

# An Average Model for the Phase-Shifted Converter

Christophe Basso

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# Introduction

- ❑ A small-signal model has been described by Fu-Sheng Tsai in 1993
- ❑ Despite all efforts, I could not make it work (sorry about that).
- ❑ Based on the PWM switch, I modified the auto-toggling model described in my last book to cope with phase-shifted requirements.
- ❑ The model is now fully auto-toggling between CCM and DCM.

# Cycle-by-cycle simulation to understand the signals

- ❑ The switching model uses the simplified version of a phase shift controller

## parameters

$f_c = 1k$   
 $pm = 60$   
 $G_{fc} = 7$   
 $p_{fc} = 81$   
 $V_{out} = 12$

$G = 10^{-(G_{fc}/20)}$   
 $boost = pm - (p_{fc} - 90)$   
 $pi = 3.14159$   
 $K = \tan((boost/2 + 45) \cdot \pi / 180)$

$F_{zero} = f_c / k$   
 $F_{pole} = f_c \cdot k$

$R_{LED} = CTR_{min} \cdot R_{pullup} / G$   
 $C_{zero} = 1 / (2 \cdot \pi \cdot F_{zero} \cdot R_{upper})$   
 $C_{pole} = 1 / (2 \cdot \pi \cdot F_{pole} \cdot R_{pullup})$   
 $C_{pole} = C_{pole} \cdot X_{Copto}$

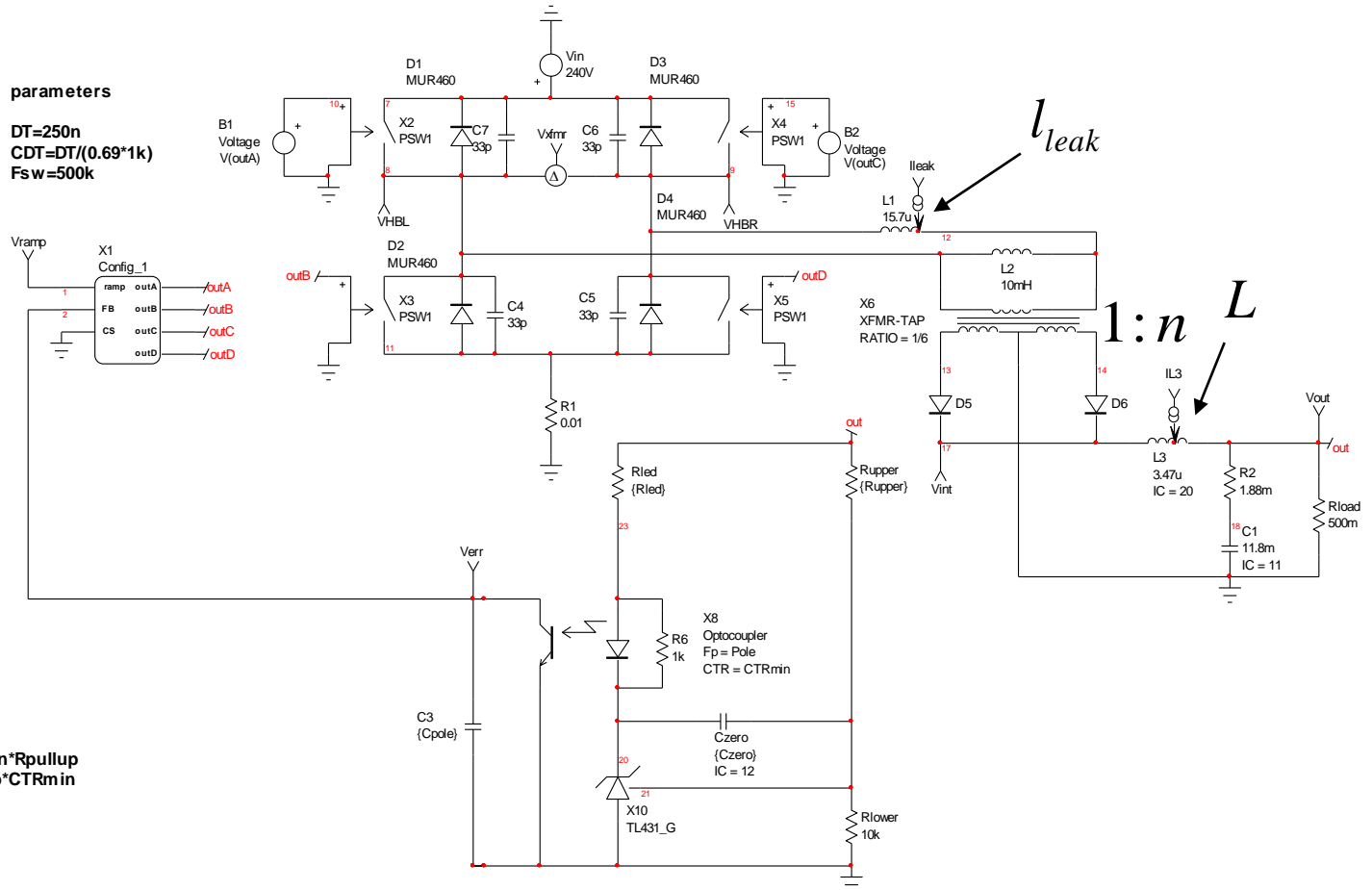
$V_{TL431min} = 2.5$   
 $V_f = 1$   
 $V_{dd} = 5V$   
 $V_{cesat} = 300m$   
 $I_{bias} = 1m$

$CTR_{min} = 0.3$   
 $Pole = 4k$   
 $C_{opto} = 1 / (2 \cdot \pi \cdot Pole \cdot R_{pullup})$

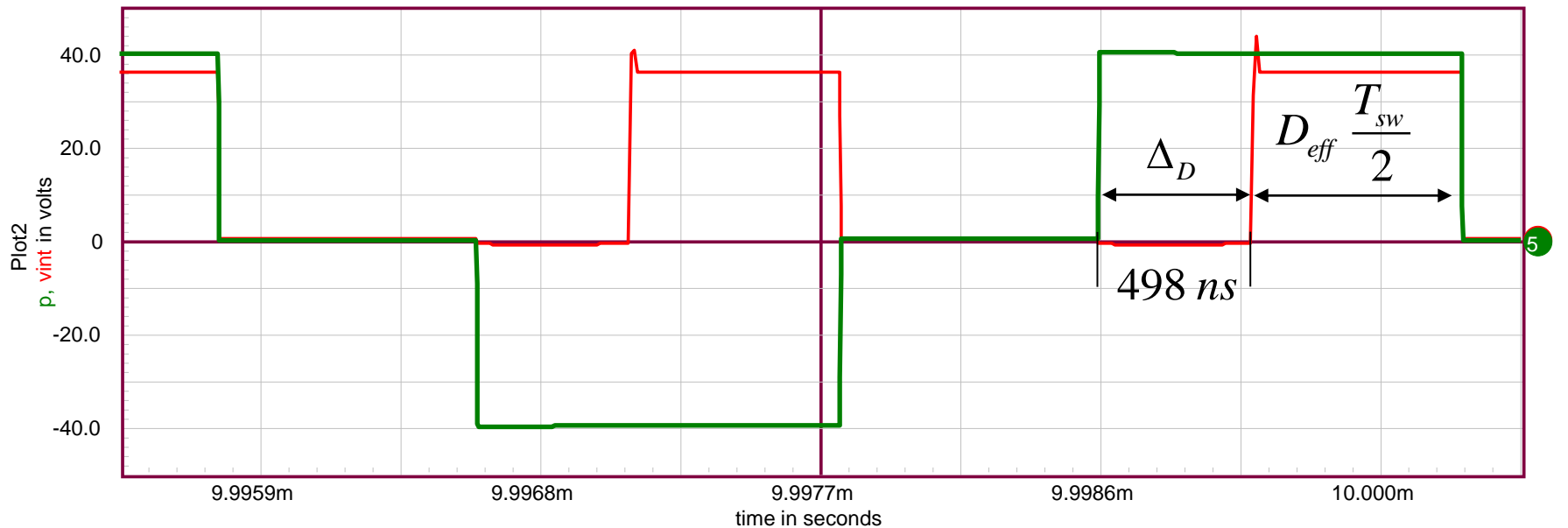
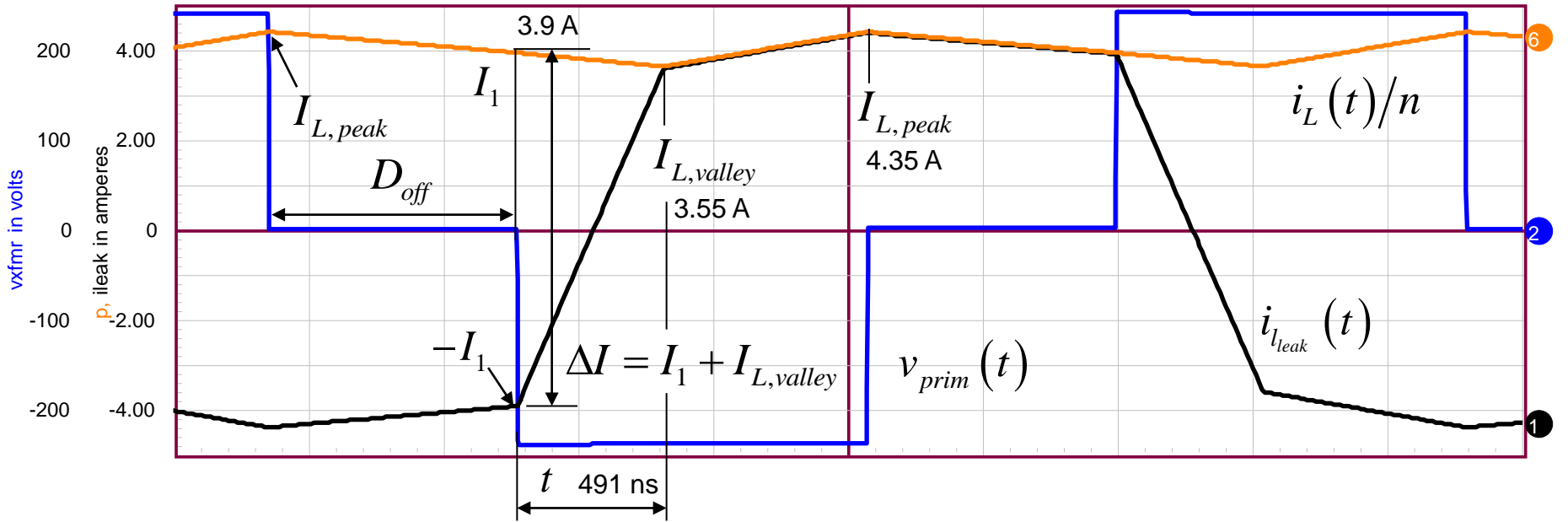
$R_{pullup} = 4.7k$   
 $R_{upper} = (V_{out} - 2.5) / 250\mu$   
 $R_{LED1} = V_{out} - V_f - V_{TL431min}$   
 $R_{LED2} = V_{dd} - V_{cesat} + I_{bias} \cdot CTR_{min} \cdot R_{pullup}$   
 $R_{LEDmax} = (R_{LED1} / R_{LED2}) \cdot R_{pullup} \cdot CTR_{min}$

