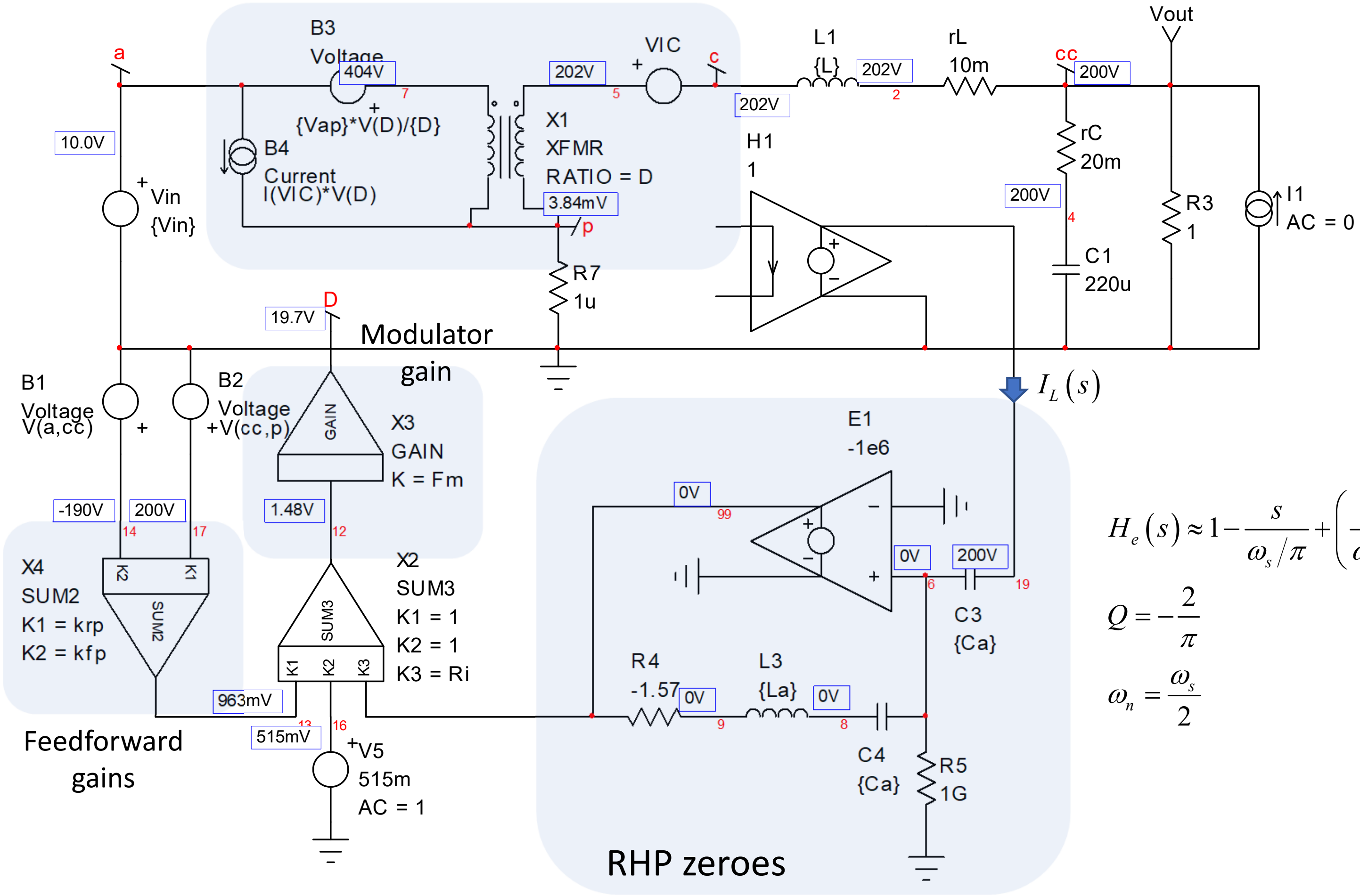
Voltage-Mode PWM switch model

parameters

Vin=10
Vout=5
Vap=Vin
Vac=Vin-Vout
RL=1
L=100u
Ri=100m
D=Vout/Vin
Mc=1.5
Fs=100k
Ts=1/Fs
pi=3.14159
Ca=(1/Fs)/pi
La=(1/Fs)/pi
kfp=-(D*(1/Fs)*Ri/L)*(1-D/2)
krp=((1-D)^2*(1/Fs)*Ri)/(2*L)
Sn=((Vin-Vout)/L)*Ri
Fm=1/((Sn+(Mc-1)*Sn)*Ts)
Fm2=L/(Vac*Mc*Ts*Ri)



$$H_e(s) \approx 1 - \frac{s}{\omega_s/\pi} + \left(\frac{s}{\omega_s/2}\right)^2 = 1 + \frac{s}{\omega_n Q} + \left(\frac{s}{\omega_n}\right)^2$$

$$Q = -\frac{2}{\pi}$$

$$\omega_n = \frac{\omega_s}{2}$$

Ridley's original current-mode subcircuit models the sampling block $H_e(s)$ via a pair of RHP zeroes. These RHP zeroes are modeled with a second-order polynomial form featuring a fixed negative Q tuned at half the switching frequency.

