – October 2012

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Corrections of typos, mistakes and errors found by readers or by the author himself.  
Special thanks go to Mr Tomas Gubek from Czech Republic who really scrutinized the text line by line and caught a lot of typos and errors!

**Page 9:** Please note that the rise time is measured from 10 to 90 percent of the output waveform,  $y(t)$ , in the case of the damped response. For an under-damped response, like what we will study in Chapter 3, the rise time is measured from 0 to 100 percent.

Contributed by Krishnakumar Gopalakrishnan, May 2013

**Page 34:** equation (2.34) is reversed:  $Q = \frac{\sqrt{10.25}}{5} \approx 0.64$

Contributed by Tomas Gubek, October 2013

**Page 37:** ...in (2.42) its denominator has indeed the dimension of a time constant.

**Page 56:** 10<sup>th</sup> line from the top: ...before transmitting it to the **load** (not source)

Contributed by Tomas Gubek, October 2013

**Page 61:** Figure 2.25: lower side: the label is not  $v_{out}(t)$  but must be  $i_{out}(t)$

Contributed by Tomas Gubek, October 2013

**Page 62:** Figure 2.26: upper side: the 200  $\mu\text{s}$  label must be 300  $\mu\text{s}$ .

Contributed by Tomas Gubek, October 2013

**Page 65:** Figure 2.27, caption: ...if the zero of the **numerator** would lie...

Contributed by Tomas Gubek, October 2013

**Page 65:** in Figure 2.27, the total phase lag reaches  $180^\circ$  and not  $-180^\circ$

**Page 68:** some of the expression could include brackets to avoid confusion:

$$Z_{in}(s) = \frac{1}{sC} + R_4 \parallel (R_3 + R_2 \parallel R_1) = \frac{1 + sC[R_4 \parallel (R_3 + R_2 \parallel R_1)]}{sC} \quad (2.136)$$

$$\omega_{z_1} = \frac{1}{sC[R_4 \parallel (R_3 + R_2 \parallel R_1)]} \quad (2.138)$$

Contributed by Raymond Carr, February 2014

**Page 69:** the final equation should be labeled (2.142) and one of its terms subscript is wrong

$$Z_{in}(s) = R_1 + (R_3 + R_4) \parallel R_2 \frac{1 + sC[R_4 \parallel (R_3 + R_2 \parallel R_1)]}{1 + sC[R_1 + R_3 \parallel (R_4 + R_2)]} \quad (2.142)$$

Contributed by Raymond Carr, February 2014

**Page 73:** Equation (2.160) should be  $\mathcal{L}^{-1}\left\{\frac{1+s}{s^2+4}\right\} = -\frac{1-2j}{4j}e^{-2jt} + \frac{1+2j}{4j}e^{2jt}$

Contributed by Tomas Gubek, October 2013

**Page 81:** in Figure 3.5, the middle graph caption should be 29.32 dB and not 29.6 dB

**Page 84:** in Figure 3.7, the phase margin symbol should be  $\varphi_m$  and not  $\phi_m$

**Page 84:** in Figure 3.7, the lower right comment should be  $\angle T(f) = -180^\circ$

**Page 98:** Equation (3.32) should be  $Q = \frac{1}{2\zeta} = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{RC\omega_0} = \frac{L\omega_0}{R}$

Contributed by Tomas Gubek, October 2013

**Page 99:** last bullet,  $Q > 0.5$ , it should be corrected to:  $Q > 0.5$  or  $\zeta < 1$ :

Contributed by Tomas Gubek, October 2013

**Page 105:** in (3.50) and (3.51), the Greek letter is sigma while it should be zeta:

$$t_r = \frac{\pi - \cos^{-1}(\zeta)}{\omega_0 \sqrt{1 - \zeta^2}} \quad (3.50)$$

$$\lim_{\zeta \rightarrow 0} \frac{\pi - \cos^{-1}(\zeta)}{\omega_0 \sqrt{1 - \zeta^2}} \approx \frac{\pi - \frac{\pi}{2}}{\omega_0} \approx \frac{1.6}{\omega_0} \quad (3.51)$$