## parameters

$DT = 250n$   
 $CDT = DT / (0.69 \cdot 1k)$   
 $F_{sw} = 500k$

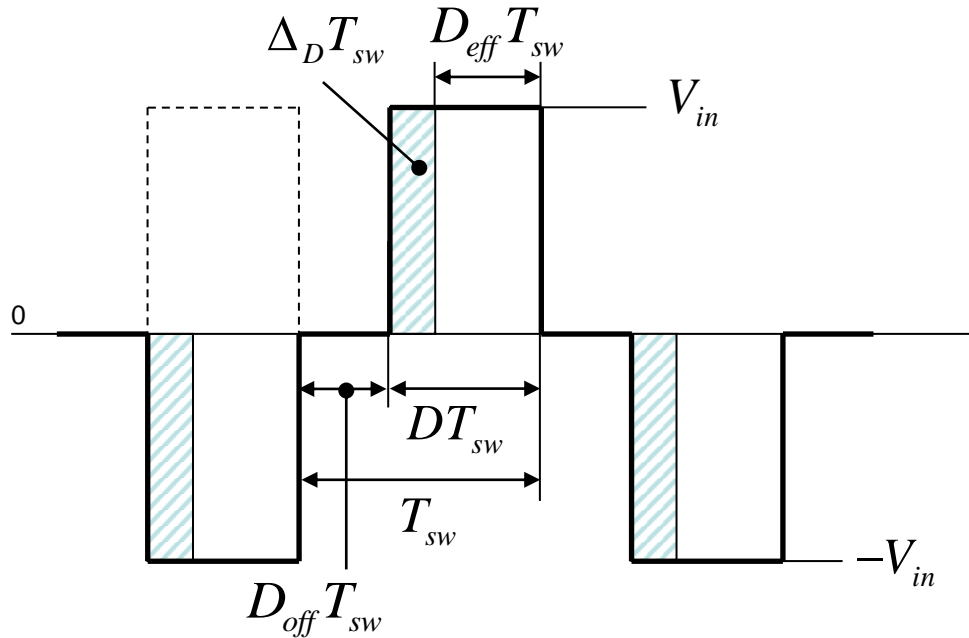


# Resulting signals from the switching simulation



# Equations to derive the leakage inductance reset time

Voltage across the transformer primary



$$D_{off} + D = 1$$

$$D_{off} = 1 - D$$

$$D = \Delta D + D_{eff}$$

$$D_{eff} = \frac{V_{out}}{nV_{in}}$$

Output inductor equations

$$\Delta I_L = \frac{V_{out} (1 - D_{eff})}{LF_{sw}}$$

$$I_{L,peak} = n \left( I_{out} + \frac{\Delta I_L}{2} \right) = n \left( I_{out} + \frac{V_{out} [1 - D_{eff}]}{2F_{sw}L} \right)$$

$$I_{L,valley} = n \left( I_{out} - \frac{\Delta I_L}{2} \right) = n \left( I_{out} - \frac{V_{out} [1 - D_{eff}]}{2F_{sw}L} \right)$$

$F_{sw}$  is the output ripple period which is twice the primary leakage period

## Equations to derive the leakage inductance reset time

$$I_1 = I_{L,peak} - D_{off} T_{sw} S_f = I_{L,peak} - (1-D) n T_{sw} \frac{V_{out}}{L}$$

$$I_1 = n \left( I_{out} + \frac{\Delta I_L}{2} \right) - (1-D) n T_{sw} \frac{V_{out}}{L}$$

$$I_1 = n \left( I_{out} + \frac{V_{out} (1-D_{eff})}{2 L F_{sw}} - (1-D) T_{sw} \frac{V_{out}}{L} \right)$$

The current excursion across the leakage inductor is  $\Delta I = I_{L,valley} + I_1$

The voltage excursion across the leakage inductor is  $V_{in}$

The current slope across the leakage inductor is  $V_{in} / l_{leak}$

The leakage inductor reset time is then:  $t = \Delta I l_{leak} / V_{in}$

$$\Delta I = n \left( I_{out} - \frac{V_{out} [1-D_{eff}]}{2 F_{sw} L} \right) + n \left( I_{out} + \frac{V_{out} (1-D_{eff})}{2 L F_{sw}} - (1-D) T_{sw} \frac{V_{out}}{L} \right)$$

## Equations to derive the leakage inductance reset time

$$\Delta D = \frac{\Delta I}{T_{sw} \frac{V_{in}}{l_{leak}}}$$

$$\Delta D = \frac{n \left( I_{out} - \frac{V_{out} [1 - D_{eff}]}{2 F_{sw} L_{out}} \right) + n \left( I_{out} + \frac{V_{out} (1 - D_{eff})}{2 L_{out} F_{sw}} - (1 - D) T_{sw} \frac{V_{out}}{L} \right)}{\frac{V_{in}}{l_{leak}} T_{sw}}$$

$$\Delta D = \frac{n \left[ 2 I_{out} - T_{sw} \frac{V_{out}}{L} (1 - D) \right]}{\frac{V_{in}}{l_{leak}} T_{sw}}$$

$$\Delta D = \frac{n}{\frac{V_{in}}{L_{lk}} \frac{T_s}{2}} \left( 2 I_L - \frac{V_{out}}{L} (1 - D) \frac{T_s}{2} \right)$$

The two formulas exactly match,  $T_s$  is the bridge switching period whereas  $T_{sw}$  is the controller switching period:  $T_s = 2 T_{sw}$

# Equations to derive the leakage inductance reset time

□ Numerical application:

$$n = \frac{1}{6}, I_{out} = 24 \text{ A}, F_{sw} = 500 \text{ kHz}, l_{leak} = 15.7 \mu\text{H}$$

$$L = 3.43 \mu\text{H}, V_{in} = 240 \text{ V}, V_{out} = 12 \text{ V}$$

$$\Delta D = \frac{n \left[ 2I_{out} - T_{sw} \frac{V_{out}}{L} (1 - D) \right]}{\frac{V_{in}}{l_{leak}} T_{sw}} = 0.245$$

$$\Delta D T_{sw} = 490 \text{ ns} \quad \longrightarrow \quad \text{Simulation gives 491 ns}$$

$$D = \Delta D + D_{eff} = \Delta D + \frac{V_{out}}{nV_{in}} = 0.245 + \frac{12}{166m \times 240} = 0.54$$

$$I_{L,peak} = 4.408 \text{ A} \quad I_{L,valley} = 3.591 \text{ A} \quad I_1 = 3.9 \text{ A}$$



# In the PWM switch model, the ripple is neglected

- ❑ In the previous expression, the leakage inductor current swung between the peak value and  $I_2$ .
- ❑ On average, we consider the inductor current ripple free
- ❑ Therefore, the leakage current swings between  $-nI_{out}$  and  $+nI_{out}$
- ❑ In the buck average model,  $I_{out} = I_c$ .
- ❑ The average equation for  $\Delta D$  is therefore:

$$\Delta D_{avg} = \frac{2 \cdot n \cdot l_{leak} I_{out} F_{sw}}{V_{in}} \longrightarrow \text{Gives 0.262}$$

- ❑ In the PWM switch,  $I_c$  is the output current terminal and  $V_{in}$  is applied across  $V_{ap}$ :

$$\Delta D_{avg} = \frac{2l_{leak} I_c F_{sw}}{V_{ap}} \quad \text{Because } I_c = I_{out}/n, \text{ the } n \text{ has gone in the model}$$

# Conduction Mode Transition Point Calculations

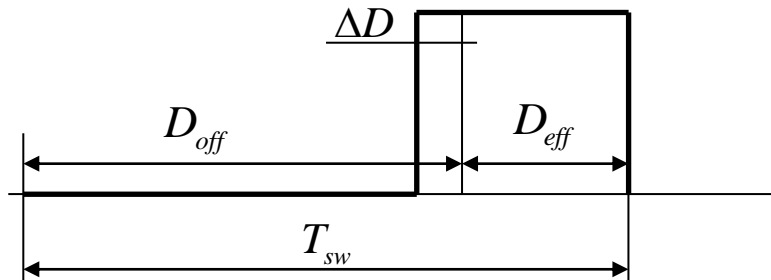
- Discontinuous Conduction Mode, classical buck:

The critical point is the same as for a classical buck except that  $D_{eff}$  must be used:

$$R_{crit} = 2LF_{sw} \frac{nV_{in}}{nV_{in} - V_{out}} = 2 \times 3.47 \mu \times 1 \text{Meg} \frac{0.166 \times 240}{0.166 \times 240 - 12} = 9.9 \Omega$$

This is the output ripple frequency or twice the oscillator value.