Christophe Basso – January 2021

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.SUBCKT PWMCCMr 1 2 3 4 5 {RI=0.33 L=37.5U FS=50K RL=1 D=0.45 VAP=11 VAC=6 IC=0.8 VP=2}
*            A P C C'Control
* .PARAM TS = {1/FS}       ; Switching time
* .PARAM PI = 3.14159     ; PI constant
* .PARAM KF= {-(D*TS*RI/L)*(1-D/2)}
* .PARAM KR= {((1-D)^2*TS*RI)/(2*L)}
* PWM Switch model *
BE2 7 1 V =  V(17)*{(VAP/D)}
BG1 1 2 I =  V(17)*{IC}
BGxf 7 2 I =  I(Vxf)*{D}
BExf 9 2 V =  V(7,2)*{D}
Vxf 9 3 0
Rvc 5 0 10MEG
* He(s) Circuit *
Hi 10 0  Vxf  1
C1 10 12 {(1/FS)/3.14159}
L1 12 13 {(1/FS)/3.14159}
C2 13 14 {(1/FS)/3.14159}
Re 14 15  -1.57
E1 15 0 12 0 -1E6
R2 12 0 10MEG
* Summing gains *
```

$V_{ac'}$ k_f' $V_{c'p}$ k_r'

```
BEd 16 0 V =  V(1,4) * {-(D*(1/FS)*RI/L)*(1-D/2)} + V(4,2) * {((1-D)^2*(1/FS)*RI)/(2*L)} + V(15) * {RI} + V(5)
Rd 16 0 10MEG
* Modulator Gain *
BEFm 17 0 V = V(16)*1/({VP}+({VAC}*{1/FS}*{RI}/{L}))
RFm 17 0 10MEG
.ENDS
```

This is the model I built in 1996 and it calculates all the necessary coefficients for using the subcircuit in all switching cells.

It is possible to replace this block by the real sampling block as described in Ridley's thesis:

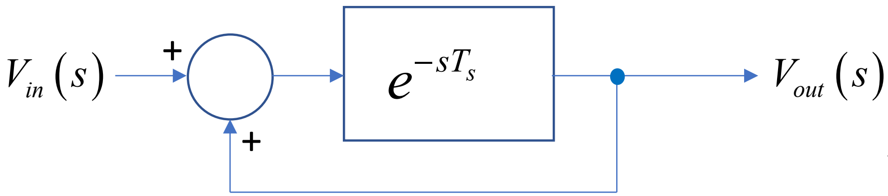
$$H_e(s) = \frac{sT_s}{e^{sT_s} - 1}$$

This expression features a zero at the origin and naturally cuts off any dc component: this is an ac-only model. Rather than using Laplace expressions not always available in a SPICE simulator, I've tried to use a delay line to exactly model this block in a simpler way. If you multiply the numerator and the denominator by e^{-sT_s} you have:

$$\frac{sT_s}{e^{sT_s} - 1}\,\frac{e^{-sT_s}}{e^{-sT_s}} = \frac{e^{-sT_s}}{1 - e^{-sT_s}}\,sT_s$$