Contributed by Tomas Gubek, October 2013

**Page 107:** Figure 3.26, caption: ...ratio when lower than 1.

Contributed by Tomas Gubek, October 2013

**Page 118:** in the text, above Figure 3.33: ...as sources **that add** and subtract...

Contributed by Tomas Gubek, October 2013

**Page 118:** the fifth reference is missing, it should be:

5. B. Erisman, R. Redl, "Optimizing the Load Transient Response of the Buck Converter", APEC'1998 proceedings, p. 170-176, vol. 1

Contributed by Tomas Gubek, October 2013

**Page 155:** equation dimension of (4.7) is wrong, it must be:  $V_D(s) = \varepsilon(s) \tau_d s = \varepsilon(s) \frac{s}{\omega_{\tau_1}}$

Contributed by Tomas Gubek, October 2013

**Page 157:** ...more friendly to us engineers than equation (4.8), isn't it?

Contributed by Tomas Gubek, October 2013

**Page 166:** ... the output inductor ( $L_1$ ) and the output capacitor ( $C_{out}$ ) in Figure 4.11.

Contributed by Tomas Gubek, October 2013

**Page 171:** Terms in the ln quotient must be swapped, no impact on result:

$$Q = \sqrt{\left(\frac{\pi}{\ln k}\right)^2 + \frac{1}{4}} = \sqrt{\left(\frac{\pi}{\ln \frac{48.4}{21.5}}\right)^2 + \frac{1}{4}} = 3.9$$

Contributed by Tomas Gubek, October 2013

**Page 181:** in Figure 4.26,  $C_1$  must be labeled 1  $\mu\text{F}$  not 1 mF

**Page 182:** Equation (4.73) gives 1.4 Hz, not 14 Hz.

Contributed by Tomas Gubek, October 2013

**Page 184:** In equations (4.77) (4.78) and (4.79), this is  $\arg H(f_c)$  and not  $\arg T(f_c)$

$$\arg H(f_c) - 270^\circ + \text{boost} = -360^\circ + \varphi_m \quad (4.77)$$

$$\text{boost} = -360^\circ + \varphi_m - \arg H(f_c) + 270^\circ = \varphi_m - \arg H(f_c) - 90^\circ \quad (4.78)$$

$$\text{boost} = \varphi_m - \arg H(f_c) - 90^\circ = 70 + 71 - 90 = 51^\circ \quad (4.79)$$

Contributed by Tomas Gubek, October 2013

**Page 194:** Figure 4.39:  $158^\circ$  or  $-202^\circ$  then  $90^\circ$  or  $-270^\circ$  at the bottom

Contributed by Tomas Gubek, October 2013

**Page 196:** in the text, above (4.127): Suppose we need a phase boost of  $120^\circ$ , then...

Contributed by Tomas Gubek, October 2013

**Page 200:** in Figure 4.44, in this particular case, the label “146 Hz” is obviously misplaced and should occur before the 480-Hz label, at 0 dB.

Contributed by Tomas Gubek, October 2013

**Page 203:** in equation (4.140), it must be  $\arg H(f_c)$  and not  $\arg T(f_c)$

$$\text{boost} = \varphi_m - \arg H(f_c) - 90^\circ = 70 + 134 - 90 = 114^\circ$$

Contributed by Tomas Gubek, October 2013

**Page 211:** in Figure 4.57,  $v_{out}(t)$  is the first curve on top, the square below is  $i_{out}(t)$

Contributed by Tomas Gubek, October 2013

**Page 215:** in (4.159), we have 151 krad/s:  $\omega_{p_G} = \omega_{z_1} = 151 \text{ krad/s}$ ,  $f_{p_G} = f_{z_1} = 24.1 \text{ kHz}$

Contributed by Tomas Gubek, October 2013

**Page 226:** a small  $s$  should be in equation (4.190) and (4.191):  $D(s) = \dots$  and  $Z_{out}(s) = \dots$

Contributed by Tomas Gubek, October 2013

**Page 226:** (4.191) zero featuring the inductance must be reversed:  $Z_{out}(s) = \dots \frac{\left(1 + s \frac{L}{r_L}\right)(1 + \dots)}{\dots}$

Contributed by Tomas Gubek, October 2013

**Page 250:** in (4.261) the “-“ sign must precede the parenthesis:

$$s_2 \approx \omega_0 \frac{-\left(1 - \frac{4Q^2}{2}\right) - 1}{2Q} = \omega_0 \left(\frac{2Q^2 - 2}{2Q}\right) \approx -\frac{\omega_0}{Q}$$