- Model transition point:



$$D_{off} = 1 - D_{eff}$$

$$D_{off} = 1 - (D - \Delta D)$$

$$D_{off} = 1 - D + \Delta D$$

# Conduction Mode Transition Point Calculations

□ The mode transition can also be derived by cancelling the equation describing  $\Delta D$ :

$$\Delta D = \frac{n \left[ 2I_{out} - T_{sw} \frac{V_{out}}{L} (1 - D) \right]}{\frac{V_{in}}{l_{leak}} T_{sw}} \Delta D + \frac{V_{out}}{nV_{in}} = 0$$

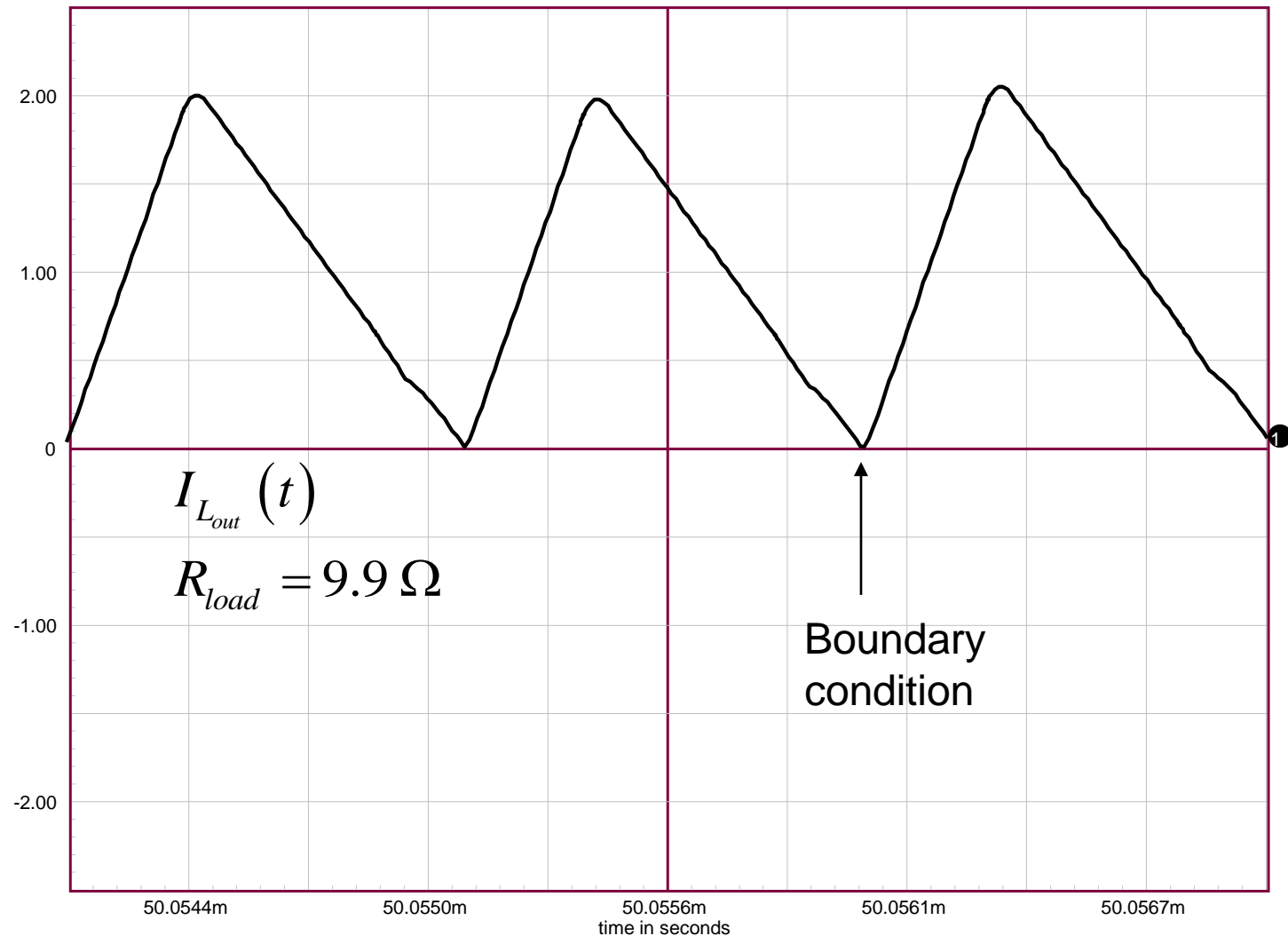
$$\Delta D = \frac{l_{leak} \left( T_{sw} V_{out}^2 - nT_{sw} V_{out} V_{in} + 2nI_{out} LV_{in} \right)}{T_{sw} V_{in} \left( LV_{in} - nl_{leak} V_{out} \right)} = 0$$

$$l_{leak} \left( T_{sw} V_{out}^2 - nT_{sw} V_{out} V_{in} + 2nI_{out} LV_{in} \right) = 0$$

$$\longrightarrow I_{out,crit} = \frac{T_{sw} V_{out} (nV_{in} - V_{out})}{2nLV_{in}}$$

$$R_{crit} = \frac{V_{out}}{I_{out,crit}} = \frac{2nLV_{in}}{T_{sw} (nV_{in} - V_{out})} = \frac{2 \times 0.166 \times 3.47 \mu \times 240}{1 \mu \times (0.166 \times 240 - 12)} = 9.9 \Omega$$

- ❑ The phase-shifted converter has been loaded by a 9.9-Ω resistor
- ❑ The switching ripple on the output inductor is 1 MHz



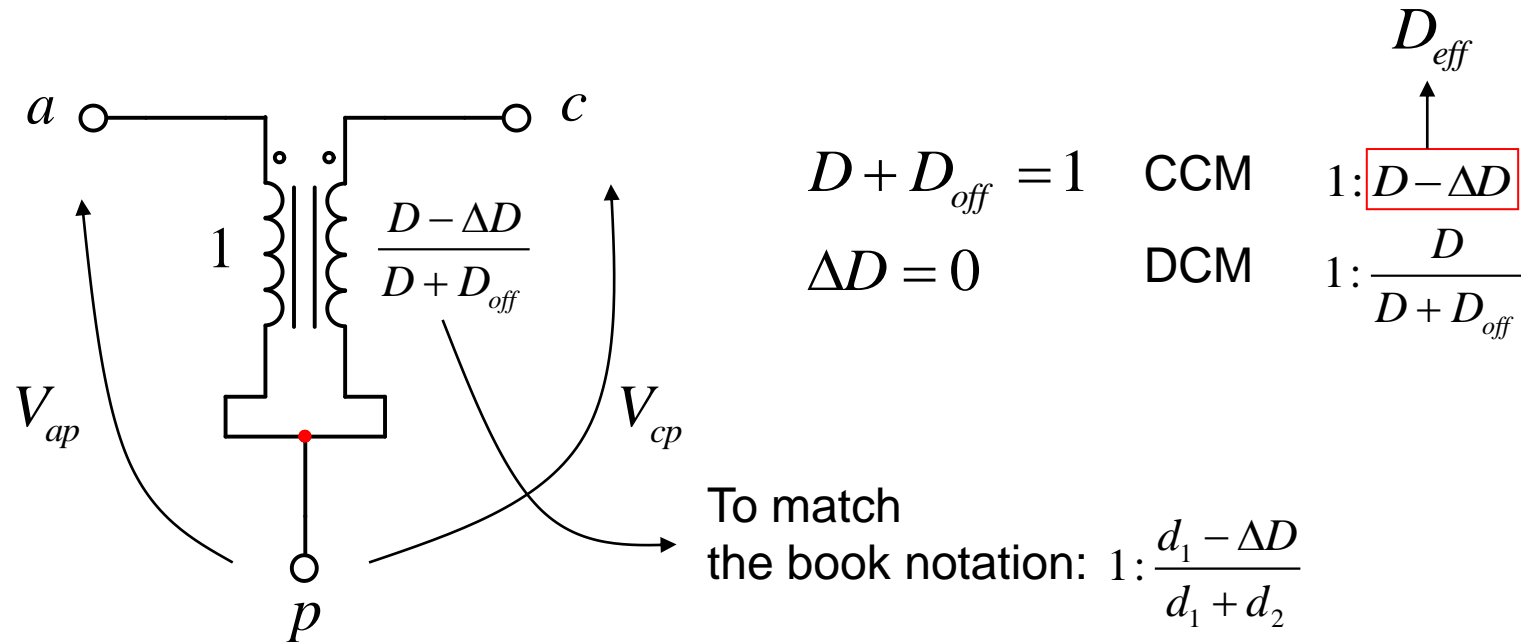
□ A formula including the effect of the leakage inductance has been derived by Monsieur Schutzen from General Electric:

$$R_{crit} = 2F_{sw} \frac{(l_{leak} n^2 + L)}{1 - \frac{V_{out}}{nV_{in}}}$$

□ It is similar to those derived before except that the total leakage (reflected to the secondary side) is accounted in series with the output inductor.