This is a delay line

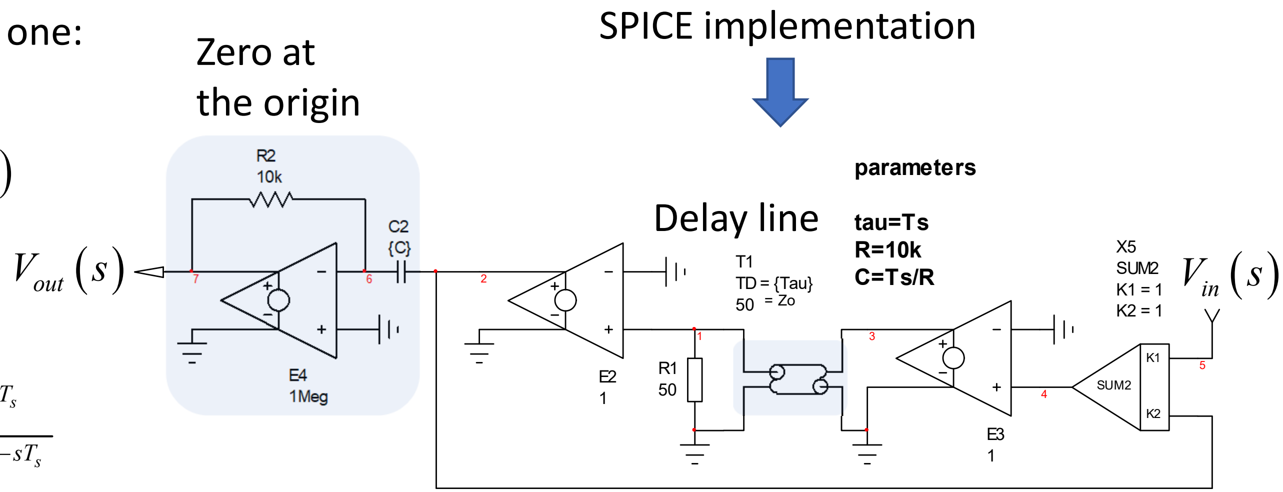
It can be modeled with a closed-loop system like this one:



$$V_{out}(s) = \left[V_{out}(s) + V_{in}(s)\right]e^{-sT_s}$$

$$V_{out}(s)\left(1 - e^{-sT_s}\right) = V_{in}(s)\,e^{-sT_s}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{e^{-sT_s}}{1 - e^{-sT_s}}$$



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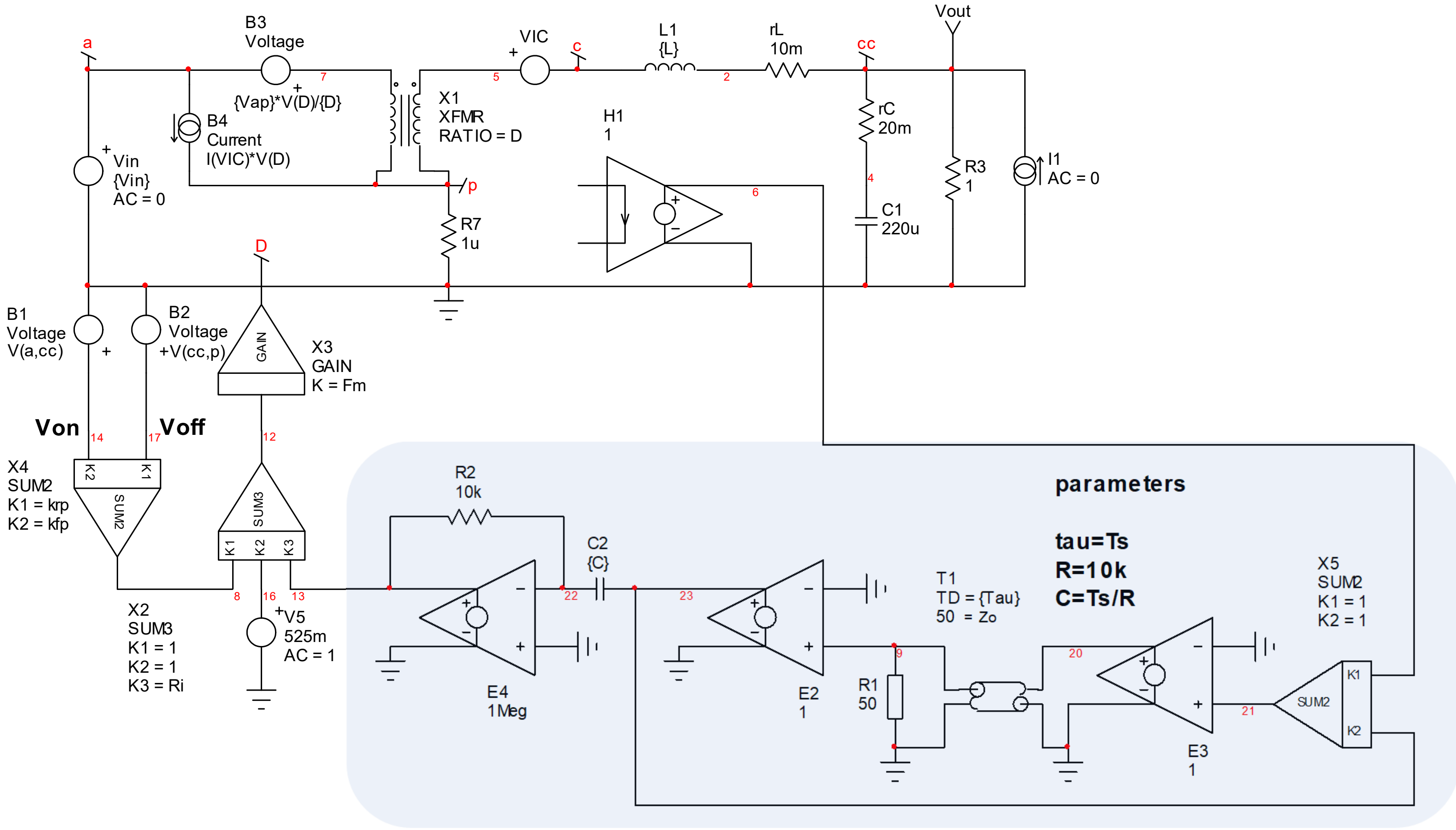
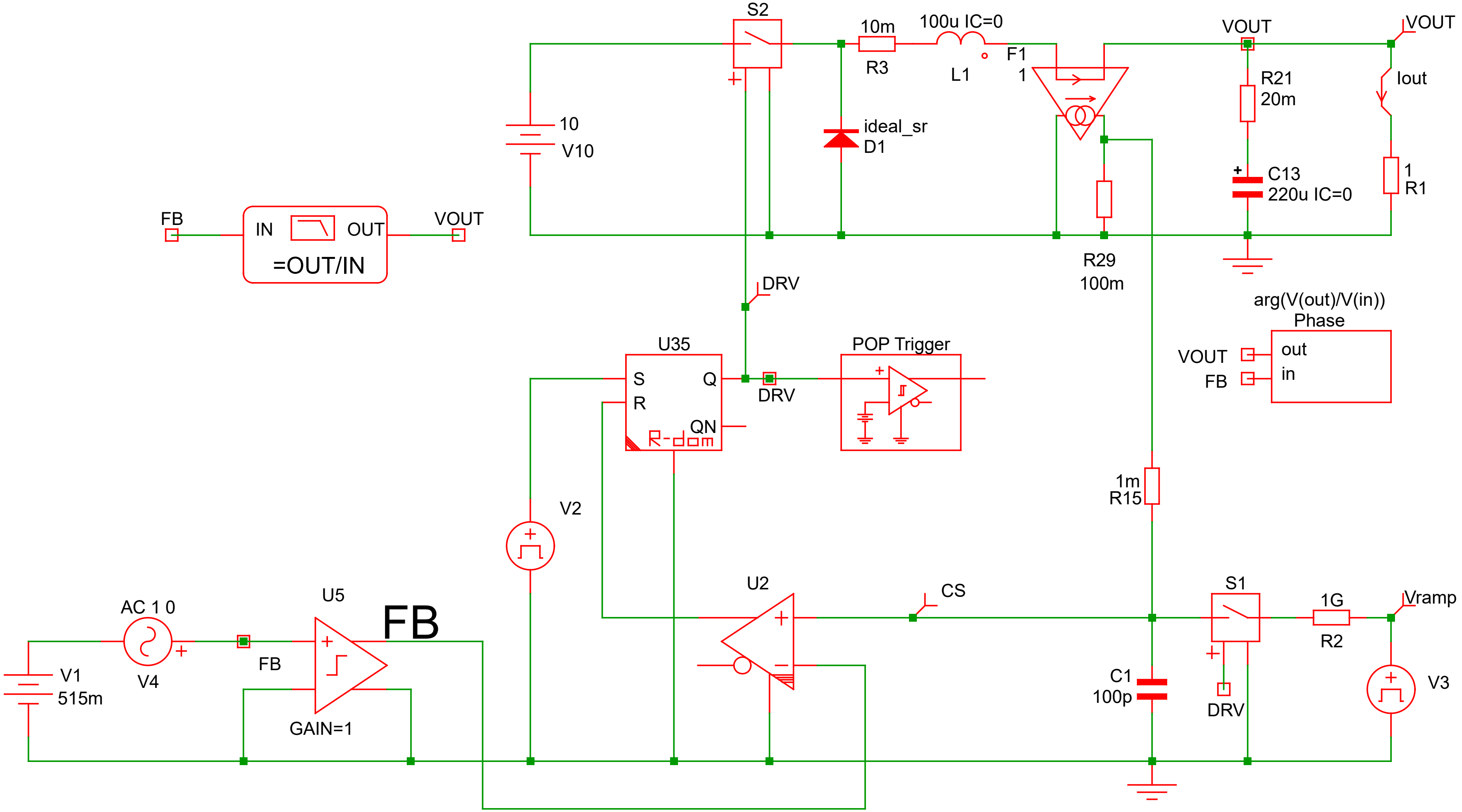
kfp=-(D*(1/Fs)*Ri/L)*(1-D/2)
krp=((1-D)^2*(1/Fs)*Ri)/(2*L)
kf=kfp
kr=-kfp+krp
Sn=((Vin-Vout)/L)*Ri
Fm=1/((Sn+(Mc-1)*Sn)*Ts)
Fm2=L/(Vac*Mc*Ts*Ri)

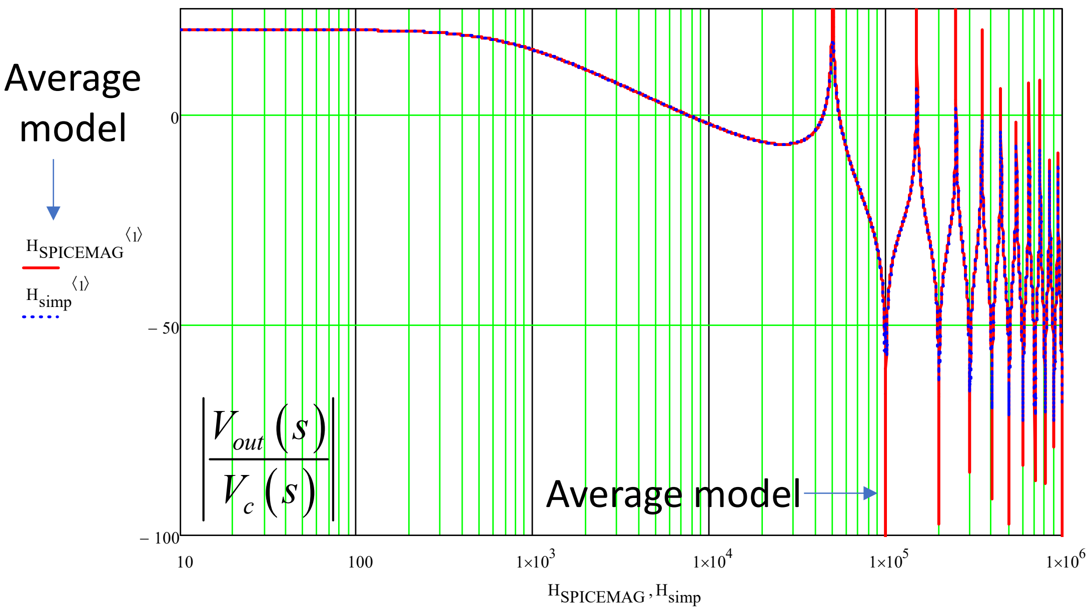


Buck converter operated in current-mode control

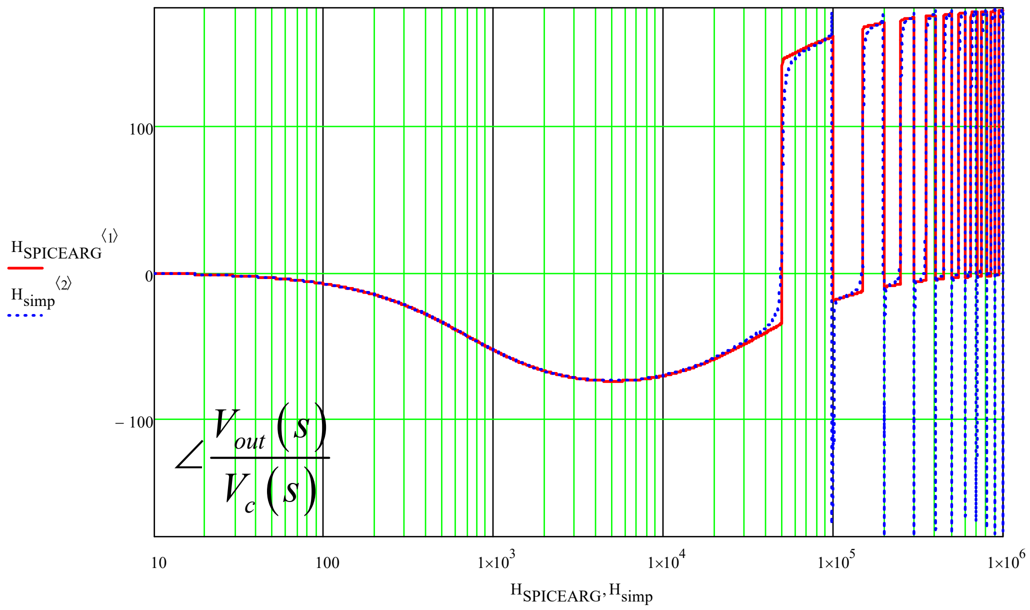
SIMPLIS model used for extracting the small-signal response



When this block is installed, the ac response of the average CM model matches that of SIMPLIS:



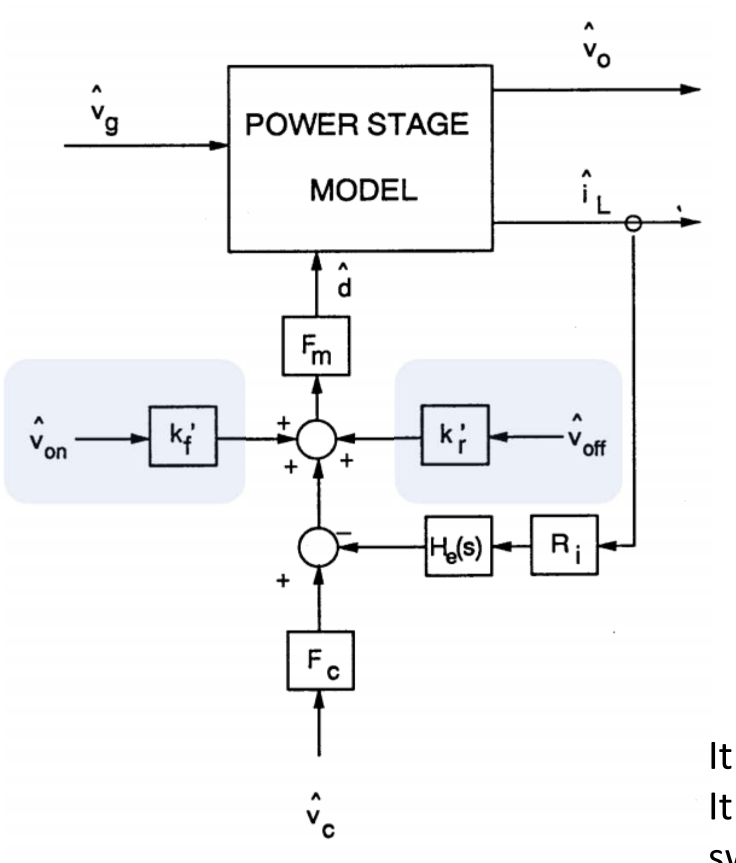
Magnitude



Phase

The average model is less damped than with SIMPLIS and explains the phase differences with a more pronounced phase response for SPICE.

I have been asked several times to explain how the feedforward gains were derived. These coefficients reflect the impact of the input and output voltages on the inductor current.



How to obtain?

	Constant Frequency Trailing Edge
k_f'	$-\frac{DT_s R_i}{L}\left[1-\frac{D}{2}\right]$
k_r'	$\frac{D'^2 T_s R_i}{2L}$

It is important to note that these coefficients k_f' k_r' are invariant. It means that their expressions are not tied to a particular switching cell and remain the same for a boost, a buck or a buck-boost converter with a properly-oriented subcircuit.