Contributed by Tomas Gubek, October 2013

**Page 251:** in Figure 4.98, the symbol next to the dB unit must be  $^\circ$  and not a crossed box!

Contributed by Tomas Gubek, October 2013

**Page 252:** in Table 4E.1, in expressions for PI, PI<sub>2</sub>, PID and PID<sub>2</sub>, to maintain a dimensionless expression, the term  $G_0$  is actually  $\omega_{p_0}/\omega_{z_1}$ , the 0-dB crossover pole divided

by the 1<sup>st</sup> zero. For clarity,  $G_0$  can go and all expressions are divided by  $-\frac{(\dots)}{\frac{s}{\omega_{po}}(\dots)}$ .

factor  $\frac{s}{\omega_{z_1}}$  in the numerator, you have  $\frac{\omega_{po}}{\omega_{z_1}} \frac{1 + \frac{\omega_{z_1}}{s}}{\dots} = G_0 \frac{1 + \frac{\omega_{z_1}}{s}}{\dots}$  where the mid-band gain  $G_0$  is now dimensionless.

Action Mode	Basic Element	Transfer Function	Implementation	Bode Plot $ G(s) $	Type
Proportional	P	$G(s) = -k_p$			
Integral	I	$G(s) = -k_i \frac{1}{s}$			1
Derivative	D	$G(s) = -s k_d$			
Proportional Integral	PI	$G(s) = -G_0 \left(1 + \omega_{z_1}/s\right)$			2a
Proportional Integral + 1 <sup>st</sup> -order lag	PI <sub>1</sub>	$G(s) = -G_0 \frac{1 + \omega_{z_1}/s}{1 + s/\omega_{p_1}}$			2
Proportional Integral Derivative	PID	$G(s) = -G_0 \left(1 + \omega_{z_1}/s\right) \left(1 + s/\omega_{z_2}\right)$			3a
Proportional Integral Derivative + 2 <sup>nd</sup> order lag	PID <sub>2</sub>	$G(s) = -G_0 \frac{(1 + \omega_{z_1}/s)(1 + s/\omega_{z_2})}{(1 + s/\omega_{p_1})(1 + s/\omega_{p_2})}$ $G_0 = \frac{\omega_{po}}{\omega_{z_1}}$			3

Contributed by Tomas Gubek, October 2013

**Page 255:** below equation (5.7): According to (5.5), to *amplify* the input signal by...

Contributed by Tomas Gubek, January 2014

**Page 266:** Figure 5.12, caption ...; this is the type *2b* argument.

Contributed by Tomas Gubek, January 2014

**Page 274:** something went wrong here:  $G_0 = G_1 G_2$

Contributed by Tomas Gubek, January 2014

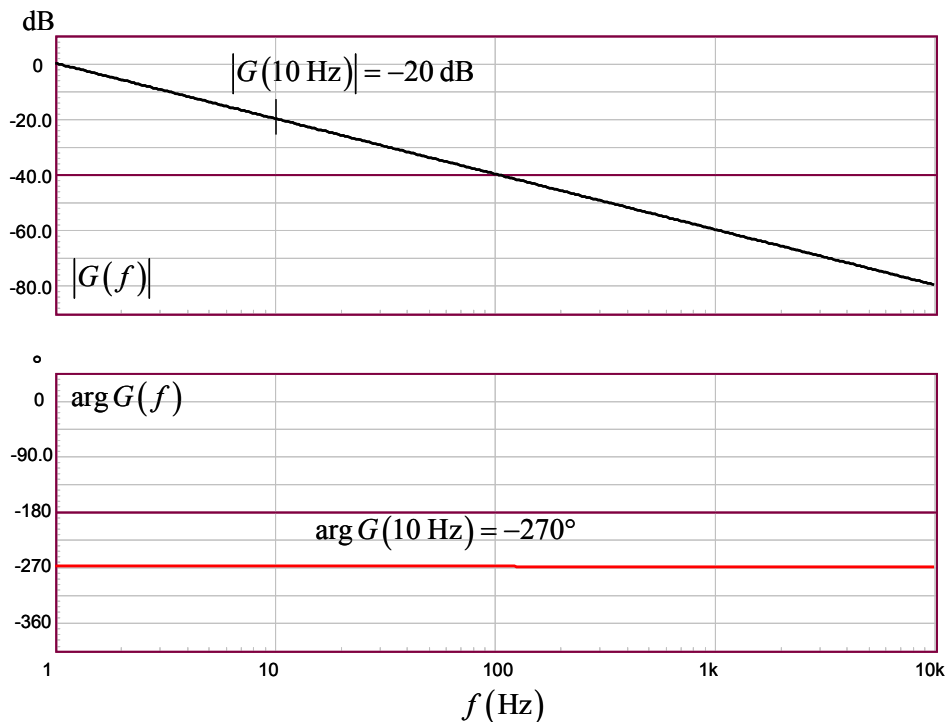
**Page 296:** in Figure 5.34, caption: ...Here, the *current* loop chain is not represented.