□ For large turns ratios, the contribution of the reflected leakage term is weak. However, for smaller ratios, it can make a difference between the results delivered by the first set of equations and Mr Schutzen's equation.

# Conduction Mode Transition Point Calculations



In the DCM PWM switch model described page 166 of my book:

$$d_2 = \frac{2LF_{sw}}{D} \frac{I_c}{V_{ac}} - d_1 \quad d_2 \text{ is the off duty-cycle in DCM, } d_1 \text{ is the on duty-cycle}$$

When  $d_2$  hits  $1 - d_1 + \Delta D$   $\longrightarrow$  The model leaves DCM and enters CCM

By clamping  $d_2$  between 0 and  $1 - d_1 + \Delta D$  the models auto-toggles.

# Average model implementation

```
.SUBCKT PWMswitchPS a c p d dL params: L=10u Ll=1u Fs=100k
```

```
*
* This subckt is DCM-CCM phase shift average model
*
* 1 - active ; 2 - passive ; 3 - common ; 4 - duty-cycle ; 5 - dL
*
```

```
.subckt limit d dc params: clampH=0.99 clampL=16m
```

```
Gd 0 dcx d 0 100u
Rdc dcx 0 10k
V1 clpn 0 {clampL}
V2 clpp 0 {clampH}
D1 clpn dcx dclamp
D2 dcx clpp dclamp
Bdc dc 0 V=V(dcx)
.model dclamp d n=0.01 rs=100m
.ENDS
```

```
.subckt limit2 d2nc d d2c dL
```

```
Gd 0 d2cx d2nc 0 100u
Rdc d2cx 0 10k
V1 clpn 0 7m
BV2 clpp 0 V=1-V(d)+V(dL)-6.687m
D1 clpn d2cx dclamp
D2 d2cx clpp dclamp
B2c d2c 0 V=V(d2cx)
.model dclamp d n=0.01 rs=100m
.ENDS
```

```
Xd d dc limit params: clampH=0.99 clampL=16m
```

```
BVcp 6 p V=((V(dc)-V(dL))/(V(dc)-V(dL)+V(d2)+1u))*V(a,p)
BIap a p I=((V(dc)-V(dL))/(V(dc)-V(dL)+V(d2)+1u))*I(VM)
Bd2 d2X 0 V=(2*I(VM)*{L}-v(a,c)*V(dc)^2*{1/Fs}) / (v(a,c)*V(dc)*{1/Fs}+1u)
Xd2 d2X dc d2 dL limit2
BdL dL 0 V=2*{L1}*I(VM)*{Fs}/(V(a,p)+1u) < 0 ? 0 : 2*{L1}*I(VM)*{Fs}/(V(a,p)+1u)
VM 6 c
*
```

```
.ENDS
```