$$\left.\frac{\partial I_L\left(v_c, v_{on}, v_{off}\right)}{\partial v_{on}}\right|_{v_{off}=v_c=0} =$$

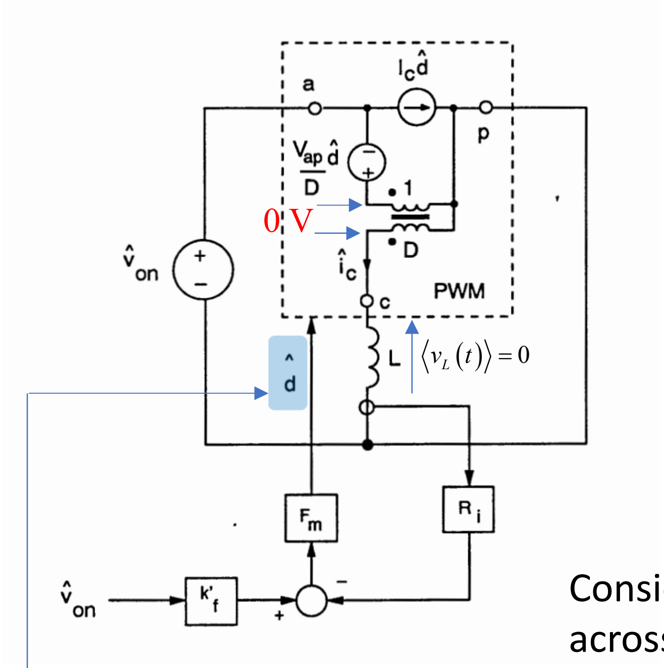
$$\frac{d}{dv_{on}}\left[\frac{v_c}{R_i} - \frac{v_{off}}{v_{on}+v_{off}}\cdot T_s\cdot\frac{S_e}{R_i} - \frac{\frac{v_{off}\cdot R_i}{L}\cdot\left(1-\frac{v_{off}}{v_{on}+v_{off}}\right)\cdot T_s}{2\cdot R_i}\right] \rightarrow \frac{S_e\cdot T_s\cdot v_{off}}{R_i\cdot\left(v_{on}+v_{off}\right)^2} - \frac{T_s\cdot v_{off}^2}{2\cdot L\cdot\left(v_{on}+v_{off}\right)^2}$$

D $\quad$ D^2

$$\frac{S_e\cdot T_s}{R_i\cdot\left(v_{on}+v_{off}\right)}\cdot\frac{v_{off}}{v_{on}+v_{off}} - \frac{T_s}{2\cdot L}\cdot\left(\frac{v_{off}}{v_{on}+v_{off}}\right)^2 = \frac{S_eT_sD}{R_iV_{ap}} - \frac{T_sD^2}{2L}$$

$V_{ac}+V_{cp}$ → V_{ap}

$$\frac{<\hat{i}_L>}{\hat{v}_{on}} = \frac{DS_eT_s}{V_{ap}R_i} - \frac{D^2T_s}{2L}$$



$$\hat{d} = \left(\hat{v}_{on}k'_f - \hat{i}_L R_i\right)F_m$$

$$\hat{i}_L = \frac{1}{R_i}\left(-\frac{\hat{d}}{F_m} + k'_f\,\hat{v}_{on}\right)$$

Considering 0 V across L:

$$\hat{v}_{on}D = -V_{ap}\hat{d}$$

$$\hat{d} = -\frac{\hat{v}_{on}D}{V_{ap}}$$

$$\hat{i}_L = \frac{1}{R_i}\left(\frac{\hat{v}_{on}D}{V_{ap}F_m} + k'_f\,\hat{v}_{on}\right)$$

$$\frac{\hat{i}_L}{\hat{v}_{on}} = \frac{1}{R_i}\left(\frac{D}{V_{ap}F_m} + k'_f\right)$$

$$\frac{<\hat{i}_L>}{\hat{v}_{on}} = \frac{1}{R_i}\left[\frac{D}{F_mV_{ap}} + k'_f\right]$$

Invariant feedforward gains derivation for Ridley's model
Christophe Basso – November 2020

$$\frac{<\hat{i}_L>}{\hat{v}_{on}} = \frac{D S_e T_s}{V_{ap} R_i} - \frac{D^2 T_s}{2L} \qquad \frac{<\hat{i}_L>}{\hat{v}_{on}} = \frac{1}{R_i}\left[\frac{D}{F_m V_{ap}} + k_f'\right]$$