**Page 300:** ...but also in the mid-band gain definition.

Contributed by Tomas Gubek, January 2014

**Page 301:** Suppose we need to create an attenuation of 20 dB at 10 Hz.

**Page 303:** Figure 5.40 is wrong. The right picture should be this one:



**Figure 5.40:** These ac results show the correct attenuation value at 10 Hz.  
Contributed by Tomas Gubek, January 2014

**Page 306:** the text below equation (5.194) must be replaced up to ...*than in first*.

Expressions and definitions now differ from what we originally derived for the classical type 2 compensator. In (5.13), assuming  $C_2$  is much smaller than  $C_1$ , then the gain  $G_0$  is defined by (5.22). In the above expressions, however, assuming that  $R_2$  is much smaller than  $R_1$ , the gain  $G_0$  in (5.189) simplifies to  $\approx C_1/C_2$ . If we equate both expressions (we want a similar mid-band gain with both configurations) and extract  $C_2$ , we have  $C_2 = C_1 R_1 / R_2$ . As  $R_2$  is smaller than  $R_1$ , then  $C_2$  will be much larger than  $C_1$ . Therefore, as the entire current defined by (5.185) will cross  $C_2$  at start-up, we can expect a longer charging time than with the traditional type 2 configuration.

Contributed by Tomas Gubek, January 2014

**Page 307:** the text after ...*and 6.5 Hz* must be updated as below:

For the first case, Figure 5-5,  $C_1$  equals 138 nF and  $C_2$  is 16 nF. As expected, for a similar pole/zero arrangement, the second configuration (Figure 5-43) leads to a  $C_2$  value of 154 nF.

Contributed by Tomas Gubek, January 2014

**Page 310:** in equation (5.209),  $R_{upper}$  is  $R_1$ : 
$$C_3 = \frac{f_{p_2} - f_{z_2}}{2\pi R_1 f_{p_2} f_{z_2}}$$

Contributed by Tomas Gubek, January 2014

**Page 313:** in equation (5.234),  $R_{upper}$  is  $R_1$ :  $C_3 = \frac{f_{p_2} - f_{z_2}}{2\pi R_1 f_{p_2} f_{z_2}} = \frac{32.5k - 769}{6.28 \times 10k \times 32.5k \times 769} = 20 \text{ nF}$

Contributed by Tomas Gubek, January 2014

**Page 317:** in equation (5.246),  $R_{upper}$  is  $R_1$ :  $C_3 = \frac{f_{p_1} - f_{z_2}}{2\pi R_1 f_{p_1} f_{z_2}}$

Contributed by Tomas Gubek, January 2014

**Page 321:** in equation (5.259),  $R_{pulldown}$  is  $R_{pullup}$

Contributed by Tomas Gubek, January 2014

**Page 332:** ...the phase must **be** boosted by  $150^\circ$ .

Contributed by Tomas Gubek, January 2014

**Page 344:** in the text below equation (5.362): ...similar to that of the type 3 definition in (4.123) that Mr. Venable purposely squared:

$$k_{venable} = \left[ \tan \left( \frac{boost}{4} + 45 \right) \right]^2 \quad (5.363)$$

**Page 345:** Figure 5.71:

$$k = \tan \left( \frac{boost}{2} + 45 \right)$$

**Page 351:** The sentence is a bit misleading I believe. I meant that the peak amplitude decreases by 1.5 division. Perhaps it is simpler like that: ...*increase it until the modulation occupies 7 divisions peak to peak. This point corresponds to a 3-dB drop from the reference point at 100 Hz:*

Contributed by Tomas Gubek, January 2014

**Page 358:** equation (6.6) needs to be cleaned up:  $V_{(-)}(s) = V_{out}(s) \frac{R_{lower}}{R_{lower} + R_1}$

Contributed by Tomas Gubek, January 2014

**Page 377:** yes, the ratio between the 5-V output and the 2.5-V reference is 2, not 2.5!

Therefore (6.90) is  $boost_{5V} = 2 \tan^{-1} \sqrt{2} - 90^\circ = 19.5^\circ$ .