Leakage inductance

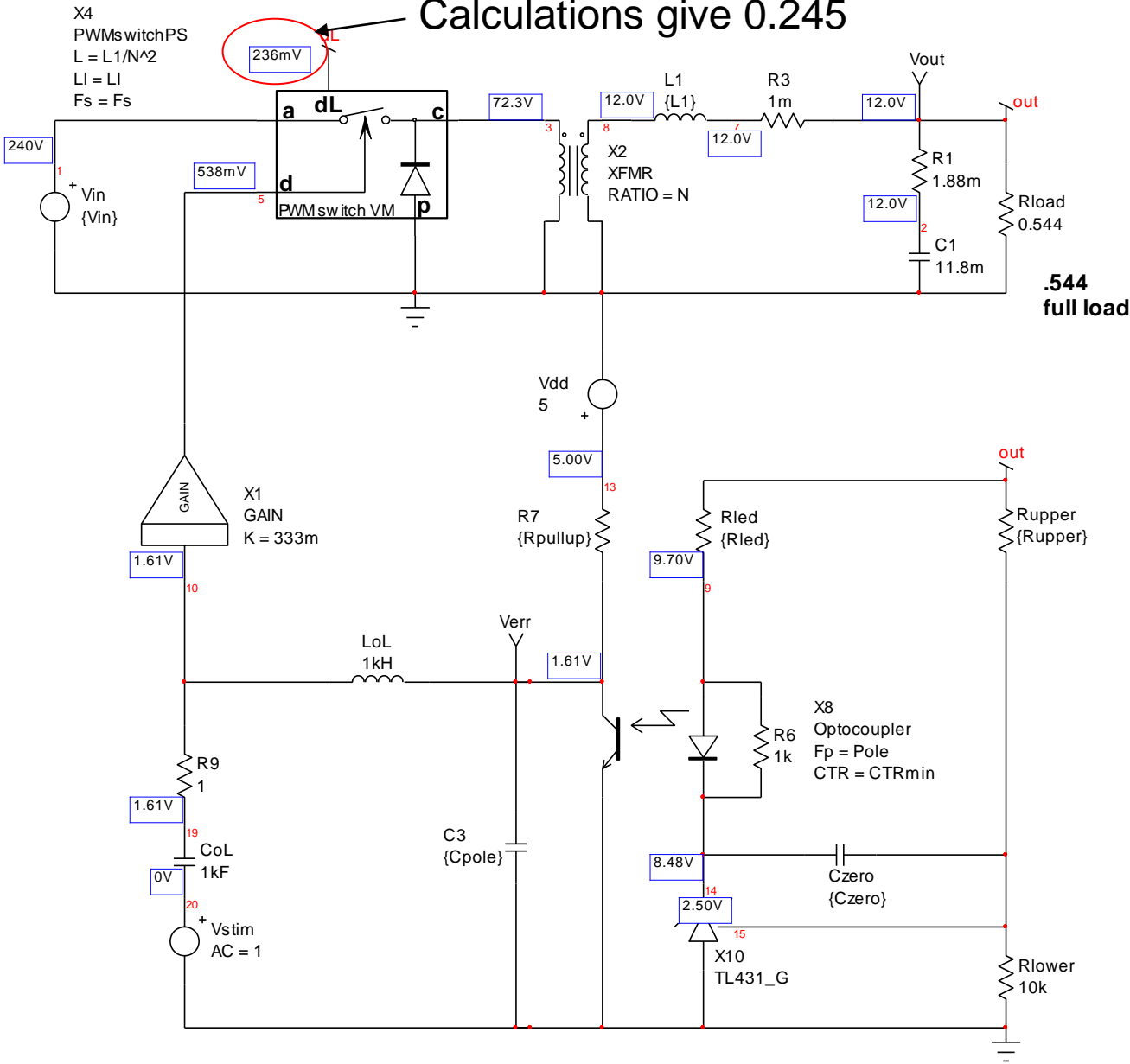
Accounts for  $\Delta D$

$$d_2 = \frac{2I_c L - V_{ac} d_1^2 T_{sw}}{V_{ac} d_1 T_{sw}}$$

$$\Delta D_{avg} = \frac{2l_{leak} I_c F_{sw}}{V_{ap}}$$

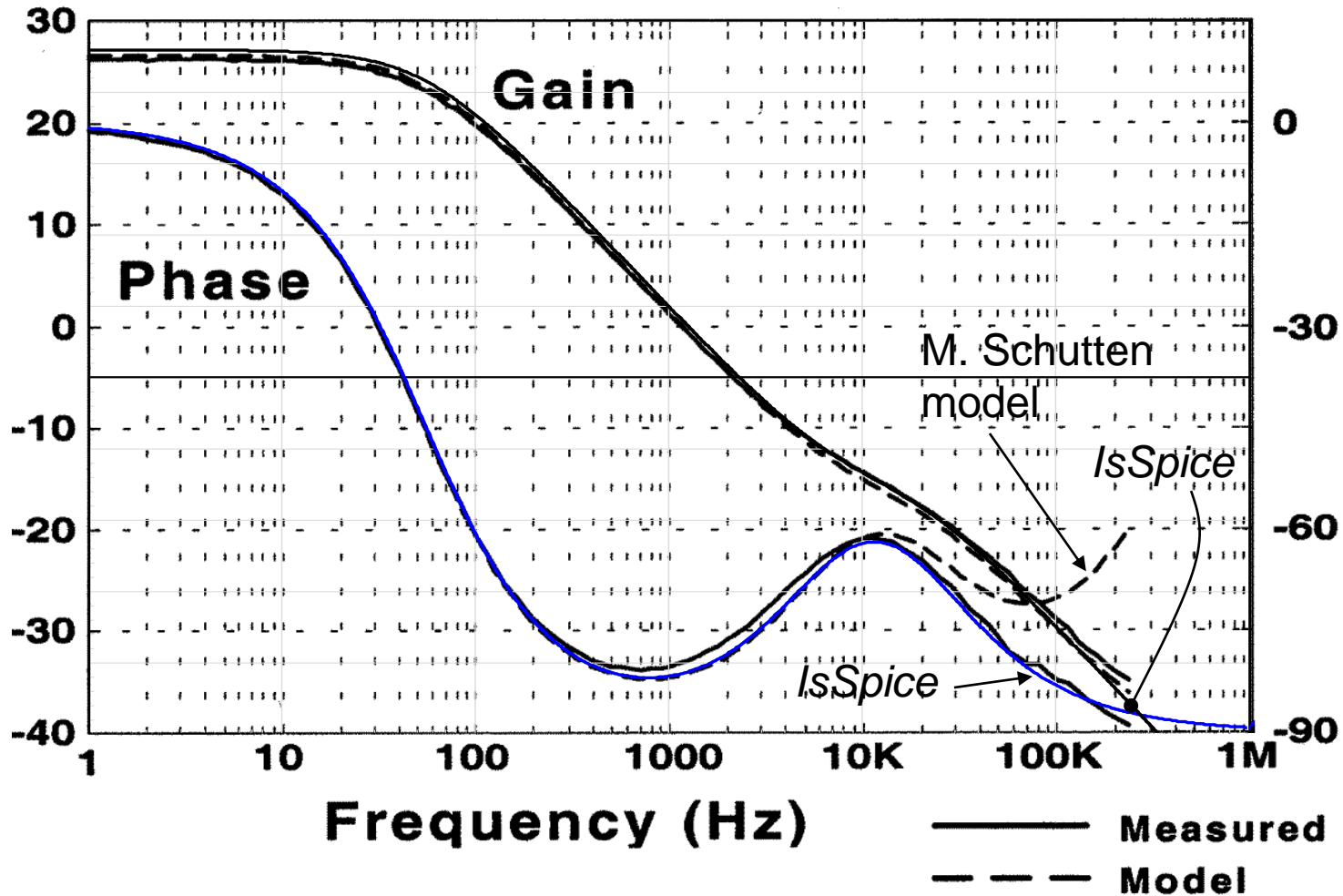
Clamp of  $d_2$

# Average model implementation, IsSpice



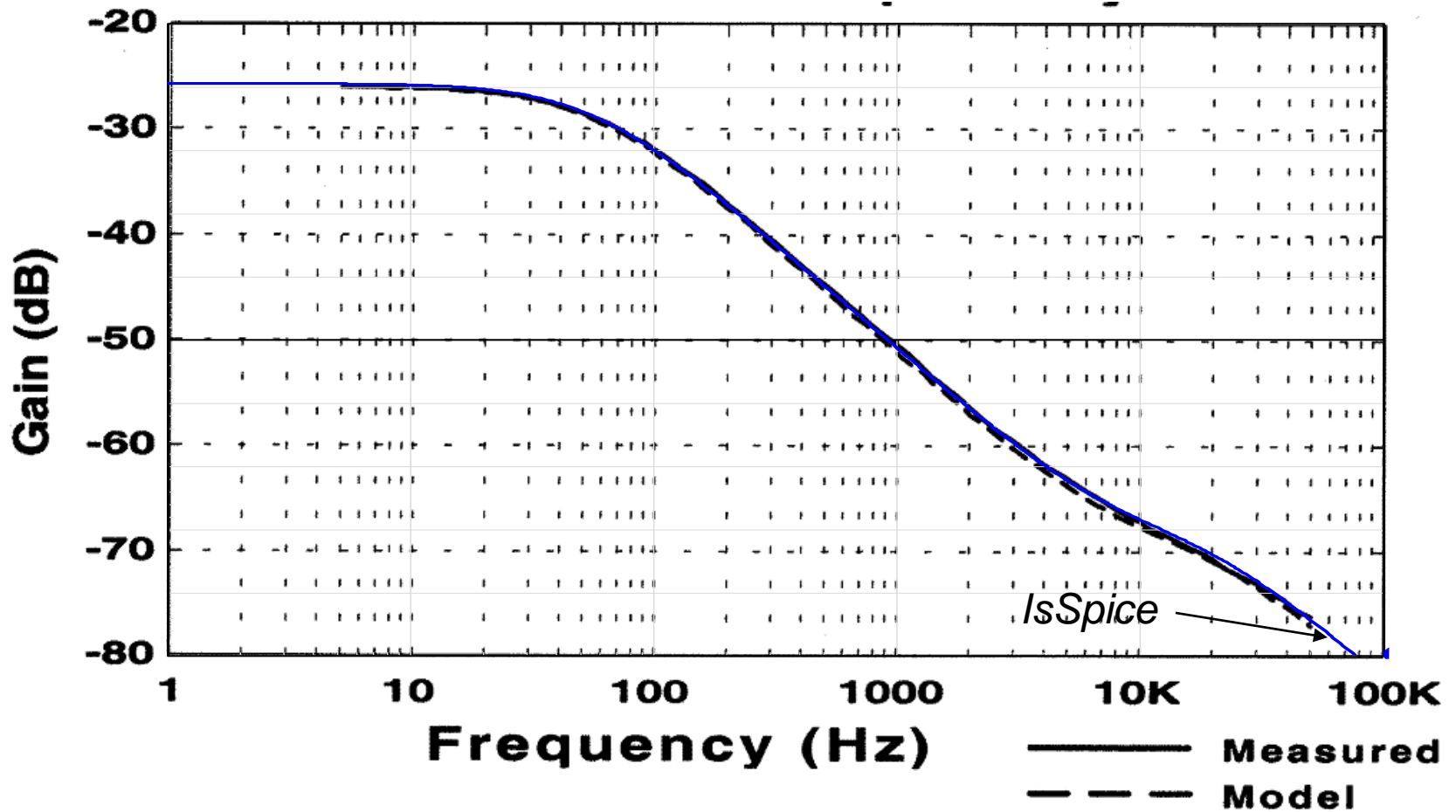


# Ac simulations versus prototype values



Control to output

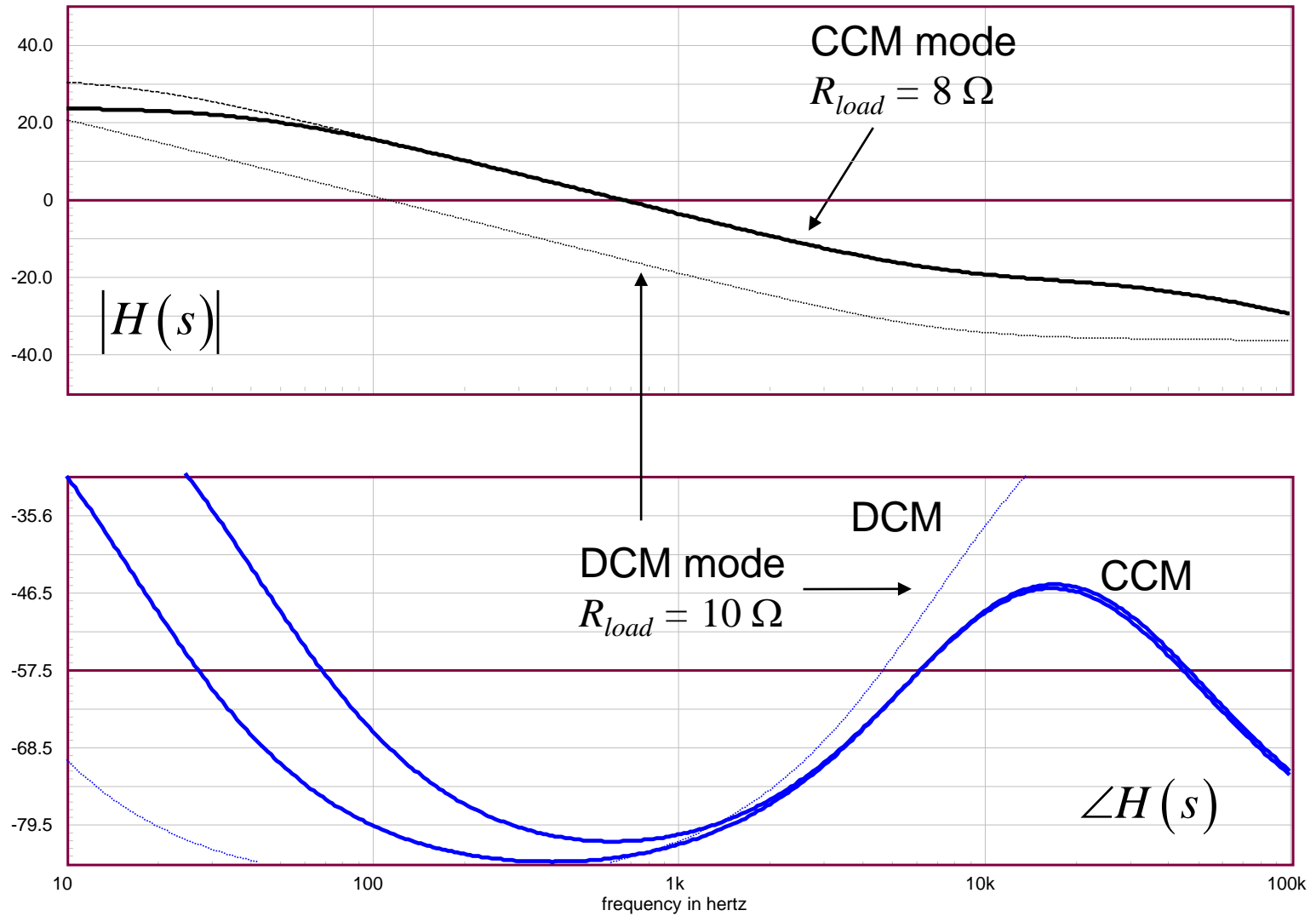
# Ac simulations versus prototype values