$$k_f' = \frac{-D T_s R_i}{L}\left[1 - \frac{D}{2}\right]$$

$$F_m = \frac{1}{(S_e + S_n)T_s} \qquad S_n = \frac{v_{on}}{L} R_i$$

$$V_{ap} = v_{on} + v_{off}$$

$$\frac{D \cdot S_e \cdot T_s}{V_{ap} \cdot R_i} - \frac{D^2 \cdot T_s}{2 \cdot L} = \frac{1}{R_i} \cdot \left(\frac{D}{F_m \cdot V_{ap}} + k_f\right)$$

$$k_f = -\frac{2 \cdot D \cdot L - 2 \cdot D \cdot F_m \cdot L \cdot S_e \cdot T_s + D^2 \cdot F_m \cdot R_i \cdot T_s \cdot V_{ap}}{2 \cdot F_m \cdot L \cdot V_{ap}}$$

$$-\frac{D \cdot T_s \cdot \left[2 \cdot L \cdot \left(\frac{v_{on}}{L} \cdot R_i\right) + D \cdot R_i \cdot (v_{on} + v_{off})\right]}{2 \cdot L \cdot (v_{on} + v_{off})} = -0.045$$

$$-D \cdot R_i \cdot T_s \cdot \frac{2}{2 \cdot L} \cdot \frac{v_{on}}{v_{on} + v_{off}} - D \cdot R_i \cdot \frac{T_s}{2 \cdot L} \cdot \frac{D \cdot v_{on}}{v_{on} + v_{off}} - D \cdot R_i \cdot \frac{T_s}{2 \cdot L} \cdot \frac{D \cdot v_{off}}{v_{on} + v_{off}} = -0.045$$

$$-D \cdot R_i \cdot T_s \cdot \frac{2}{2 \cdot L} \cdot (1 - D) - D \cdot R_i \cdot \frac{T_s}{2 \cdot L} \cdot D \cdot (1 - D) - D \cdot R_i \cdot \frac{T_s}{2 \cdot L} \cdot D^2 = -0.045$$

$$-D \cdot R_i \cdot T_s \cdot \frac{2}{2 \cdot L} \cdot (1 - D) - D \cdot R_i \cdot \frac{T_s}{2 \cdot L} \cdot D \cdot (1 - D) - D \cdot R_i \cdot \frac{T_s}{2 \cdot L} \cdot D^2 = -0.045$$

$$\frac{-D \cdot T_s \cdot R_i}{L} \cdot \left[1 - D + \frac{D \cdot (1 - D)}{2} + \frac{D^2}{2}\right] = -0.045$$

$$\frac{-D \cdot T_s \cdot R_i}{L} \cdot \left(1 - \frac{D}{2}\right) = -0.045$$

$$\frac{D}{D_p} = \frac{\frac{v_{off}}{v_{on} + v_{off}}}{1 - \frac{v_{off}}{v_{on} + v_{off}}} = \frac{v_{off}}{v_{on}}$$