This would not allow a phase boost larger than  $90^\circ + 19.5^\circ = 109.5^\circ$ .

Contributed by Tomas Gubek, January 2014

**Page 385:** In figure 7.2, the bandgap...

Contributed by Tomas Gubek, January 2014

**Page 386:** After equation (7.12): This is the value displayed in Figure 7-2 over the *ref* node.

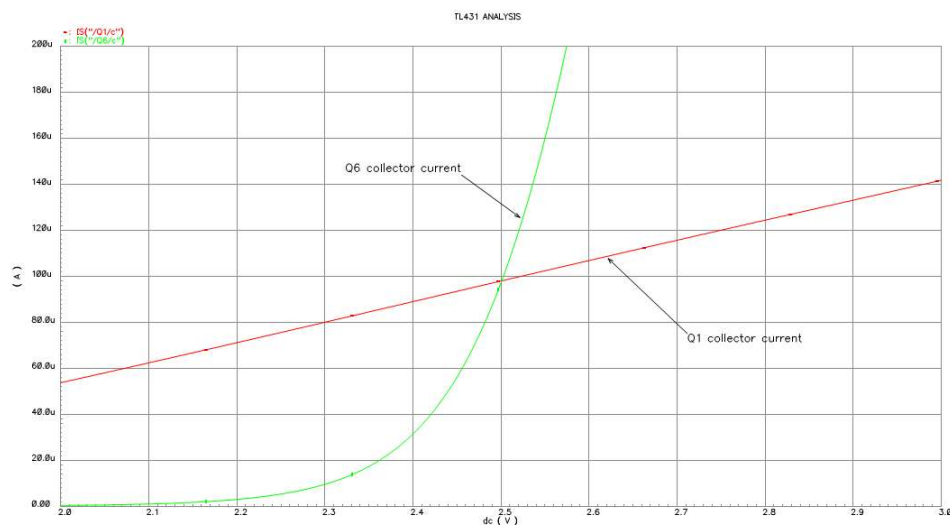
Contributed by Tomas Gubek, January 2014

**Page 387:** the paragraph below 7.1.2 needs a slight correction:

...working as a differential amplifier. However, the key point is the transconductance path from the  $V_{ref}$  node to  $Q_6$  collector (also to  $Q_9$  collector) that is much higher than the transconductance affecting the same  $V_{ref}$  node to  $Q_1$  collector. We can show that the first transconductance involving  $Q_6$  (or  $Q_9$ ) is an exponential function while that involving  $Q_1$  is a linear function. So when the loop leaves equilibrium and forces an increase of  $V_{ref}$ ,  $Q_6$  ( $Q_9$ ) conducts more current than  $Q_1$  and the cathode-anode voltage goes down via  $Q_{10}$  and  $Q_{11}$ .

Not for inclusion:

The below figure shows how these currents diverge when the 2.5-V reference level is left. Equilibrium is reached at  $V_{ref}$  equals 2.5 V, both transistors sink the exact same current and the TL431 cathode-anode voltage does not change. If  $V_{ref}$  increases, current in  $Q_6$  increases at a significantly larger pace than that of  $Q_1$ , upsetting equilibrium:  $V_{KA}$  goes down.



Book section highlighted by Tomas Gubek, January 2014

Figure update kindly contributed by Petr Kadanka, January 2014

**Page 388:** Figure 7.4: of course, the “ $\mu$ ” had to transform into m... Yes, you should read 300  $\mu$ V and 600  $\mu$ A.

**Page 394:** In (7.26) the term  $V_f$  should be replace by 1.

Contributed by Tomas Gubek, January 2014

**Page 414:** the sentence below (7.95) should be: As the total capacitance is 618.3 nF...

Contributed by Tomas Gubek, January 2014

**Page 433:** Figure 7.35: of course, the “ $\mu$ ” had to transform into m... Yes, you should read 1000  $\mu$ F and 0.1  $\mu$ F for the 2 caps.