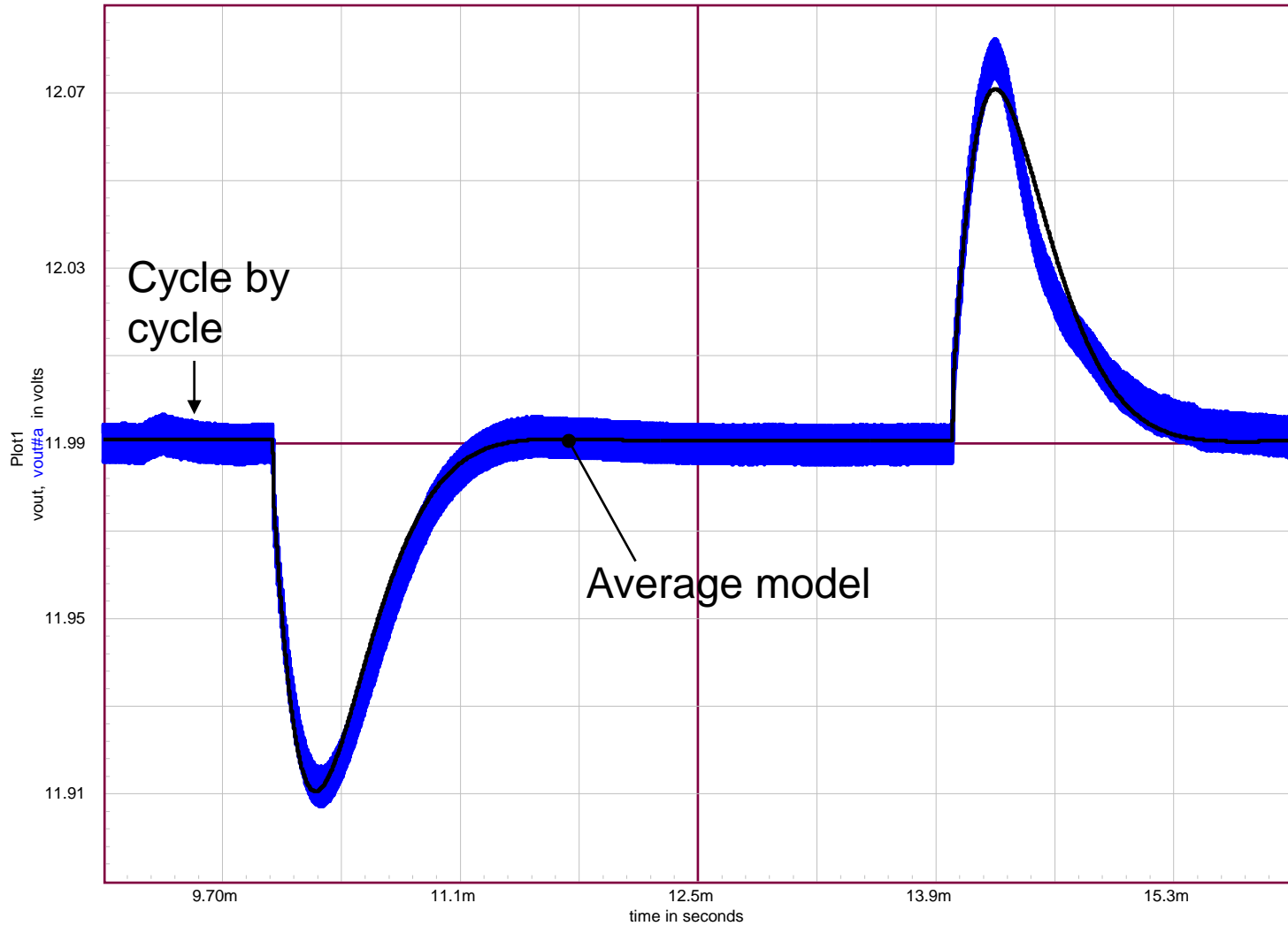
Audio susceptibility

# Mode transition point

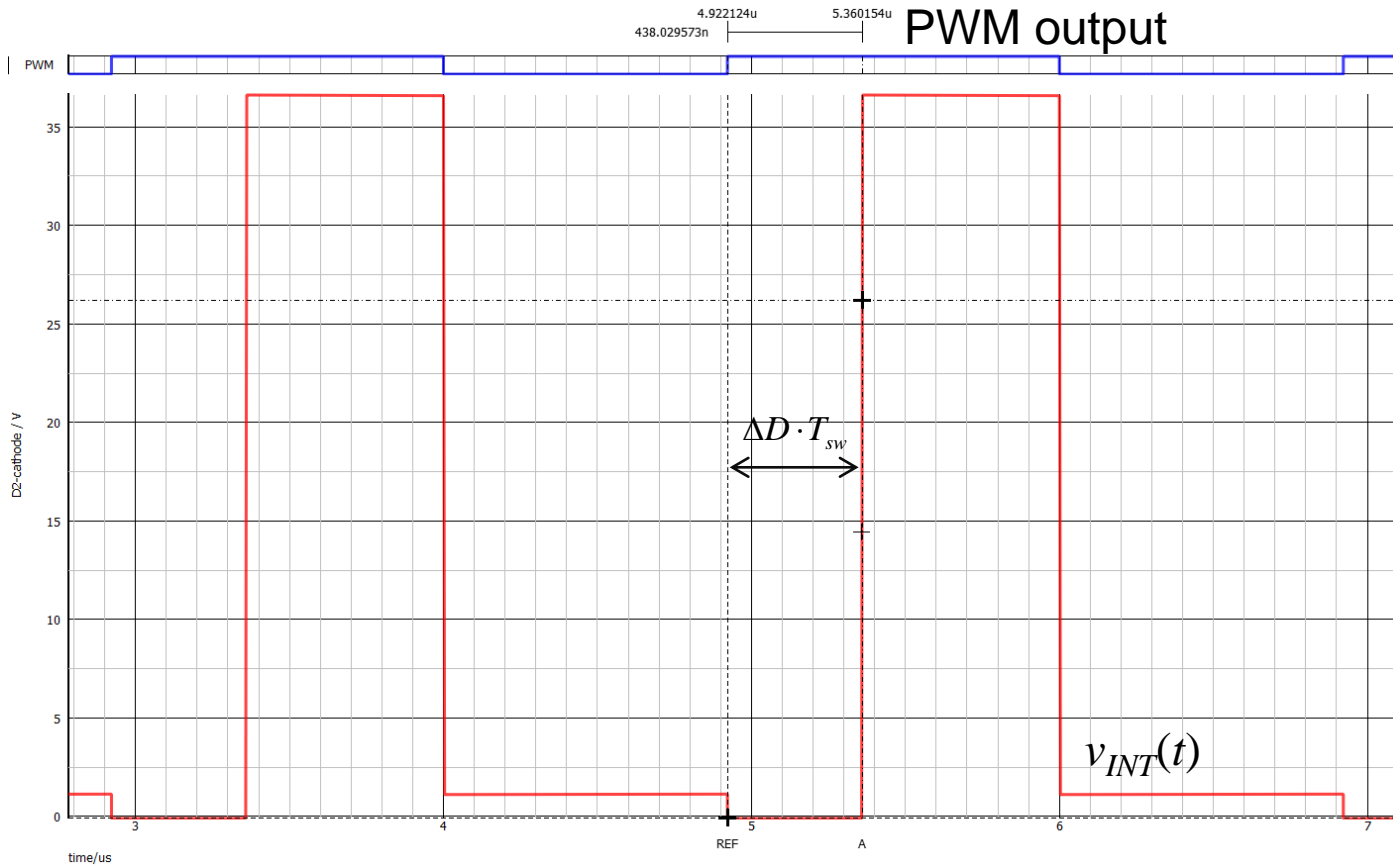
□ The load resistor is increased until the ac response changes



# Transient simulations versus cycle-by-cycle results

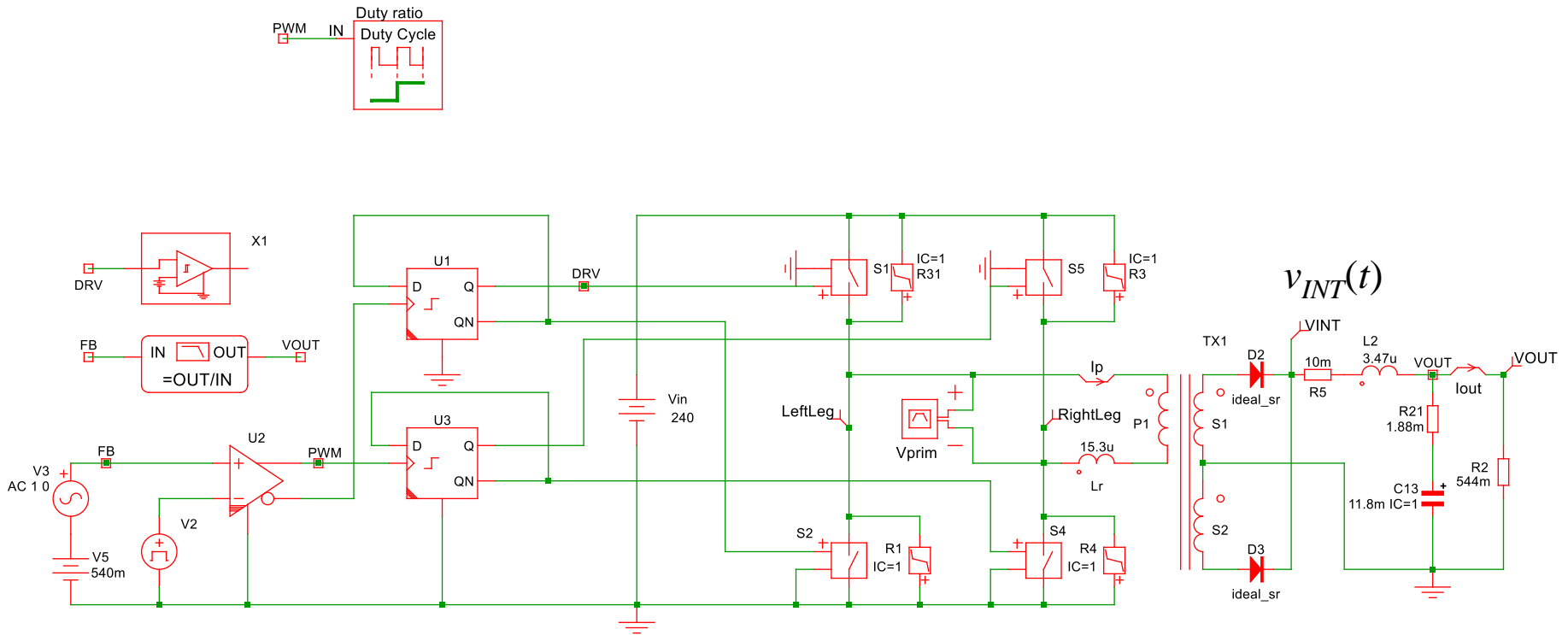


Carefully observing the switching waveforms, the voltage at the output inductor left-side connection is actually delayed by the  $\Delta DT_{sw}$  duration. This should introduce a pure delay in the transfer function which must be accounted for in the final control-to-output transfer function:



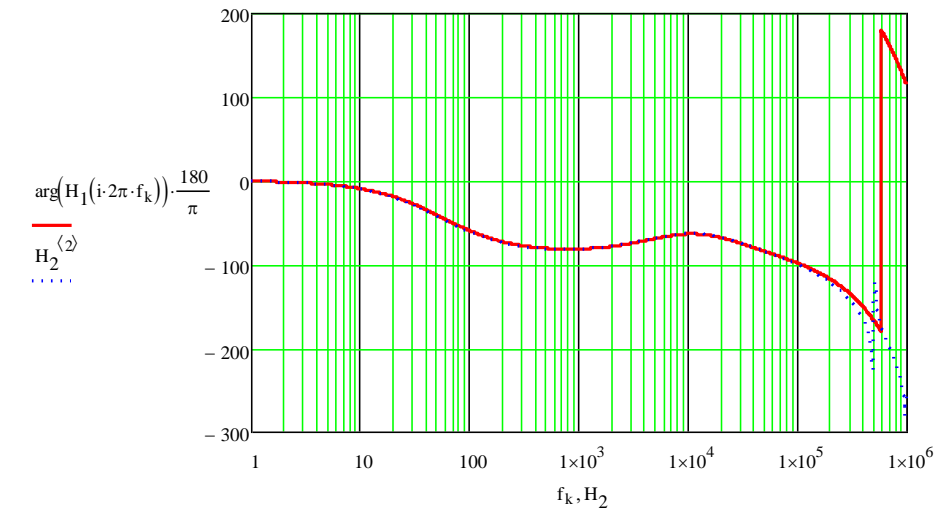
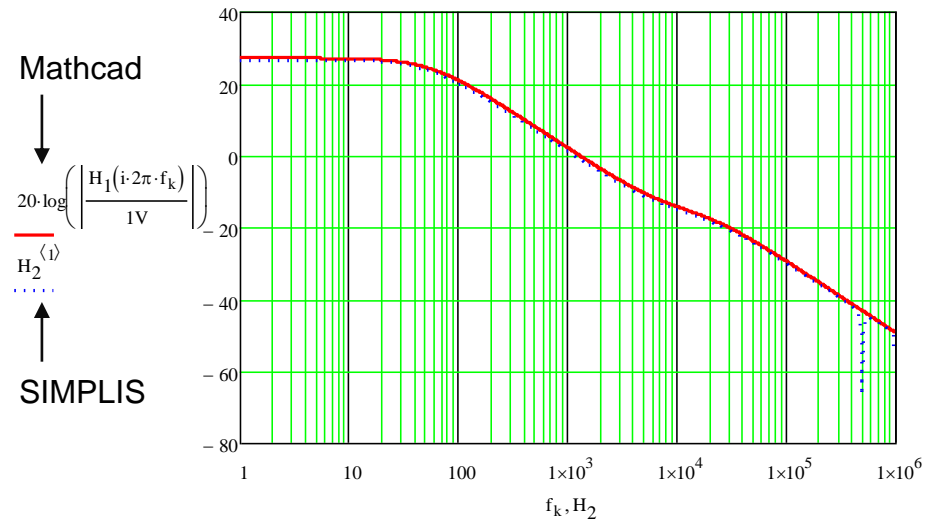
This delay is modeled by the following term:  $e^{-s \cdot \Delta D \cdot T_{sw}}$

Searching the web, I found that this [paper](#) from 2021 was proposing this option



This is a fixed-frequency voltage-mode-controlled isolated phase-shifted converter delivering 12 V 22 A from a 240-V source. This is an open-loop configuration.

- Christophe Basso - Transfer Functions of Switching Converters -

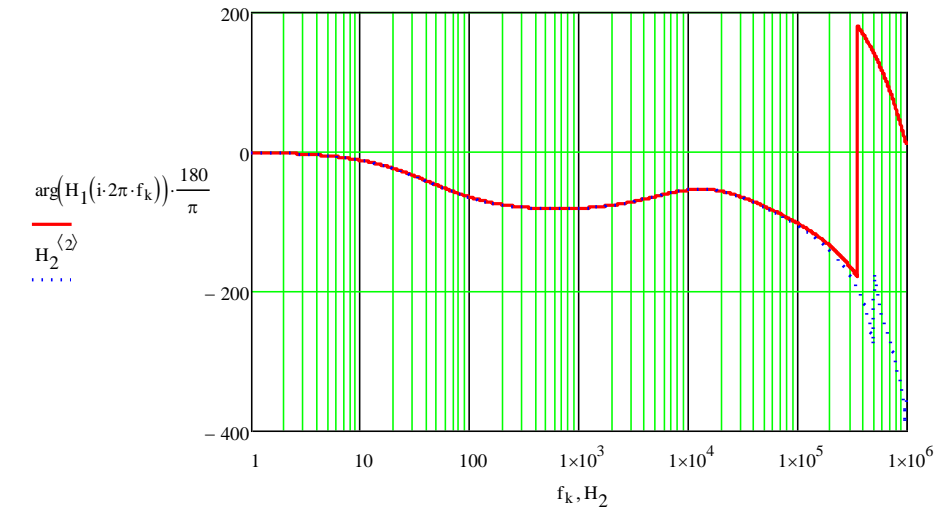
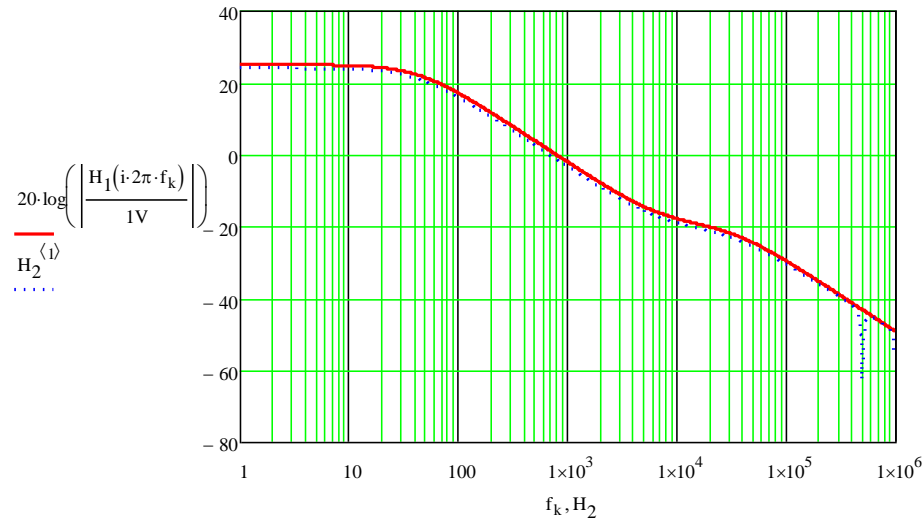


$$H_1(s) := N_1 \cdot V_{in} \cdot \frac{1 + s \cdot r_C \cdot C_1}{1 + \frac{R_d}{R_L} + s \cdot \left( \frac{L_1}{R_L} + R_d \cdot C_1 \right) + s^2 \cdot L_1 \cdot C_1} \cdot e^{-s \cdot \Delta t}$$