$v_{on} := 6V$ $\quad v_{off} := 4V$ $\quad L := 100\mu H$

$V_{ap} := 10V$ $\quad R_i := 1.2\Omega$ $\quad T_s := 10\mu s$

$D := \frac{v_{off}}{v_{on} + v_{off}} = 0.4$ $\quad S_e := 10\frac{V}{s}$

$S_n := \frac{v_{on}}{L} \cdot R_i$

$D_p := \frac{v_{on}}{v_{on} + v_{off}} = 0.6$ $\quad F_m := \frac{1}{(S_e + S_n) \cdot T_s} = 1.389\frac{1}{V}$

$D + D_p = 1$

$$\left.\frac{\partial I_L\left(v_c, v_{on}, v_{off}\right)}{\partial v_{off}}\right|_{v_{on}=v_c=0} =$$

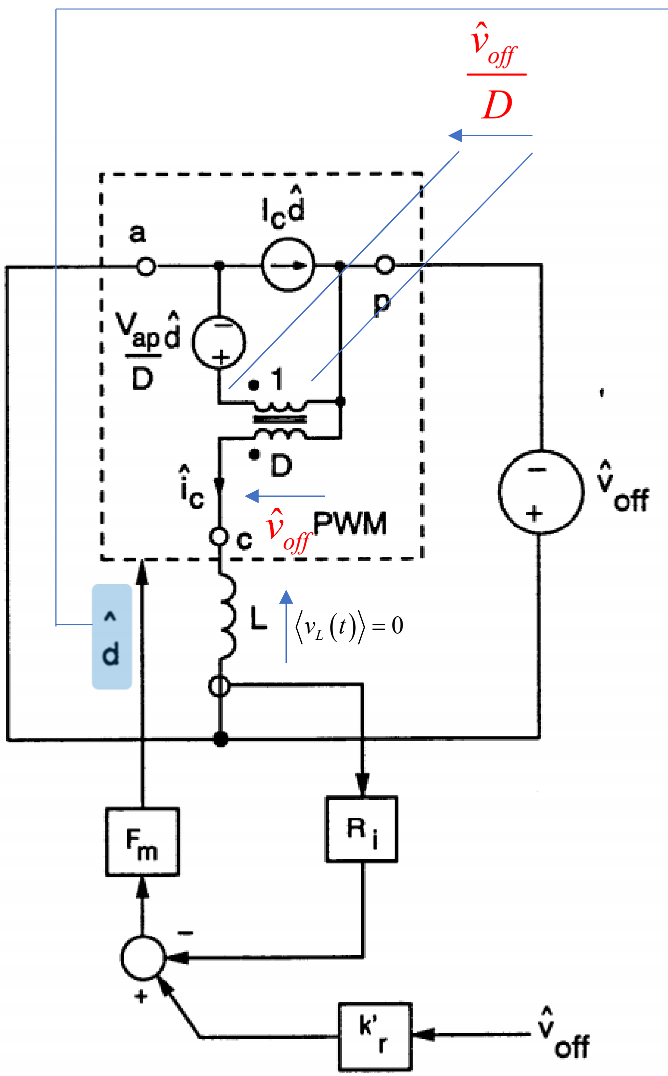
$$\frac{d}{dv_{off}}\left[\frac{v_c}{R_i} - \frac{v_{off}}{v_{on}+v_{off}}\cdot T_s\cdot\frac{S_e}{R_i} - \frac{\frac{v_{off}\cdot R_i}{L}\cdot\left(\frac{v_{on}}{v_{on}+v_{off}}\right)\cdot T_s}{2\cdot R_i}\right] \rightarrow \frac{S_e\cdot T_s\cdot v_{off}}{R_i\cdot\left(v_{on}+v_{off}\right)^2} - \frac{T_s\cdot v_{on}}{2\cdot L\cdot\left(v_{on}+v_{off}\right)} - \frac{S_e\cdot T_s}{R_i\cdot\left(v_{on}+v_{off}\right)} + \frac{T_s\cdot v_{on}\cdot v_{off}}{2\cdot L\cdot\left(v_{on}+v_{off}\right)^2} \rightarrow -\frac{T_s\cdot v_{on}\cdot\left(2\cdot L\cdot S_e + R_i\cdot v_{on}\right)}{2\cdot L\cdot R_i\cdot\left(v_{on}+v_{off}\right)^2}$$

$D' = 1 - D$ $\qquad$ D'^2

$$\frac{\hat{i}_L}{\hat{v}_{off}} = -\frac{T_s v_{on}\left(2LS_e + R_i v_{on}\right)}{2LR_i\left(v_{on}+v_{off}\right)^2} = -\frac{T_s v_{on}{}^2\, 2LS_e + T_s v_{on}{}^2 R_i}{2LR_i\left(v_{on}+v_{off}\right)^2} = -\frac{S_e T_s}{R_i\left(v_{on}+v_{off}\right)}\frac{v_{on}}{v_{on}+v_{off}} - \frac{T_s}{2L}\left(\frac{v_{on}}{v_{on}+v_{off}}\right)^2$$

$$V_{ac} + V_{cp} = V_{ap}$$