**Page 437:** the first parameter should be  $I_{Zbias}$ .

Contributed by Tomas Gubek, January 2014

**Page 443:** ... $Q_1$  (if  $V_{out}$  decreases)

Contributed by Tomas Gubek, January 2014

**Page 445:** Figure 7.47: the sign  $\leq$  has gone in the right side of  $R_{LED}$ .

**Page 446:** Figure 7.49: the sign  $\leq$  has gone in the right side of  $R_{LED}$ .

**Page 455:** middle of the page: ...account for *his* presence... → for *its* presence...

**Page 455:** in (8.2), 0.017 should be 0.018  
Contributed by Tomas Gubek, January 2014

**Page 495:** some of the signs have gone during the print:  
Figure 9.8, a minus sign is missing close the second arrow vertically pointing to the circle.

**Page 499:** the sign has gone during the print:  
Figure 9.12, a minus sign is missing in the triangle, it is  $-G(s)$  of course.  
Contributed by Tomas Gubek, January 2014

**Page 500:** a question mark is in the middle of the sentence and has nothing to do here.  
...at point B in relation? to...  
...case, **is** delayed by  $90^\circ$ .  
Contributed by Tomas Gubek, January 2014

**Page 504:** equation 9.26: the numerator is not  $Z_{out}$  of course, but  $Z_{in}$ :

$$T(s) = \frac{V_{err}(s)}{V_c(s)} = -G(s)H(s) \frac{Z_{in}(s)}{Z_{out}(s) + Z_{in}(s)}$$

Contributed by Thoralf Rosahl, February 2013

**Page 514:** equation 9.43, the denominator  $R_3$  should obviously be the result of 9.42 (1.35 k $\Omega$ ).

$$G_0 = \frac{R_{pullup} CTR_{max}}{R_{3,max}} = \frac{3.3k \times 1.4}{1.35k} = 3.42 \text{ or } 10.7 \text{ dB}$$

In the text, ...result found in (9.43) : 10.7 dB minimum gain.  
Contributed by Tomas Gubek, February 2014

**Page 515:** in the text, 2<sup>nd</sup> paragraph, there is a reference to a source  $B_1$  that is not labeled in Figure 9.29. It is actually the source right to the input source  $V_6$ ,  $(V(err)-0.6)/3...$   
Contributed by Tomas Gubek, February 2014

**Page 529:** in figure 9.41, this is 10  $\mu$ s and not 10 ms.  
Contributed by John Pearson, February 2013

**Page 531:** in the upper section, reference [17] is wrong and it should be [14]  
Contributed by John Pearson, February 2013

**Page 540:** some subscripts have disappeared when going to print. In figures 9.47/9.48/9.49 and 9.50, the  $\pi$  in  $r_\pi$  is gone, as well as  $\beta$  in  $\beta I_b(s)$ . Figure 9.51 is ok though.

**Page 542:** oops, I goofed the equation by replacing  $r_\pi$  by  $h_{11}$ :

$$\omega_z = \frac{\beta}{C_b r_\pi}$$

Contributed by Tomas Gubek, February 2014

**Page 543:** the sentence above (9.115), ...and solving for the base current, we have:  
Contributed by Tomas Gubek, February 2014



**Page 543:** in the sentence (9.116), this is  $R_{FB}$  not RFB.

Contributed by Tomas Gubek, February 2014

**Page 550:** to match equations below, in the upper component list  $\beta \approx 73$ .

Contributed by Tomas Gubek, February 2014

**Page 555:** in (9.149), it is  $P_{R_1} = R_1 I_{R_1, rms}^2 = 0.377 \times 5.98^2 = 143.467 \text{ W}$

Contributed by Tomas Gubek, February 2014

**Page 556:** in (9.155) and (9.156), it is

$$Q = R \sqrt{\frac{C}{L}} \quad (9.155)$$

$$Q = \frac{-R^2}{R - R} \sqrt{\frac{C}{L}} = \infty \quad (9.155)$$

Contributed by Tomas Gubek, February 2014

**Page 561:** in Figures 9.69/9.70,  $C$  is 100  $\mu\text{F}$  not 100 mF