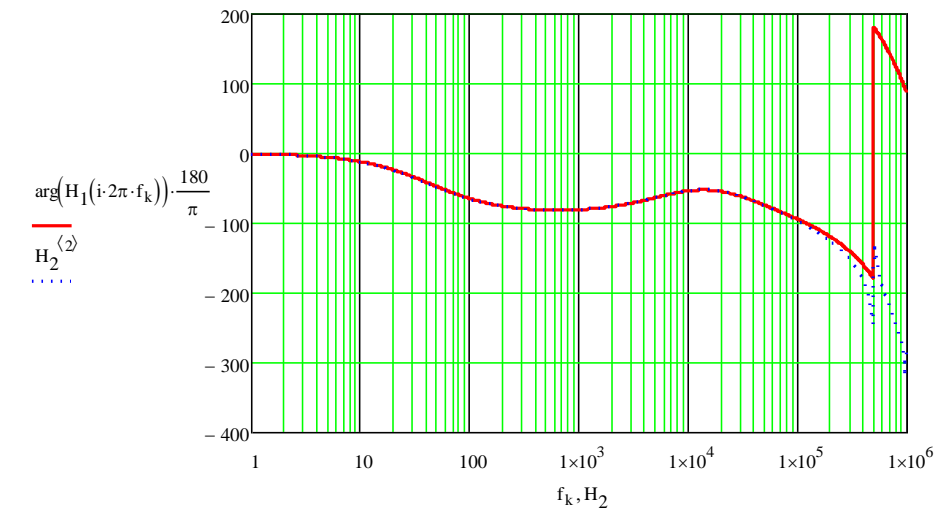
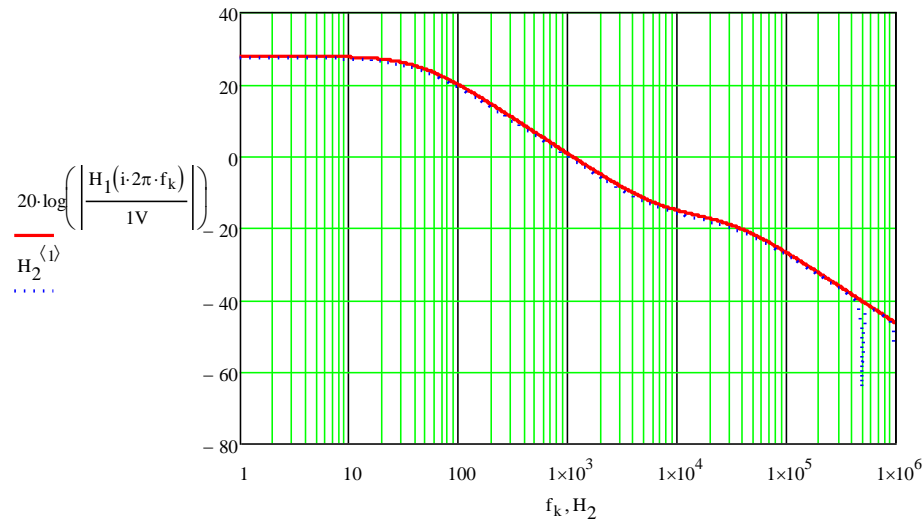
$$\Delta D := \frac{N_1 \cdot \left[ 2 \cdot I_{out} - \frac{1}{F_{sw}} \cdot \frac{V_{out}}{L_1} \cdot (1 - d_1) \right]}{\frac{V_{in}}{L_{lk}} \cdot \frac{1}{F_{sw}}} = 0.21744$$

$$\Delta t := \Delta D \cdot T_{sw} = 434.87248 \text{ ns}$$

The leakage inductance is increased to 25  $\mu\text{H}$ , duty ratio  $d_1 = 69.9\%$

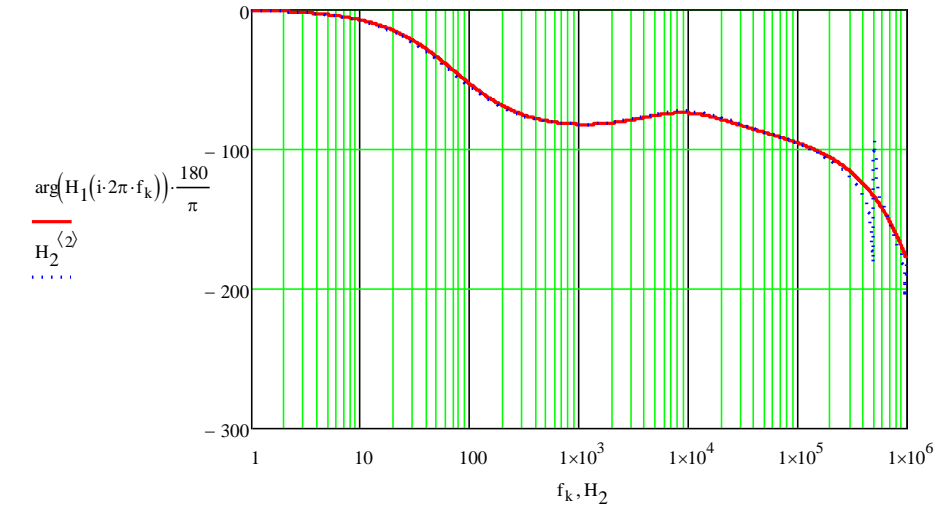
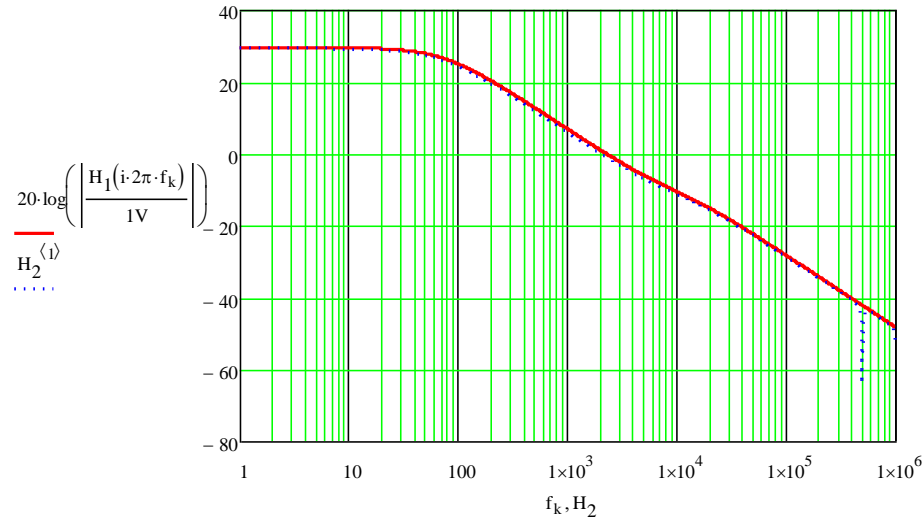


The leakage inductance is 25  $\mu\text{H}$ ,  $V_{in} = 330 \text{ V}$ , duty ratio  $d_1 = 49.9\%$

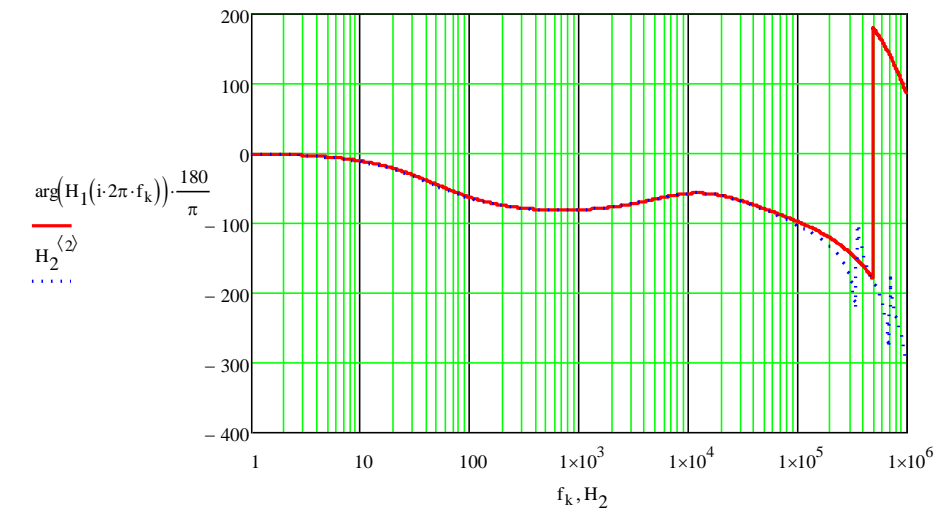
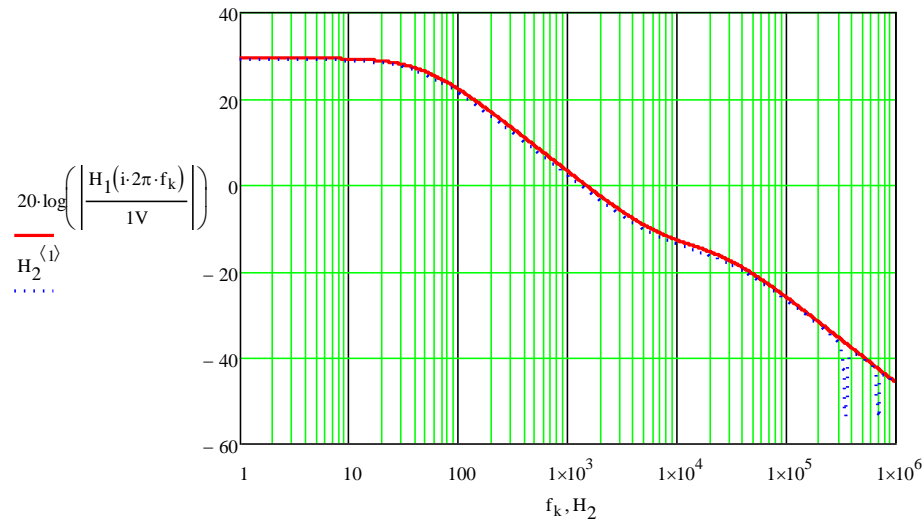




The leakage inductance is  $10\ \mu\text{H}$ ,  $V_{in} = 270\ \text{V}$ , duty ratio  $d_1 = 40.4\%$



The leakage inductance is  $39\ \mu\text{H}$ ,  $F_{sw} = 350\ \text{kHz}$ ,  $V_{in} = 356\ \text{V}$ , duty ratio  $d_1 = 40.4\%$



## Conclusion

- ❑ A simple and efficient average model has been derived
- ❑ It automatically toggles between CCM and DCM
- ❑ It predicts the mode transition point
- ❑ It matches, in CCM, ac prototype measurements
- ❑ It can easily be ported to different simulator languages
- ❑ Simulation results are in agreement with the transfer function