$$\frac{\hat{i}_L}{\hat{v}_{off}} = -\frac{S_e T_s}{R_i\left(v_{on}+v_{off}\right)}\frac{v_{on}}{v_{on}+v_{off}} - \frac{T_s}{2L}\left(\frac{v_{on}}{v_{on}+v_{off}}\right)^2 = -\frac{S_e T_s D}{R_i V_{ap}} - \frac{T_s D'^2}{2L} \quad\Longleftrightarrow\quad \frac{\langle \hat{i}_L \rangle}{\hat{v}_{off}} = \frac{-D' S_e T_s}{V_{ap} R_i} - \frac{D'^2 T_s}{2L}$$



$$\hat d = \left(\hat v_{off} k'_r - \hat i_L R_i\right) F_m$$

Considering 0 V across L: $\frac{V_{ap}\hat d}{D} - \frac{\hat v_{off}}{D} = -\hat v_{off}$ ⇒ $\hat d = \frac{v_{off}}{V_{ap}} D'$

$$\frac{v_{off}}{V_{ap}} D' = \left(\hat v_{off} k'_r - \hat i_L R_i\right) F_m$$

⇒ Solve for i_L: $\frac{\hat i_L}{\hat v_{off}} = \frac{1}{R_i}\left(k_r - \frac{D'}{F_m V_{ap}}\right)$

$$\frac{<\hat i_L>}{\hat v_{off}} = \frac{1}{R_i}\left[\frac{-D'}{F_m V_{ap}} \overset{+}{-} k_r'\right]$$

Solve for k_r

$$-\frac{S_e T_s D}{R_i V_{ap}} - \frac{T_s D'^2}{2L} = \frac{1}{R_i}\left(k_r - \frac{D'}{F_m V_{ap}}\right)$$

⇒ $R_i \cdot \left[\frac{D_p}{F_m \cdot R_i \cdot V_{ap}} - \frac{T_s \cdot v_{on} \cdot (2 \cdot L \cdot S_e + R_i \cdot v_{on})}{2 \cdot L \cdot R_i \cdot (v_{on} + v_{off})^2}\right] = 0.022$

$\frac{D_p^2 \cdot T_s \cdot R_i}{2 \cdot L} = 0.022$ $k_r' = \frac{D'^2 T_s R_i}{2L}$ ok

$$\frac{v_{off}}{V_{ap}} \cdot D_p = \left(v_{off} \cdot k_r - i_L \cdot R_i\right) \cdot F_m$$

$$\frac{F_m \cdot k_r \cdot v_{off} - \frac{D_p \cdot v_{off}}{V_{ap}}}{F_m \cdot R_i}$$

$$-\frac{T_s \cdot v_{on} \cdot \left(2 \cdot L \cdot S_e + R_i \cdot v_{on}\right)}{2 \cdot L \cdot R_i \cdot \left(v_{on} + v_{off}\right)^2} = \frac{F_m \cdot k_r - \frac{D_p}{V_{ap}}}{F_m \cdot R_i}$$

$$R_i \cdot \left[\frac{D_p}{F_m \cdot R_i \cdot V_{ap}} - \frac{T_s \cdot v_{on} \cdot \left(2 \cdot L \cdot S_e + R_i \cdot v_{on}\right)}{2 \cdot L \cdot R_i \cdot \left(v_{on} + v_{off}\right)^2}\right] = 0.0216$$

$$R_i \cdot \left[\frac{D_p}{\frac{1}{\left(S_e + \frac{v_{on}}{L} \cdot R_i\right) \cdot T_s} \cdot R_i \cdot \left(v_{on} + v_{off}\right)} - \frac{T_s \cdot v_{on} \cdot \left(2 \cdot L \cdot S_e + R_i \cdot v_{on}\right)}{2 \cdot L \cdot R_i \cdot \left(v_{on} + v_{off}\right)^2}\right] = 0.0216$$

$$R_i \cdot \left[\frac{D_p}{\frac{1}{\left(S_e + \frac{v_{on}}{L} \cdot R_i\right) \cdot T_s} \cdot R_i \cdot \left(v_{on} + v_{off}\right)} - \left[\frac{T_s \cdot 2 \cdot L \cdot S_e}{2 \cdot L \cdot R_i \cdot \left(v_{on} + v_{off}\right)} \cdot D_p + \frac{T_s \cdot R_i}{2 \cdot L \cdot R_i} \cdot \left(D_p\right)^2\right]\right] = 0.0216$$

$$R_i \cdot \left[D_p \cdot T_s \cdot \frac{2 \cdot v_{on} - D_p \cdot v_{off} - D_p \cdot v_{on}}{2 \cdot L \cdot \left(v_{on} + v_{off}\right)}\right] = 0.0216$$

$$R_i \cdot \left(D_p \cdot T_s \cdot \frac{v_{on}}{v_{on} + v_{off}} \cdot \frac{2 - D_p \cdot \frac{v_{off}}{v_{on}} - D_p}{2 \cdot L}\right) = 0.0216$$

$$\frac{R_i \cdot T_s}{2 \cdot L} \cdot \left[D_p \cdot D_p \cdot \left(2 - D_p \cdot \frac{v_{off}}{v_{on}} - D_p\right)\right] = 0.0216$$

$$\frac{R_i \cdot T_s \cdot D_p^{\,2}}{2 \cdot L} \cdot \left(2 - D_p \cdot \frac{v_{off}}{v_{on}} - D_p\right) = 0.0216$$

$$\frac{R_i \cdot T_s \cdot D_p^{\,2}}{2 \cdot L} \cdot \left(2 - D_p \cdot \frac{D}{D_p} - D_p\right) = 0.0216$$

$$\frac{R_i \cdot T_s \cdot D_p^{\,2}}{2 \cdot L} = 0.0216 \quad \Leftrightarrow \quad k_r' = \frac{D'^2 T_s R_i}{2L}$$